

# Stochastic Models in Techno-Economic Analysis of Broadband Access Networks

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**Abstract**—Development of networks, specially access networks, is very important and urgent task nowadays. However, it turns out that this segment of telecommunication networks is the most expensive and complicated part of this undertaking. Therefore, the thorough analyses are carried out to determine the best solution under specific circumstances before any decisions are made. This paper presents techno-economic model, which was implemented and used to carry out analyses for one of the biggest city in Poland. To take uncertainty into consideration the stochastic approach was applied providing more robust solution, therefore, improving the safety of investment. Analyses concern FTTH (fibre to the home) technology, type of generic FTTx network architecture. It uses optical fibre in local telecommunication loop, what is becoming more and more popular. Presented results show the usefulness of techno-economic surveys in planning access networks development. The appropriate choice of network parameters, such as the aggregation ratio, is essential and could significantly influence the investment profitability.

**Keywords**—broadband, decision support, FTTH, network planning, stochastic programming.

## 1. Introduction

Demand for network bandwidth has been rapidly growing for several years and telecommunication networks become basic information exchange channel for information society and knowledge-based economy. If majority of backbone network has been upgraded already, then access networks are usually based on old infrastructure and are a bottleneck in subscriber connections. Therefore, development of this segment became crucial in improving broadband accessibility. On the other hand, however, it is the most expensive network layer, hence detailed analyses are carried out before any decisions are made. It allows decision-makers to find out possible future results of network development venture. To perform such surveys in reasonable time and comparable way it is necessary to use dedicated tools. Projects were conducted to develop a methodology for such analyses. One of them was Tool for Introduction Scenario and Techno-Economic Evaluation of Access Network (TITAN) executed within the Research in Advance Communications in Europe Programme (RACE). It addressed different aspects of the access networks' development: geometrical models for the distribution, evolution of component price in time or operational cost approach [1]. The methodology was implemented basing on Excel spreadsheet software. Excel was also used in implementation of

worldwide interoperability for microwave access (WiMAX) model created in National Institute of Telecommunications (Warsaw). This model consists of 3 parts: demand, technological and economic. It is thoroughly described in [2]. There were also different cost analyses, concerning fiber networks either [3], [4].

Observing situation on the market it seems there is still a need to develop such applications for open access, for example, in case of public tender to verify and compare received proposals. Private companies do not usually make their tools available or it is very expensive. They would rather offer execution of the analysis and provide results. There is a risk of subjectivity. For example, some equipment can be favoured in respect of marketing connections.

Surveys show the nature of network planning process is strongly affected by uncertainty. Presented idea tries taking this aspects into consideration within analysis. Inclusion of random elements in techno-economic model ensures reflecting real circumstances in a better way and leads to more robust solution. Analysis of different scenarios is also helpful in making the decision maker aware of potential threats. Furthermore, mentioned research are mainly based on simulation. Here an optimization approach is partly suggested to obtain the best possible solution.

As network technology fibre to the home (FTTH) was assumed, which meets all demands of the newest services (video on demand, video conference, online games, etc.). Although it seems the most expensive technology it could turn out the most profitable and safest solution in time perspective. It results from laborious and costly installation of passive telecommunication infrastructure and its long depreciation period. For this reason hybrid fiber/copper network could become investment trap because of insufficient bandwidth in few years, what will forced operators to replace copper cables with fibres [5].

The characteristic of network development process and general conception of uncertainty modeling are described in Section 2. Section 3 is devoted to FTTH technology. Structure of the model and its particular modules are presented in Section 4, and Section 5 discusses experiments that were carried out and their results. Summary and potential future research are outlined in Section 6.

## 2. Dynamics and Uncertainty

The development of access network is usually a large-scale process and lasts several of years. In view of this speci-

ficity presented model takes into consideration long-lasting investment character, where the time period is discrete. For that reason some of future data have to be assumed, what of course may bring inaccuracy. It is even more obvious when one considers what kind of data is used in model. For example, demand which determine how many users will be eager to be connected to the network. It is very difficult to precisely forecast such data with regard to the subjective individual potential subscribers' decisions. This uncertainty should be considered to receive more appropriate results.

To address the above-mentioned issues multistage stochastic programming was used. It is a generalized form of more popular two stage approach, where in first stage decision is made before, and in second stage after an outcome of the random process. The second stage decision can be treated as a response to the realization of the stochastic process aimed at minimizing possible losses, caused by first stage decision. Multistage problem is similar, but the outcomes are revealed and decisions are made sequentially (Fig. 1). The general explanation of stochastic programming is in [6]. More detailed and extensive description can be found in [7], [8].

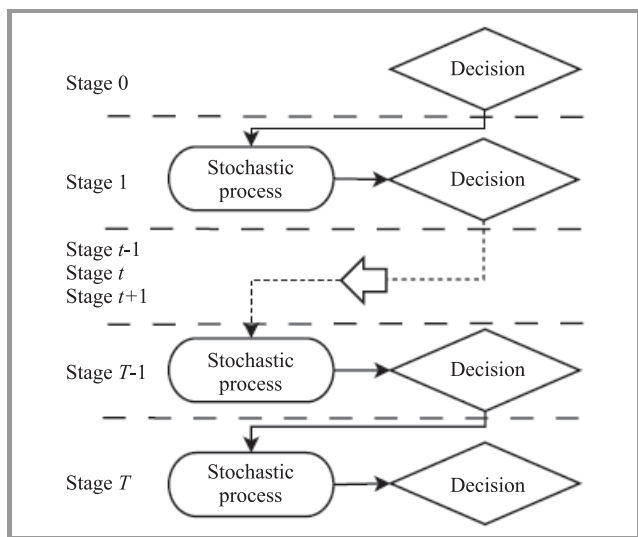


Fig. 1. Decision process in multistage stochastic programming.

As random variable, in the presented model, the take up rate was assumed (which, as mentioned before, seems the most unpredictable). The stochastic process was expressed as a finite set of different scenarios. To formulate the problem as typical mathematical programming a scenario tree was applied. It is a standard tree with a single root, where each node has unique predecessor (except the root) and possibly several successors (leaves have not successors). The nodes at the same level correspond to various scenarios in one period of time. Assuming that the root corresponds to 0 period and venture lasts  $T$  periods, there are  $T + 1$  levels in the scenario tree.

**Construction of scenario tree.** In the presented model transition-based approach is used to build the scenario tree.

This method was described in detail in [9], so here main concept will be explained and a way to use it in assessing the demand.

This approach is based on transitions (branches) between nodes from different levels. A number of branches and their probability are specified to determine the possible realizations of the stochastic process. Starting from the root node this information allows to define the nodes at further levels. To simplify, a symmetrical tree was assumed, what results in the same number and probability distribution of transitions for the nodes at the same level.

To formally define the transitions between  $t$  and  $t + 1$  stages the following functions are defined:

- $f(t)$ : the number of transitions from each node at stage  $t$ ;
- $p(t, i)$ : the probability of transition  $i \in \{1, \dots, f(t)\}$  from each node at stage  $t$ ;
- $d(t, i)$ : the change of take up rate corresponding to transition  $i \in \{1, \dots, f(t)\}$  at stage  $t$ .

Giving the above values explicit for  $0 \leq t < T$  and  $i \in \{1, \dots, f(t)\}$  is sufficient to build the scenario tree.

Each node can be unambiguously determine by a pair  $(t, n)$ , where  $t$  is a number of the stage and  $n$  is a number of the node in this particular stage. Moreover, because any node has a unique predecessor it is possible to determine a whole path from the root node to the specific  $(t, n)$  node. To navigate through the tree some auxiliary functions have to be defined, like a number of the nodes in the given period, predecessor function, which for  $(t, n)$  node gives the number of the predecessor node in  $t - 1$  period or a number of the last transition from the predecessor to the actual node. To give some idea about the nature of these functions, one of them is explained below. Let us consider the first mentioned function, the number of the nodes in the given period. According to prior assumption that the scenario tree is symmetrical and its consequences, the number of the nodes in period  $t$  equals a product of  $f(t - 1)$  and the number of the nodes in period  $t - 1$ . Bearing in mind that there is single root node the function can be defined as

$$N(t) = \begin{cases} 1 & \text{for } t = 0 \\ N(t - 1)f(t - 1) & \text{for } 0 < t \leq T. \end{cases} \quad (1)$$

Finally, there are two the most important values corresponding to each node: take up rate  $D(t, n)$  and unconditional probability  $P(t, n)$ .  $D(t, n)$  is a possible value of total take up rate, which can occur with  $P(t, n)$  probability in period  $t$ .  $D(t, n)$  is a simple sum of the changes of take up rate  $d(t, i)$ , which belong to the path from the root to node  $(t, n)$  and initial take up rate in period 0. The probability  $P(t, n)$  can be computed by multiplication the probability of predecessor and the probability of transition, which leads from predecessor to current node  $(t, n)$ . These two parameters determine all possible scenarios in a given time horizon.

Of course in reality, construction of accurate scenario tree, or more general demand model, which takes into consideration all possible scenarios, is hard problem itself. Therefore, author is aware of some simplifications in the above scenario representation. However, presented approach allows to model in a simplified way different demand forecast evolutions, both total in whole time horizon and dynamics in particular stages.

### 3. The Outline of FTTH Technology

Fibre technology is getting more and more popular in access networks. There are various types of FTTx networks depending on fibre saturation of connection between central office and subscriber [10]. FTTH is characterized by bringing the fibre medium directly to the end-user and for this reason is considered to be the most future-proof access technology and hence also as the target for evolutionary path of access networks. This expectation is supported by unlimited bandwidth of passive infrastructure, because bandwidth of such network is limited only by active equipment like transmitters and receivers [11].

**Types of architectures.** Two main different FTTH architectures can be distinguished: point-to-point and star. In point-to-point architecture there is a dedicated fibre to each subscriber, which connects him directly to the central office. It is the simplest FTTH network to design and maintain. Star architecture, on the other hand, is characterized by sharing one fibre by many customers through a remote node, which is located between subscriber's household and the central office. This node aggregates/splits network traffic from/to different customers. Star architecture can be active, when the remote node is powered, or passive, in other case. Further, the passive star can be divided into single wavelength system (all subscribers use a common wavelength) and wavelength division multiplexed (WDM) system (each subscriber uses a different wavelength).

Main components of FTTH connection are: optical line termination (OLT), optical network unit (ONU) and obviously optical fibres. OLT is a device with optical transceiver and it is usually located in the central office. In point-to-point architecture one OLT port is dedicated to single subscriber, when in star case it is used by group of subscribers. ONU, called also optical network termination (ONT), is situated on the customer premises. This device converts optical signal from the central office to subscriber into its electronic form and electronic signal into optical in the opposite direction. Optical path is composed of fibres and is usually divided into 3 sections:

- feeder cable: first section of connection from the office center, ends in the remote node in the star network;
- distribution cable: links the remote node with network access point;
- drop cable: direct connection between the subscriber and the access point.

In point-to-point network these are all main components and division of the path is obviously quite artificial, because of dedicated fibre to each customer.

The star architecture apart from above mentioned components needs also a few more devices. Mainly it concerns the remote node equipment and specially switches/splitters. In the active star case in the aggregation node there are switches and temperature stabilization system, therefore, the node have to be powered. In the passive star in the remote node splitters are used to distribute optical signal among subscribers.

Presented model assumes the passive star architecture. It is the most popular FTTH architecture today what may result from no need to power the remote nodes and therefore, economic character of such network. Furthermore, the star network has more complex structure in view of feeder cable sharing and for that reason is more complicated to model in comparison to dedicated architecture.

## 4. Model Specification

### 4.1. General Structure

Analysis of research, which were carried out so far, led to some assumptions, which had a significant influence on presented model structure. Apart from dealing with uncertainty there were also two others important premises: optimization character and linearity. The former assures the best solution in specific circumstances, in contrast to simulation, which only gives answers to fixed inputs, causing searching process of satisfactory solution very arduous. What even more important it is not known if calculated solution can be improved. The latter was assumed in order to simplify computation, because stochastic problems are usually quite big, and to use freely available software to solve linear and integer programming problems.

Based on the above assumptions, to reflect characteristic of the passive optical network (PON), a compromise was

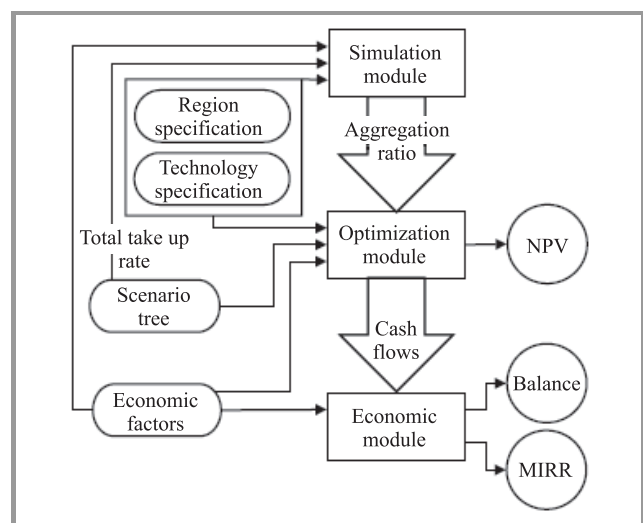


Fig. 2. General structure of model.

made, which resulted in module-based architecture of the model (Fig. 2). There are 3 basic modules:

- nonlinear simulation module: calculates the optimal aggregation rate in the remote node;
- optimization module: computes how many and which customers should be connected to maximize objective function (net present value – NPV, was chosen), using optimal aggregation rate;
- economic module: the simplest part, which calculates additional economic parameters helpful in assessing investments, using cash flows from optimization module.

#### 4.2. Network Dimensioning

Before particular modules are described some information will be given about geometric model used for network dimensioning. One should realize that cost of passive infrastructure, specially for cable networks, constitutes a quite big part of total costs, specially when trenches have to be dug. Thus it is important to determine basic measurements of access network.

Presented model has a hierarchical structure with four levels (to simplify square areas on each level were assumed) and is an extended approach, which has been proposed in [3]:

- access region,
- access area,
- aggregation area,
- connection zone.

The access region is a whole geographic region, where access network is built. Roughly in the middle there is the central office connected to the backbone network. The access region is composed of the access areas, which in reality correspond to districts, estates or small rural regions and are characterized by a given area, household density, distance to the central office and numbers of different kinds of potential subscribers. Uniform distribution of subscribers density within individual kind of the customers in the access areas was assumed.

Each access area is divided into the same size aggregation areas exact to multiple size of this aggregation area. In the middle there is the remote node which aggregates network traffic from subscribers and links them to the central office. Next the aggregation area is divided into connection zones, where single building with potential subscribers was assumed. In the building there might be different number of the subscribers depending on their kind and type of access area (precisely on its population density).

According to above it is possible to determine basic measurements of the network. Trenches schema is shown in Fig. 3(a). When  $u$  stands for length of connection zone side,  $a$  for length of aggregation area side (in  $u$  unit)

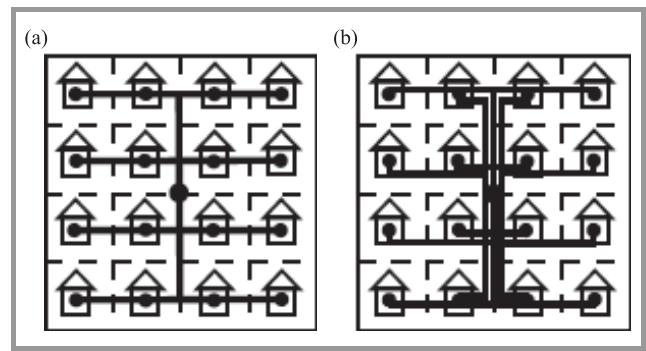


Fig. 3. Schema of aggregation area: (a) trenches; (b) fibres.

and  $n$  is a number of the buildings in this aggregation area, length of the trenches  $L_D$  can be calculated as

$$L_D = (n - 1)u, \quad (2)$$

where  $n = a^2$ . Next, for specific network topology, i.e., star topology (Fig. 3(b)), length of the fibre used to connect customers to the remote node can be calculated. Assuming initially single fibre to each building and even value of  $a$  it could be done as

$$L_f = \left(n^{\frac{3}{2}}/2\right)u. \quad (3)$$

Keeping in mind that in one building there can be several customers, above simple dependency should be modified to take into account bringing several fibres to single building. Denoting set of customers types as  $K$ , it is possible to express total number of customers in the given access area as  $C = \sum_{k \in K} c_k$ . Assuming  $g_k$  as an average number of  $k$ -type customers per building, total number of buildings can be calculated as  $B = \sum_{k \in K} b_k$ , where  $b_k$  represents a number of buildings occupied by  $k$ -type customers  $b_k = c_k/g_k$ . Further, determining  $c_a$  as a number of customers in the aggregation area the number of buildings in the aggregation area  $b_a$  can be calculated as  $b_a = \frac{c_a}{C}B$ . These lead to complete formula determining length of the fibre in the aggregation area:

$$L_f = \frac{(b_a)^{\frac{3}{2}}}{2}u \left( \sum_{k \in K} \frac{b_k}{B} g_k \right), \quad (4)$$

what can be simplified to

$$L_f = \frac{(b_a)^{\frac{3}{2}}}{2}u \left( \sum_{k \in K} \frac{c_k}{B} \right), \quad (5)$$

where  $u = \sqrt{\frac{A}{B}}$  and  $A$  is an area of the access area.

#### 4.3. Simulation Module

The aim of the simulation is to calculate the optimal aggregation rate in the remote node. It is aggregation rate which minimize expected infrastructure cost for the given scenario tree in the access area. In other words it has to be

decided how many subscribers should be connected to one aggregation node, therefore, how many splitters should be installed. Because this decision has to be made ultimately at the beginning of investment but it should minimize final network, the total take up rate in different scenarios at the last stage are considered only.

As it was considered in [4], there are two opposite aims which determine the optimal aggregation ratio. It results from the cost of optical fibres, splitters and OLT ports. On the one hand the less splitters in the remote node, the smaller aggregation area, therefore shorter distance from the node to the subscriber and lower cost of fibres. On the other hand, however, the less splitters in the remote node, the greater number of nodes and the higher probability of using greater total number of splitters in the access area. Obviously it is true only for take up rate less than 100%. To better understand the second dependency one can imagine small access area which is divided into two aggregation areas, each with the remote node with single splitter. In such situation each splitter has to be installed even if there are only two subscribers, each in different aggregation area. If these two remote nodes are replaced by one node with two splitters, the second splitter will be used only if there are at least  $r+1$  subscribers, where  $r$  is the split ratio. The number of splitters determines also the number of OLT port in the central office, thus higher aggregation ratio decreases cost of splitters and OLT ports.

Assuming  $m$  as the aggregation ratio (it means that there would be installed  $m$  splitters in the remote node if take up rate was 100%) and  $r$  as the split ratio, there will be  $mr$  customers in the aggregation area. Assuming further binomial distribution of customers take up rate, what means each customer takes a decision independently, keeping in mind uniform distribution of customers density, and using dependency (Eq. 5), length of the fibre can be calculated as

$$L_{f,m,n} = \frac{1}{2} \left( \frac{mr}{C} \right)^{\frac{3}{2}} \sqrt{A} \left( \sum_{k \in K} c_k h_{k,n} \right), \quad (6)$$

where  $h_{k,n}$  is a fraction of  $k$ -type customers, which want to be connected to the network, when  $n$  scenario occurs.

It is left to calculate the real number of splitters, which are used in the remote node, at  $h_{k,n}$  take up rate for  $k \in K$  and  $n$  scenario. Because there are  $mr$  customers in the aggregation area and fraction of  $k$ -type customers equals  $c_k/C$ , the number of subscribers is

$$N_{ConSub_{m,n}} = mr \left( \sum_{k \in K} \frac{c_k}{C} h_{k,n} \right). \quad (7)$$

Hence keeping in mind  $r$  as the split ratio, the number of splitters in the remote node can be expressed as

$$N_{spl_{m,n}} = \left\lceil m \left( \sum_{k \in K} \frac{c_k}{C} h_{k,n} \right) \right\rceil, \quad (8)$$

where  $\lceil x \rceil$  is the smallest integer not less than  $x$ . To be precisely it should be add there will be  $\lceil \frac{C}{mr} \rceil$  aggregation areas, when the aggregation ratio is  $m$ .

To find the optimal aggregation ratio the average infrastructure cost per connected subscriber is calculated. Assuming that  $C_{OLT}$ ,  $C_{spl}$ ,  $C_f$  are, respectively, costs of OLT port, splitter and fiber (length unit, e.g., 1 km) this can be done as

$$C_{m,n} = \frac{(C_{OLT} + C_{spl}) N_{spl_{m,n}} + C_f L_{f,m,n}}{N_{ConSub_{m,n}}}. \quad (9)$$

Calculating this cost for each scenario  $n$  in the last period, it is possible to determine its expected value for the given scenario tree and aggregation ratio  $m$ :

$$E(C_m) = \sum_n C_{m,n} P_n, \quad (10)$$

where  $P_n$  is probability of scenario  $n$ .

Assuming  $M_{\max}$  as maximal number of splitters that can be installed in the remote node, module computes expected cost  $E(C_m)$  for aggregation ratio  $1 \leq m \leq M_{\max}$  and returns one which minimizes it.

#### 4.4. Optimization Module

The most important and complicated part of the presented model is optimization module, which merges technological and economic aspects. All details of this module was described thoroughly in [12], thus here some basic assumptions and general structure will be only explained.

Module on the basis of given demand calculates the necessary amount of equipment, its cost and possible revenue. Users demand is estimated using the scenario tree, which was described in Section 2. Further, the geometric model and the optimal aggregation rate value from simulation module are used. As decision variables the number of connected and non-connected customers of each type were assumed. The objective function is net present value at the end of assumed time horizon

$$NPV = \sum_t \frac{CF(t)}{(1+r)^t}, \quad (11)$$

where  $r$  is the depreciation value and  $CF(t)$  is the cash flow in period  $t$  (all costs and revenues between  $t$  and  $t+1$ ).

For the sake of linear assumption, what was mentioned earlier in the model structure description, amount of all network components, which have nonlinear dependency on the number of subscribers, are calculated to satisfy total demand (to connect all eager customers). Model takes into account both, capital (CAPEX) and operational (OPEX) expenditure. CAPEX includes costs of:

- trenches and ducts,
- fibres,
- central office,
- remote nodes,
- customer premises equipment.

OPEX is calculated proportional to CAPEX, according to classes suggested in TITAN methodology [1]. Revenue comes from customers connection and monthly fees.

#### 4.5. Economic Module

In third module selected economic factors are computed on the basis of cash flows from optimization module. Presented model takes into consideration two additional values:

- balance,
- modified internal rate of return (MIRR).

Balance is a simple sum of revenues and costs. Due to this simplicity it is a very common economic factor to assess various ventures. It could be treated as down limitation of investment profitability – if balance is less than zero, all values, which take into consideration discount aspect (i.e., NPV), will also show unprofitability of such undertaking (typical investment is the most costly at the beginning). Moreover, the balance is objective in contrast to discounted values, which depend on depreciation rate. For these reasons it is probably the most universal measure to compare different investments, specially those with similar time horizon.

The second measure is modified internal rate of return, which as IRR is relative value of venture efficiency, unlike absolute values as balance or NPV, which reflect size differences between various investments. MIRR in contrast to IRR assumes the positive cash flows from a project are reinvested at the rate equates usually to cost of capital (depreciation rate used in NPV calculation). In case of IRR the positive cash flows are reinvested at the internal return rate. For that reason MIRR more accurately reflects the nature of access network development project. As it was described before development of network is a long-lasting undertaking, specific rather repeatable, what explains use of different reinvested rate instead of common internal return rate.

In other words MIRR is the depreciation rate for which discounted total future value of all positive cash flows equals total present value of all negative cash flows reached from investment. Formally this can be expressed as

$$\sum_{t=0}^T \frac{CFI_t}{(1+r)^t} = \frac{\sum_{t=0}^T CFO_t (1+r)^{T-t}}{(1+MIRR)^T} \quad (12)$$

and therefore

$$MIRR = \sqrt[T]{\frac{\sum_{t=0}^T CFO_t (1+r)^{T-t}}{\sum_{t=0}^T \frac{CFI_t}{(1+r)^t}}} - 1, \quad (13)$$

where  $CFI_t$  is the negative cash flow in period  $t$ ,  $CFO_t$  is the positive cash flow in period  $t$ ,  $r$  is the depreciation rate (cost of capital) and  $T$  is the time horizon of investment. As it can be seen MIRR is not only more appropriate to network development investment in due to its specificity but is also simpler to compute in comparison to IRR, for which it is necessary to use reverse-simulation method. Decision is made by analogy with IRR, therefore the investment is profitable when MIRR is greater than cost of capital and obviously the greater it is, the better.

## 5. Techno-Economic Analysis

Described model was applied to perform techno-economic analysis for the city Łódź (Poland) and surroundings. The region was chosen with regard to available data, which were obtained thanks to [2] authors' kindness. That study also concerns development of the access network but for WiMAX technology, thus it was possible to compare some of the results in order to general verification. Moreover, the region of Łódź varies widely in population density, therefore it is good for analysis. Three types of the access areas were distinguished: urban, suburban and rural.

Time horizon of the investment was assumed to be 6 years. It was subjective decision on the basis of telecommunications market profile. On the one hand it should be horizon long enough to develop the access network but on the other hand short enough to make economic and demand forecasts reliable as much as possible and to convince investors to wait for profits. All presented below experiments were carried out using the hypothetical scenario tree with 24 scenarios in the last period, where take up rate varies from 10% to 22% for individual and business customers (public customers like schools and municipality offices were connected in 100%). This simple estimation was a result of study various demand forecasts and experience from FTTH project in other countries, which were obtained mainly from the Internet.

### 5.1. Aggregation Ratio

Results from the simulation module will be discussed at first. Optimal aggregation ratio for 3 selected access areas with different population density were calculated. For each access area expected cost was computed for 3 different demand cases:

- set of possible scenarios with their probability in the last period;
- the lowest take up rate (pessimistic scenario);
- the highest take up rate (optimistic scenario).

In the second and third cases there was assumed only one possible scenario with 100% probability. In Fig. 4 results for low population density area are illustrated.

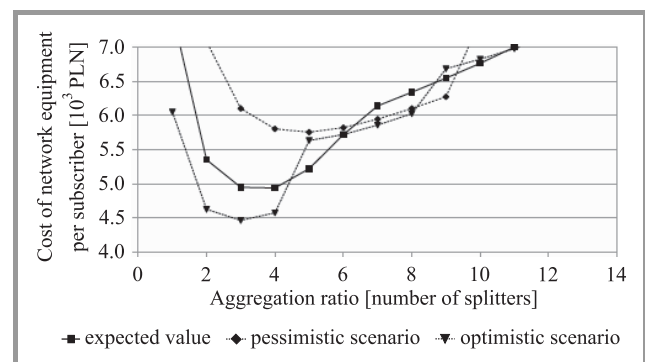


Fig. 4. Simulation module results for low population density area.

First, one can see that single splitter in the remote node is the worst solution, giving significant higher costs than higher aggregation ratio in all aggregation areas, although it could seem intuitively that installing splitters the closest as possible to the subscribers to reduce the use of fibres would be the best approach.

Further, it turns out that generally the aggregation ratio for pessimistic scenario is greater than its equivalent for optimistic one. Therefore, it seems the lower the take up rate, the higher aggregation ratio is more profitable. Because the aggregation ratio is increasing with the reduction of the subscribers number, this means the costs of fibres have less impact than the costs in the remote nodes and central office. It is different when changes of the population density are considered. In rural areas the optimal aggregation ratio is lower than in areas with greater population density. Therefore, one can come to a conclusion the main factor, which affects the costs this time, is the distance between the subscribers and the remote node.

Moreover, growth of population density, as well as take up rate, decreases the costs per subscriber, thus the higher payback from investment can be expected. To sum up it seems FTTH access network investment is positive correlated with the number of subscribers.

**5.2. Economic Analysis**

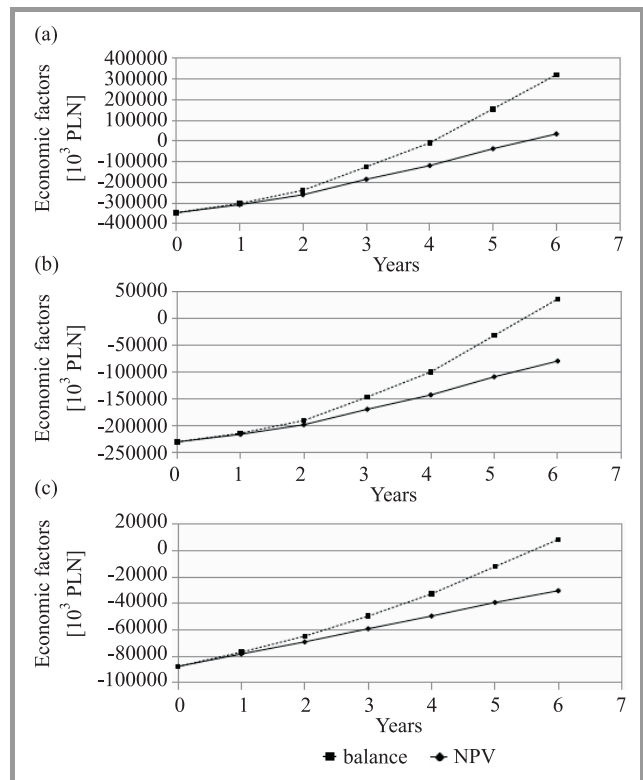
On the basis of optimal aggregation ratio from simulation module further experiments were carried out. Some economic values were calculated to assess investment profitability for various cases.

At the beginning different population density access areas were compared (Fig. 5). As it was mentioned before 3 types of areas were considered: low population density rural area, average population density suburban area and high population density urban area. As it can be seen in Table 1 only urban area is economically profitable, what is confirmed by both NPV greater than 0 and MIRR greater than assumed 15% discount rate (cost of capital). However, balance is positive for all areas what can mean that in longer time horizon each of these investments can be profitable, especially as maintain costs of fibre network are lower in comparison to copper one.

It is also interesting to compare suburban and rural areas. Although NPV is about 2.5 times lower for suburban area, MIRR are very similar and balance is much greater than for rural area. It results mainly from different scale of investment, because in suburban area initial expenditures are much higher. For this reason it seems that for longer time horizon the suburban area will be more profitable.

All these observations lead to the conclusion that investment in FTTH access network is positive correlated with the number and density of subscribers.

Similar analysis was carried out for the case when all interested customers were connected to the access network (additional constraint on the unsatisfied demand). Such requirement seems rational for example when a public organization subsidizes access network investment in non-



**Fig. 5.** Economic factors for different population density areas: (a) high density; (b) medium density; (c) low density.

profitable areas. In this particular situation such assumption aggravated economic results only slightly. It also allowed to estimate the value of subsidy required to make investment profitable in suburban and rural areas. It equals about 80 and 30 millions PLN, respectively.

**Table 1**  
Economic factors for different population density areas

Area	Balance [10 <sup>3</sup> PLN]	NPV [10 <sup>3</sup> PLN]	MIRR [%]
Rural	8481.9	-30347.8	7.14
Suburban	35205.7	-80540.6	7.08
Urban	319654.7	33828.8	16.80

Another experiment concerned duct system. Because trenching primary duct system is very expensive and the city Łódź keeps control of the sewage system, the possibility to use it was considered to reduce capital expenditure. It was assumed the sewage system exists in suburban and urban areas. As it is shown in Fig. 6 this approach improves significantly efficiency of investments and makes the access network development profitable also in medium population density area. Payback period (time for which NPV = 0) is about 4 years long in suburban area and shortens to about 3.5 years in urban area.

The City Hall of Łódź had a clear vision of network ownership and management, where the city was an owner of the network infrastructure but direct customers services

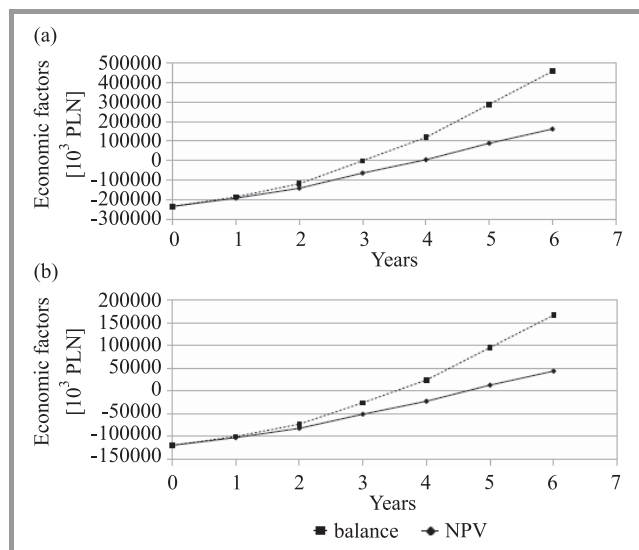


Fig. 6. Economic factors using the sewage system: (a) high density; (b) medium density.

were offered by the service operators [2]. Due to this business model of investment, there was a need to decide how to share income between the network owner and the service providers. In previous experiments it was assumed 20% of income goes to the service operators. Here (Table 2) results for two other values, 35% and 50%, are presented. In case of 35% value investment is still relatively profitable, but when profit margin reaches 50% undertaken will cross the profitability limit. Presented results concern high population density area with the primary duct system.

Table 2

Economic factors for high population density area and different service operators profit margin

Profit margin [%]	Balance [10 <sup>3</sup> PLN]	NPV [10 <sup>3</sup> PLN]	MIRR [%]
35	269781.1	62105.1	19.64
50	101031.6	-21429.0	12.64

Income division ratio plays an important role in business model, thus in reality it should be subject of thorough negotiations, taking into account various aspects, for example, competition.

## 6. Conclusions and Future Work

Presented results have shown the usefulness of techno-economic analyses in planning access networks development. The appropriate choice of network parameters, such as the aggregation ratio, is essential and could significantly influence the investment profitability. Additionally stochastic approach, which takes uncertainty into consideration, provides for a more robust solution due to changes in

the volume of customer demand. Suggested decisions are optimal in sense of expected value of NPV. Such method improves the safety of investment as opposed to the formulation in which random variables are replaced by expected values what may lead to infeasible solution. Analysis of different scenarios can also make the decision maker aware of potential threats.

One of the biggest problem in creating the model was limited availability of specific data. It caused some simplifications and common sense assumptions. As it turned out, detailed demographic data were not freely available and as it was mentioned before, they were provided by [2] authors. Information about the cost of components was hard to obtain as well, what could be a consequence of quite new FTTH technology.

The most important findings from experiments are not absolute exact value of individual factors, which in view of assumptions and difficulties could be questioned, but the general dependencies and characteristics follow from received results. From economic point of view, it seems the fibre access networks are more appropriate, when there is a significant number of subscribers. It results mainly from high initial investments, which are necessary to provide required infrastructure to cover the given access area, and this is related to costly civil work and trenching. For this reason, the FTTH network is unprofitable in low population density area and needs subsidy under such circumstances. On the other hand, there is an interesting opportunity for the areas where there is some kind of ducts system which can be easily adopted to distribute fibre cables. In this case, the development of the FTTH network is much less expensive, shortening significantly the payback period. From decision maker point of view, such property is also a guidance to develop the passive infrastructure within other investments like road construction, although one has to realize it requires a long-term planning period.

Apart from data availability difficulties it is necessary to mention other shortcomings and possible improvements to deal with. First is the computational complexity. One should realize the number of scenarios grows exponentially with the number of periods. Presented experiments were carried out using small scenario tree, what in view of small range of take up rate parameter was not the problem, but in general it poses the computational limits when the parameters vary greatly. To overcome this limitation the decomposition method should be considered. Secondly it should be also remembered about some simplifications in the scenario tree building method. It is the important issue which requires further detailed research. Some algorithms to build the scenario tree should be work out to ensure the most appropriate representation of the possible trends of events.

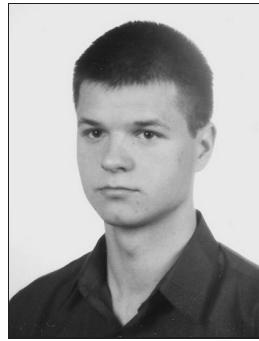
The issue of robustness in such applications may be also an interesting subject of further studies. It concerns, e.g., the choice of objectives. Some measure of risk could be applied as decision maker might be interested in the variability of returns associated with a plan. It may lead



to multicriteria optimization and more complex utility function.

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