Paper

Coordination Games with Communication Costs in Network Environments

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Abstract—In this paper, we deal with a coordination game in a network where a player can choose both an action of the game and partners for playing the game. In particular, a player interacts with players connecting through a path consisting of multiple links as well as with players directly connecting by a single link. We represent decay or friction of payoffs with distance as communication costs, and examine the effect of the communication cost on behavior of players in the game and network formation. We investigate properties of equilibrium networks by classifying the link cost and the communication cost, and show diversity of the equilibrium networks.

Keywords—communication costs, coordination games, equilibrium, networks.

1. Introduction

Studies on formation of social systems and conventions have been accumulated by mathematically modeling social interaction between individuals through sequences of playing a game. In such game theoretic approaches, interaction between individuals is represented as playing coordination games, and selection of equilibria is considered in a dynamic process with perturbations or mutations. Recently, several articles have been devoted to similar attempts in network environments by allowing players to choose partners for playing the game as well as actions in the game [1], [2], [3], and our concern is also to consider this topic.

To examine formulation of conventions, Kandori *et al.* [4] and Young [5] deal with 2×2 coordination games which are repeatedly played by randomly matched pairs of players in a population. Ellison [6], Droste *et al.* [7], and Fagiolo [8] focus on locality of interaction between players. Oechssler [9], Ely [10], and Bhaskar and Vega-Redondo [11] consider location models where the population is divided into several groups and players choose which group to join. In the recent years, similar attempts in network environments have been attracting attention [1], [2], [3]. In such models, players are allowed to choose partners for playing the game as well as actions in the game. It should be noted that the network game models are related with studies on formation of networks [12], [13], [14].

Assuming that the link formation can be realized by an unilateral decision of a player and the player pays all the link cost, Goyal and Vega-Redondo [2] define Nash equilibrium networks, and consider stability of networks in the long run. For the case where interaction is restricted to a pair of two players connected by a direct link, they show the following result. For games where two players can obtain positive payoffs even in disequilibrium, the completely connected network is in equilibrium, and when the link cost is higher than the level of the payoff, networks with two completely connected components and the empty network are also in equilibrium. For the stability in the long run, if the link cost is smaller than a certain threshold, the risk dominant equilibrium networks are stochastically stable, and if the link cost is larger than it, the payoff dominant equilibrium networks are stochastically stable. Furthermore, they consider a network game model where any two players without a direct link are allowed to interact through two or more links connecting them. In this setting, they find that the unique stochastically stable structure of networks is a minimally connected network called a center-sponsored star network, and also find that there exists a certain threshold dividing two types of coordination: the risk dominant and the payoff dominant actions.

A study of Hojman and Szeidl [3] deals with a network game model with directed links similar to that of Goyal and Vega-Redondo [2]. In their model, it is assumed that interaction between two players connected not only by a single link but also by a path of multiple links is allowed, but a player can obtain payoffs only from interaction with other players to whom there are directed paths of links. They show that the structure of Nash equilibrium networks is wheel-shaped. For the long run stability, when the link cost is small and the disequilibrium payoff is positive, the risk dominant equilibrium is a unique stochastic stable network, and otherwise the payoff dominant equilibrium is uniquely stochastically stable on the condition that the payoff dominant equilibrium creates quite high positive gain or the degree of the risk dominance is small.

Although Goyal and Vega-Redondo [2] and Hojman and Szeidl [3] deal with interaction through a path, i.e., multiple consecutive links connecting players, it is assumed that the interaction between players connected by a path does not require any communication cost or such interaction is frictionless. However, it is natural to think that interaction with distant players costs and/or takes time much more than interaction between directly connected players. In this paper, we assume that a payoff arising from interaction through a path decreases with distance. This can be represented by a discount of the payoff or a communication cost of the network. While a discounted payoff is always nonnegative, the payoff from which the communication cost is subtracted may be negative. In this paper, employing a representation of decreasing the payoff by the communication cost, we deal with a network game model with interaction between distant players. Assuming that a link between two players is formed or maintained if the payoffs of both players do not decrease and they equally pay the link cost, we examine equilibrium networks. In Section 2, we introduce a network game model with the communication costs, and the equilibrium networks are shown in Section 3. Some concluding remarks are given in Section 4.

2. A Model

We deal with a network game model where a player chooses partners for interacting through direct links or paths consisting of multiple consecutive links. The interaction through a path requires the communication cost which increases with a distance between two players.

Let $N = \{1,\dots,n\}$ be the set of players which is called a population. The interaction between two players is represented by a 2×2 coordination game shown in Table 1. Let $a_i \in \{\alpha,\beta\}$ denote an action selected by player i. Each entry of the payoff table is a 2-dimensional vector and, the first element of the vector is a payoff of the row player and the second one is that of the column player. A vector $a = \{a_1,\dots,a_n\}$ of actions selected by all the players is called a profile of actions.

Table 1
Payoff table of a coordination game

Row player	Column player α	Column player β
α	(a,a)	(f,e)
β	(e,f)	(b,b)

Because we deal with a coordination game with conflict between the risk dominant equilibrium and the payoff dominant equilibrium, it is assumed that the following conditions are satisfied for the parameters of the payoffs given in Table 1.

$$a > b, \quad a > e, \quad b > f, \quad a+f < b+e. \tag{1}$$

Thus, the outcome (α,α) is the payoff dominant equilibrium, and (β,β) is the risk dominant equilibrium.

The following notation is used. If there exists link ij between players i and j, $l_{ij} = 1$, and otherwise $l_{ij} = 0$. A set of links of player i is expressed by $l_i = (l_{i1},\dots,l_{i,i-1}, l_{i,i+1},\dots,l_{i,n}) \in \{0,1\}^{n-1}$. Because player i can decide to choose which links to maintain, the link set l_i of player i can be interpreted as a strategy for choices of links. A vector $l = (l_1,\dots,l_n)$ of link sets of all the players is a profile of links, and a set of links in the population is expressed by $g(l) = \{ij \mid l_{ij} = l_{ji} = 1\}$.

If $l_{ij} = 1$ or there exist a series of players $j_1,\dots,j_m$ such that $l_{ij_1} = \cdots = l_{j_k j_{k+1}} = \cdots = l_{j_m j} = 1$, it is said that there exists a path between players i and j. The existence of the path is denoted by $\bar{l}_{ij} = 1$, and the path is also expressed by $i \leftrightarrow j$. A set of paths of player i is expressed by $\bar{l}_i = (\bar{l}_{i1},\dots,\bar{l}_{i,i-1},\bar{l}_{i,i+1},\dots,\bar{l}_{in})$. For a path $i \leftrightarrow j$, the length of the path or the distance between players i and j is defined as the minimal number of links connecting i and j, and it is denoted by L_{ij}. A set of nodes corresponding to $g(l)$ is expressed by $N(g(l)) = \{i \mid \exists j, ij \in g(l)\}$. For a subset of links $g(l') \subset g(l)$, $g(l')$ is called a component of $g(l)$ if, for any pair of $i \in N(g(l'))$ and $j \in N(g(l'))$, there exists a path $i \leftrightarrow j$, and $ij \in g(l)$ implies $ij \in g(l')$.

To form or maintain link ij, players i and j need to pay the cost c. When there exits a path between players i and j, i.e., $\bar{l}_{ij} = 1$, they play the 2×2 coordination game paying the communication cost $d(L_{ij})$, where $d(\cdot)$ is a strictly increasing monotone function with the length of the path, and $d(1) = 0$. It is assumed that player i takes the same action a_i for all games with partners connected by direct links or paths. Let $a_{-i} = (a_1,\dots,a_{i-1},a_{i+1},\dots,a_n)$ and $l_{-i} = (l_1,\dots,l_{i-1},l_{i+1},\dots,l_n)$ be, respectively, a profile of actions and a profile of links for $N_{-i} = N \setminus \{i\}$ which is the set of players except for player i. Then, a utility of player i with a strategy (a_i,l_i) is written by

$$\pi_i((a_i,l_i),(a_{-i},l_{-i})) = \sum_{j:\bar{l}_{ij}=1} u(a_i,a_j) - \sum_{j:l_{ij}=1} c - \sum_{j:\bar{l}_{ij}=1} d(L_{ij}), \tag{2}$$

where $u(a_i,a_j)$ is player i's payoff of the game shown in Table 1 when player i chooses action $a_i \in \{\alpha,\beta\}$ and player j chooses action $a_j \in \{\alpha,\beta\}$. For concise representation, the part of costs in (2) is defined by

$$D_i = \sum_{j:l_{ij}=1} c + \sum_{j:\bar{l}_{ij}=1} d(L_{ij}). \tag{3}$$

Let $A \triangleq \{\alpha,\beta\}$ and $L \triangleq \{0,1\}^{n-1}$ denote the strategy sets of actions and links, respectively. The strategy set of a player is represented by $X \triangleq A \times L$. A strategy of player i is represented by a pair of an action and a set of links, and it is denoted by $s_i = (a_i,l_i) \in X$, $a_i \in A$, $l_i \in L$. A strategy profile $s = (s_1,s_2,\dots,s_n) \in X^n$ of all the players indicates a state of the network, and we call a strategy profile s a network or a state. We assume that each of the players knows a state s and can calculate the utilities of the other players $\pi_j(s)$, $j \neq i$ as well as the utility $\pi_i(s)$ of self. Moreover, we assume that link ij is formed only if both of the utilities of i and j do not decrease by forming link ij. Namely, when player i is selected to revise a strategy, for any player j such that $l_{ij} = l_{ji} = 0$ at the state s before revising the strategy of i and $l_{ij} = l_{ji} = 1$ at the state s' after revising it, the condition $\pi_j(s') \geq \pi_j(s)$ must be satisfied. It is said that the strategy s'_i of player i is feasible if this condition is satisfied, and a set of feasible strategies of player i is denoted by $X_i \subset X$. It follows that player i chooses a strategy s'_i among the feasible strategy set X_i, i.e., $s'_i \in X_i$. Because we assume that each player always chooses a strategy among the feasible strategy set, Nash equilibrium networks can be defined as follows.

Definition 1: A state $\hat{s} = (\hat{s}_1, \ldots, \hat{s}_n)$ is said to be an equilibrium state or an equilibrium network if, for any player $i \in N$ and any feasible strategy $s_i \in X_i$, the following condition holds:

$$\pi_i(\hat{s}_i, \hat{s}_{-i}) \geq \pi_i(s_i, \hat{s}_{-i}). \tag{4}$$

Such a strategy $\hat{s}_i = (\hat{a}_i, \hat{l}_i)$ is called an equilibrium strategy of player i, and a component in an equilibrium network is called an equilibrium component.

In this paper, focusing on actions of players and the formation of links, we examine static equilibrium networks in the network game with the communication cost. At the beginning of the examination, we give some definitions on actions of players and structures of networks. We call a player who chooses action α an α-player, and a player who chooses action β a β-player. A player holding only one link is called a leaf player, and a player holding no link is called an isolated player. If all of players who belong to a component $g(l')$ choose action α, the component $g(l')$ is called an α-component, and similarly a β-component is defined. If any pairs of players $i, j \in N(g(l'))$ are connected by a direct link, i.e., $l'_{ij} = l'_{ji} = 1$, the component $g(l')$ is said to be completely connected. If a component is divided by severing any link in the component, the component is said to be minimally connected. If after severing a ceratin link in a component, the component is not divided, there should exist a loop of links in the component. A component without any leaf player is called a leafless component. In particular, if all the players in a component have only two links and they are arranged like a circle, the component is called a ring component. These definitions are given for components, and similar definitions are also given for networks.

3. Equilibrium Networks

We deal with coordination games in network environments and examine equilibria of networks in this section. If an equilibrium network is not the empty network, there exists at least one component. Then, we first characterize equilibrium components.

Lemma 1: In any equilibrium component, all the players in the component choose the same action.

The proof of Lemma 1 is given in Appendix, and the proofs of the subsequent lemmata and theorem are also given in Appendix. While we focused on behavior of players in Lemma 1, the next lemma deals with structures of equilibrium components. Let C and $|C|$ denote a component and the number of players in the component, respectively. As shown in Lemma 1, all players choose the same action in an equilibrium component, and let the action be $x \in A = \{\alpha, \beta\}$. First, we consider the case where the link cost is smaller than the payoff obtained by coordination of choices, i.e., $c < u(x,x)$. Let L denote a distance between players i and j, and let k denote any player on path $i \leftrightarrow j$. Because a distance between players i and k decreases or does not change when link ij is formed, the sum of the communication costs over path $i \leftrightarrow j$ decreases, and the reduced cost is calculated as follows:

$$RC(L) = \sum_{k=\lceil L/2 \rceil + 1}^{L} d(k) - \sum_{k=1}^{\lfloor L/2 \rfloor} d(k), \tag{5}$$

where $\lceil x \rceil$ and $\lfloor x \rfloor$ mean the minimal integer larger than or equal to x and the maximal integer smaller than or equal to x, respectively.

Lemma 2: For a given action $x \in A = \{\alpha, \beta\}$, if $c < u(x,x)$, an equilibrium x-component C has the following structures.

(1) If $c < d(2)$, an equilibrium component is completely connected, and vice versa.

(2) If $d(2) \leq c$ and $c \leq RC(|C|-1)$, there exists an equilibrium component which is not completely connected. The maximal length of a path in the equilibrium component is the largest number L satisfying $RC(L) \leq c$ and $L < |C|$.

(3) If $d(2) \leq c$ and $RC(|C|-1) < c$, an equilibrium component is minimally connected.

From $b < a$, $c < u(\beta,\beta)$ implies $c < u(\alpha,\alpha)$. Then, the result of Lemma 2 is valid for α- and β-components if $c < b$, it is valid for α-components if $b < c < a$, and it is not valid for either of them if $a < c$.

For (3) of Lemma 2, any minimally connected component is not always an equilibrium component. As a counterexample, consider a component where 4 players are in line. From $|C| = 4$ and the condition of (3) of Lemma 2, the inequality $c > d(3) - d(2)$ holds. The utility of player i at the end of the line is $\pi_i = 3u(x,x) - c - d(2) - d(3)$, and from $c > d(3) - d(2)$, it satisfies the following inequality.

$$\begin{aligned}
\pi_i &= 3u(x,x) - c - d(2) - d(3) \\
&< 3u(x,x) - (d(3) - d(2)) - d(2) - d(3) \\
&= 3u(x,x) - 2d(3).
\end{aligned}$$

Then, the utility π_i of player i is negative when $3u(x,x) - 2d(3) < 0$. For example, when $c = 4.5, u(x,x) = 5$, $d(2) = 4, d(3) = 8$, because the condition of Lemma 2: $d(2) \leq c < u(x,x)$ and $d(3) - d(2) < c$ and the above condition: $3u(x,x) - 2d(3) < 0$ are satisfied, the utility π_i of player i is negative. Thus, the best response of player i is to sever the link, and it follows that the component is not an equilibrium component.

We also show an example of a minimally connected component which is an equilibrium component. Consider a star-shaped minimally connected component where player 1 is the center and players 2, 3, and 4 are peripheries. Because a distance between any pair of players is at most two and $d(2) \leq c$, a new link is never formed. From $c < u(x,x)$, the utility of player 1, $\pi_1 = 3(u(x,x) - c)$, is positive, and those of the other players, $\pi_2 = \pi_3 = \pi_4 = 3u(x,x) - c - 2d(2)$, are positive. When any link is severed, the utility of player 1 decreases by $u(x,x) - c$. At this moment, the counterpart is isolated and her utility becomes zero. Thus, because in the star-shaped component, a new link is not formed and any of the existing links is not severed, it is an equilibrium component.

In Lemma 2, we have considered the structure of components, and in the next lemma, we examine whether or not there exist multiple components in an equilibrium network.

Lemma 3: For a given action $x \in A = \{\alpha, \beta\}$, if $c < u(x,x)$, in an equilibrium network, there exists only one x-component, and any x-player is connected with some other player.

Next, we consider the case where the link cost is larger than the payoff obtained by coordination of choices, i.e., $c > u(x,x)$. In general, an x-component is not likely to be formed in this case, but it could be maintained because players can obtain a positive payoff arising from interaction between distant players through paths if the communication cost is relatively small.

Lemma 4: For a given action $x \in A = \{\alpha, \beta\}$, if $c > u(x,x)$, in an equilibrium network, the necessary condition for existing one or more x-components is that the condition $d(2) \leq c \leq RC(2l)$ is satisfied, and there exists an integer $l \in (1, n/2]$ satisfying the condition

$$c - u(x,x) \leq (l-1)u(x,x) - \sum_{k=1}^{l} d(k). \qquad (6)$$

Moreover, such equilibrium x-components are leafless.

From the above discussion, we have the following results.

Theorem 1: Assuming that the communication cost $d(\cdot)$ is a strictly increasing monotone function with a distance between any pair of players, we can characterize equilibrium networks as follows.

(1) In the case of $c < b$:

(a) If $c < d(2)$, an equilibrium network is the completely connected α-network or the completely connected β-network.

(b) If $d(2) \leq c$ and $c \leq RC(n-1)$, an equilibrium network is an incompletely connected α-network or an incompletely connected β-network. The maximal length of a path in the equilibrium network is the largest number L satisfying the conditions $RC(L) \leq c$ and $L < |C|$.

(c) If $d(2) \leq c$ and $c > RC(n-1)$, an equilibrium network is a minimally connected α-network or a minimally connected β-network.

(2) In the case of $b < c < a$:

(a) If $c < d(2)$, an equilibrium network is a completely connected α-network or an empty β-network.

(b) If $d(2) \leq c$ and $c \leq RC(n-1)$, an equilibrium network is an incompletely connected α-network or an empty β-network. Moreover, if there exists an integer $l \in (1, n/2]$ satisfying the condition $c - b \leq (l-1)b - \sum_{k=1}^{l} d(k)$, a network with one or more leafless β-components can be an equilibrium network, and it may include one α-component. The maximal length of a path in the equilibrium network is the largest number L satisfying the conditions $RC(L) \leq c$ and $L < |C|$.

(c) If $d(2) \leq c$ and $c > RC(n-1)$, an equilibrium network is a minimally connected α-network or an empty β-network.

(3) In the case of $a < c$:

(a) If $c < d(2)$, an equilibrium network is an empty network.

(b) If $d(2) \leq c$ and $c \leq RC(n-1)$, an equilibrium network is an empty network. Moreover, if there exists an integer $l_a \in (1, n/2]$ satisfying the condition $c - a \leq (l_a - 1)a - \sum_{k=1}^{l_a} d(k)$, a network with one or more leafless α-components can be an equilibrium network; and if there exists an integer $l_b \in (1, n/2]$ satisfying the condition $c - b \leq (l_b - 1)b - \sum_{k=1}^{l_b} d(k)$, a network with one or more leafless β-components can be an equilibrium network.

(c) If $d(2) \leq c$ and $c > RC(n-1)$, an equilibrium network is an empty network.

The structures of equilibrium networks shown in Theorem 1 are summarized in Table 2. In Theorem 1, the structures of equilibrium networks are characterized by the relation between the link cost c and the communication cost $d(2)$ of distance 2. In general, when the link cost c is smaller than the payoff of the game such as the payoff dominant equilibrium payoff a and the risk dominant equilibrium payoff b, a link is formed. Moreover, when the communication cost $d(\cdot)$ is large, compared to the link cost c, a link between any pair of players is likely to be formed, and when the communication cost $d(\cdot)$ is small, the number of links decreases because not direct links but paths allow players to interact with other players at lower costs. Moreover, comparing the link cost c and the communication cost $d(2)$ of distance 2 reveals whether or not players should form a link. By analysis taking into account the communication cost of distances larger than 2, we can examine the length of a path in a network.

In (1) of Theorem 1, because the link cost c is smaller than the payoff b of the risk dominant equilibrium, an equilibrium network is either an α-network which means the risk dominant equilibrium or a β-network which means the payoff dominant equilibrium. For the case of (a), because interaction by using direct links is profitable due to $c < d(2)$, the completely connected network is formed. In the case of (c), interaction by using direct links is costly

Table 2
Equilibrium networks

Case	Condition	(1) $c < b$	(2) $b < c < a$	(3) $a < c$
(a)	$c < d(2)$	complete α complete β	complete α empty β	empty
(b)	$d(2) \leq c$ $c \leq RC(n-1)$	incomplete α incomplete β	incomplete α empty β (leafless β)	empty (leafless α) (leafless β)
(c)	$d(2) \leq c$ $c > RC(n-1)$	minimal α minimal β	minimal α empty β	empty
(leafless α) or (leafless β) is conditional.				

compared to interaction through paths because of $d(2) \leq c$, and from $c > RC(n-1)$, any loop is not formed. Thus, an equilibrium network is minimally connected.

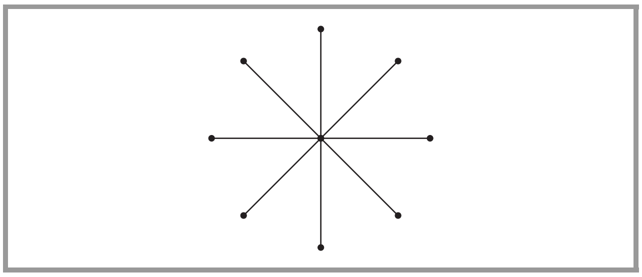

Fig. 1. A minimally connected (star) network.

As an example of a minimally connected network, a star-shaped network is given in Fig. 1. In this network, a new link between players of two units of distance is not formed because forming such links results in decrease of the utilities of the players. Moreover, any existing link is not severed because by severing a link the utilities of two players decrease by at least $b-c$. Thus, this type of networks are in equilibrium.

In the case of (b), because of $d(2) \leq c$ and $c \leq RC(n-1)$, an equilibrium network is not completely connected but it includes a loop. The density of networks, i.e., the number of links depends on the relation between the link cost c and the communication cost $d(\cdot)$, and it can be characterized by the maximal length of paths.

An example of an incompletely connected equilibrium network is given in Fig. 2. The length of a path in this network is at most 3, and if $\max\{d(2), RC(3)\} \leq c$, a new

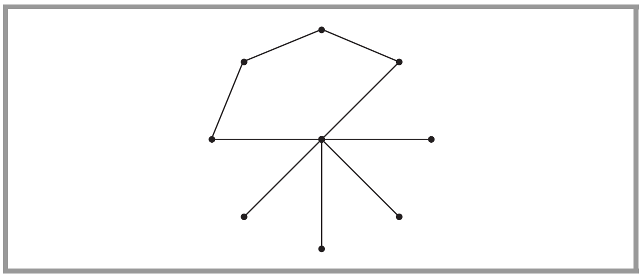

Fig. 2. An incomplete network.

link is not formed. Because severing any link in the loop makes a pair of players of 4 units of distance, the utilities of them decrease by severing the link if $c < RC(4)$. For example, let $d(k) = k$. Then, from $RC(3) = 1$ and $RC(4) = 5$, this network is an equilibrium network if $2 \leq c < 5$. On the other hand, the condition of Theorem 1, $d(2) \leq c \leq RC(8)$, can be written as $2 \leq c \leq 16$. Then, when $c = 2,3,4$, both of the equilibrium condition of this network, $\max\{2, RC(3)\} \leq c < RC(4)$, and the condition of Theorem 1, $d(2) \leq c \leq RC(8)$, are satisfied simultaneously. With the above mentioned parameters, the incompletely connected network given in Fig. 2 is an equilibrium network.

In (2) of Theorem 1, because the link cost c is larger than the payoff b of the risk dominant equilibrium and it is smaller than the payoff a of the payoff dominant equilibrium, it is supposed that an equilibrium network is an α-network but a β-network is not the case. However, it is shown that if the condition given in the theorem is satisfied, some β-network can be an equilibrium. For the case of (a), from $c < d(2)$, the completely connected network is formed when all the players choose action α, and as a special case, a state where all the players choose action β in the empty network is an equilibrium network. In the case of (c), because interaction by using direct links is costly compared to interaction through paths for the same reason as in (1), a minimally connected equilibrium α-network is formed. The empty β-network is also an equilibrium. In the case of (b), because of $d(2) \leq c$ and $c \leq RC(n-1)$, an equilibrium network is not completely connected. Moreover, as special structures of equilibrium networks, besides the empty β-network, networks with one or more leafless β-components and networks with one α-component and one or more leafless β-components can be equilibria. However, an equilibrium network with a β-component can exist only when the condition given in the theorem is satisfied. Even if the link cost c is larger than the payoff b of the game, the utilities of players may become positive because of interaction between distant players, and then leafless β-components can be included in an equilibrium network. It is noted that there does not exist a leaf player in a β-component because the utility of a player who shares a link with the leaf player increases by severing the link.

We give some examples of equilibrium networks with multiple components in Figs. 3 and 4. In a network shown in Fig. 3, assume that the payoffs of the game are set at $a = 11$, $b = 6.71$, $e = 1$, and $f = 0$, and the costs of the link and the communication are set at $c = 10$, $d(2) = 5$, $d(3) = 5.1$, and $d(4) = 20$, respectively.

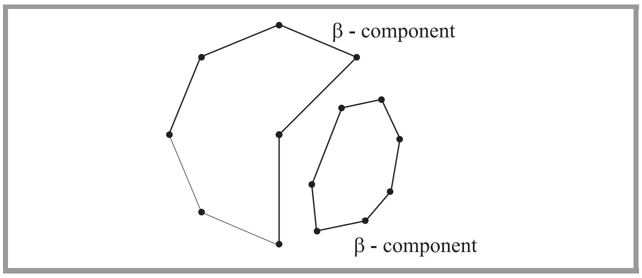

Fig. 3. A network with two β-components.

In Fig. 3, a β-network with two β-components is given. Forming a link within a component results in decrease of the utility, and therefore a new link within a component is not formed. Because by severing a link the distance between players becomes more than or equal to 4 and then the cost of a player increases, any link is not severed. Next, consider formation of a link between components. Because the utilities of players decrease by forming a link connecting two players in different components, a link between two components is not formed. In the case where links within a component and between components are formed simultaneously, no player is better off. Moreover, by moving to the other component, the utility of any player does not increase. Thus, the network with two β-components in Fig. 3 is an equilibrium network.

A network with coexistence of an α-component and a β-component is shown in Fig. 4. Let the payoffs of the game be $a = 8$, $b = 6$, $e = 1$, and $f = 0$, and suppose that the costs of the link and the communication are $c = 7$, $d(2) = 3$, $d(3) = 4$, and $d(4) = 12$. With these parameter values, by a similar consideration, one finds that the network with coexistence of an α-component and a β-components in Fig. 4 is an equilibrium network.

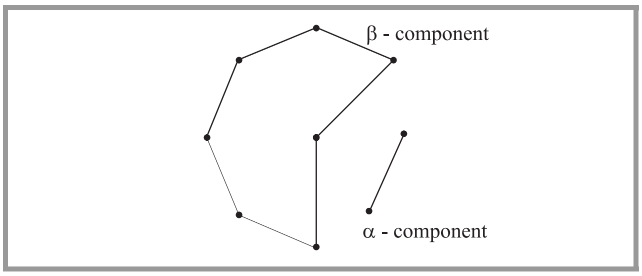

Fig. 4. A network with α- and β-components.

In (3) of Theorem 1, because of $c > a$, an equilibrium network is generally an empty network where the actions of players are unspecified. However, if $d(2) \leq c \leq RC(n-1)$ is satisfied, a leafless component can be included in an equilibrium network. In the case of (a), from $c < d(2)$, one finds that there does not exist an integer l satisfying the condition (6) in Lemma 4 because the left hand side of (6) is negative, and then only the empty networks are equilibria. For the case of (c), similarly, the condition of Lemma 4 is not satisfied due to $c > RC(n-1)$, and then there does not exist any nonempty equilibrium network. In contrast, in the case of (b), from $d(2) \leq c \leq RC(n-1)$, if players can obtain larger payoffs from the interaction between distant players through paths, besides the empty equilibrium networks, there can exist an equilibrium network with a leafless component.

4. Conclusions

In this paper, we dealt with the network game model in which a player can choose partners for playing the underlying coordination games as well as an action of the game, assuming that interaction between distant players is possible, but it requires the payment of the communication cost. We examined influence of the communication cost on the behavior of players in the game and the structure of networks. We showed a diversity of the equilibrium networks. The relevant studies [1], [2], [3] examine the long-run stability of the equilibrium networks. Naturally, it is interesting to investigate the stability of the equilibrium networks in the network game model considered in this paper, and we will intend to deal with this topic.

Appendix

Proof of Lemma 1

Assume that a component with α-players and β-players is an equilibrium component, and let n_α and n_β denote the numbers of α-players and β-players, respectively. Then, the utility of α-player i defined by (2) is expressed by

$$\pi_i^\alpha = \left\{(n_\alpha - 1)u(\alpha,\alpha) + n_\beta u(\alpha,\beta)\right\} - D_i\,, \qquad (7)$$

where D_i is the total cost defined by (3). Assume that player i changes his action of the game from α to β, and let π_i^β denote the utility of player i at the time. Because the structure of the component is the same as before, D_i does not change, and then the utility π_i^β is expressed by

$$\pi_i^\beta = \left\{(n_\alpha - 1)u(\beta,\alpha) + n_\beta u(\beta,\beta)\right\} - D_i\,. \qquad (8)$$

Because the component is in equilibrium before player i changes his action, the inequality $\pi_i^\alpha \geq \pi_i^\beta$ holds, and from Eqs. (7) and (8), one finds that

$$(n_\alpha - 1)a + n_\beta f \geq (n_\alpha - 1)e + n_\beta b\,. \qquad (9)$$

Similarly, for β-player j in the component, the following inequality holds.

$$n_\alpha e + (n_\beta - 1)b \geq n_\alpha a + (n_\beta - 1)f\,. \qquad (10)$$

From Ineqs. (9) and (10), one finds that

$$-(b-f) \geq a-e\,. \quad (11)$$

Because the inequality (11) is inconsistent with the assumption (1) of the payoffs of the game, $a > e$ and $b > f$, the component with α-players and β-players is not an equilibrium component. ■

Proof of Lemma 2

Let L be the distance between players i and j, i.e, $L_{ij} = L$, $2 \leq L < |C|$. First, we show the condition that link ij is not formed. The total cost of player i for path $i \leftrightarrow j$ is calculated as follows:

$$c + \sum_{k=1}^{L} d(k)\,. \quad (12)$$

If link ij is formed, i.e., $l_{ij} = 1$, the total cost of player i changes to

$$c + \sum_{k=1}^{\lceil L/2 \rceil} d(k) + \sum_{k=1}^{\lfloor L/2 \rfloor} d(k) + c\,. \quad (13)$$

Thus, if the cost (13) after link ij is formed is larger than the original cost (12), i.e.,

$$c \geq \sum_{k=\lceil L/2 \rceil+1}^{L} d(k) - \sum_{k=1}^{\lfloor L/2 \rfloor} d(k)\,, \quad (14)$$

then link ij is not formed.

To prove (1), assume that an equilibrium component is not completely connected. Then, there exists at least one pair of players i and j such that $L_{ij} \geq 2$. In this case, because link ij is not formed, the condition (14) is satisfied. When $L = 2$, one finds that $c \geq d(2) - d(1) = d(2)$, which is inconsistent with the assumption of (i): $c < d(2)$. Thus, if there exists an equilibrium network, its component should be completely connected.

Consider a completely connected component. The utility of player i arising from the interaction with player j is $u(x,x) - c - d(1)$. Then, if link ij is severed, the utility of player i changes to $u(x,x) - d(2)$. Because, from $d(1) = 0$ and $c < d(2)$, severing link ij results in decrease of the utility, the completely connected component is an equilibrium component.

Consider the case of (3) before the case of (2), and assume that there exists an equilibrium component with a loop consisting of L players. Let i and j be adjacent players in the loop. Because of the assumption of equilibrium, severing link ij results in increase of the cost, and one finds that the following inequality holds:

$$2c + \sum_{k=1}^{\lceil L/2 \rceil} d(k) + \sum_{k=1}^{\lfloor L/2 \rfloor} d(k) \leq c + \sum_{k=1}^{L} d(k)\,.$$

Namely, we have

$$c \leq \sum_{k=\lceil L/2 \rceil+1}^{L} d(k) - \sum_{k=1}^{\lfloor L/2 \rfloor} d(k) = RC(L)\,. \quad (15)$$

Because $d(\cdot)$ is a strictly monotone increasing function and the right hand side of (15) is at most $RC(|C|-1)$, for any $L < |C|$, the inequality (15) is inconsistent with the assumption of (3): $c > RC(|C|-1)$. Thus, there does not exist any loop in an equilibrium component, and then it is a minimally connected component.

Finally, for the case of (2), assume that an equilibrium component is completely connected. Because severing a link results in increase of the cost, it follows that $2c < c + d(2)$. This is inconsistent with the assumption of (2): $d(2) \leq c$, and therefore an equilibrium component is not completely connected. To show that an equilibrium component is not restricted to be minimally connected, we demonstrate that a ring shaped component can be an equilibrium component.

Let C and $|C|$ denote a ring shaped component and the number of players in the component, respectively. First, we give the condition that severing a link of player i results in increase of the cost of player i. The total cost D_i of player i in the component is

$$D_i = c + \sum_{k=1}^{\lceil (|C|-1)/2 \rceil} d(k) + \sum_{k=1}^{\lfloor (|C|-1)/2 \rfloor} d(k) + c\,. \quad (16)$$

When player i severs a link, the total cost of player i changes to

$$D_i^- = c + \sum_{k=1}^{(|C|-1)} d(k)\,. \quad (17)$$

Thus, the condition that severing a link of player i results in increase of the cost of player i is $D_i < D_i^-$, i.e.,

$$c < \sum_{k=\lceil (|C|-1)/2 \rceil+1}^{|C|-1} d(k) - \sum_{k=1}^{\lfloor (|C|-1)/2 \rfloor} d(k)\,. \quad (18)$$

Second, consider the condition that forming a new link results in increase of the cost. Let the distance between players i and j be $L_{ij} = L$. When player i is directly connected with player j, two loops are formed; one loop has $L+1$ players, and the other has $|C| - L + 1$ players. At this time, the cost of player i changes to

$$D_i^+ = c + \sum_{k=1}^{\lceil L/2 \rceil} d(k) + \sum_{k=1}^{\lfloor L/2 \rfloor} d(k) + c + \sum_{k=1}^{\lceil (|C|-L)/2 \rceil} d(k) + \sum_{k=1}^{\lfloor (|C|-L)/2 \rfloor} d(k) + c\,. \quad (19)$$

Thus, the condition that forming a new link of player i results in increase of the cost of player i is $D_i < D_i^+$, i.e.,

$$c > \sum_{k=\lceil L/2 \rceil+1}^{\lceil (|C|-1)/2 \rceil} d(k) - \sum_{k=1}^{\lfloor L/2 \rfloor} d(k) + \sum_{k=\lceil (|C|-L)/2 \rceil+1}^{\lfloor (|C|-1)/2 \rfloor} d(k) - \sum_{k=1}^{\lfloor (|C|-L)/2 \rfloor} d(k)\,. \quad (20)$$

Third, suppose that player i severs link ij and forms a new link is, $s \neq j$, where $L_{is} = L$. At this time, the cost of player i changes to

$$D_i^{-+} = \sum_{k=1}^{L} d(k) + c + \sum_{k=1}^{\lceil(|C|-L)/2\rceil} d(k) + \sum_{k=1}^{\lfloor(|C|-L)/2\rfloor} d(k) + c\,. \quad (21)$$

Because $d(\cdot)$ is a strictly monotone increasing function, one finds that $D_i^{-+} < D_i$, namely, the cost of player i decreases by this operation. As for the cost of player s, it is

$$D_s^{-+} = c + \sum_{k=1}^{L} d(k) + c + \sum_{k=1}^{\lceil(|C|-L)/2\rceil} d(k) + \sum_{k=1}^{\lfloor(|C|-L)/2\rfloor} d(k) + c\,, \quad (22)$$

and therefore $D_s^{+} < D_s^{-+}$. Because if (20) is satisfied, $D_s < D_s^{+} < D_s^{-+}$ holds, player s rejects player i's offer to form link is. Therefore, if both Ineqs. (18) and (20) are satisfied simultaneously, the ring shaped component C is an equilibrium component. Moreover, because from the proof of (1), if $c < d(2)$, an equilibrium component is completely connected, $d(2) \leq c$ should be also satisfied. Thus, under the following condition, a component which includes a loop but is not completely connected can be an equilibrium component.

$$\max\left\{ d(2), \sum_{k=\lceil L/2\rceil+1}^{\lceil(|C|-1)/2\rceil} d(k) - \sum_{k=1}^{\lfloor L/2\rfloor} d(k) + \sum_{k=\lceil(|C|-L)/2\rceil+1}^{\lfloor(|C|-1)/2\rfloor} d(k) - \sum_{k=1}^{\lfloor(|C|-L)/2\rfloor} d(k) \right\} < c < \sum_{k=\lceil(|C|-1)/2\rceil+1}^{|C|-1} d(k) - \sum_{k=1}^{\lfloor(|C|-1)/2\rfloor} d(k) = RC(|C|-1)\,. \quad (23)$$

As for the maximal length of a path, because (14) is the condition that link ij is not formed when the distance between players i and j is L, the maximal length is derived straightforwardly from the condition. ■

Proof of Lemma 3

Assume that a network with two or more x-components is in equilibrium, and select any two components C_1 and C_2 in them. Let $|C_1|$ and $|C_2|$ be the numbers of players in C_1 and C_2, respectively. Consider player i in C_1 who selects action a_i and player j in C_2 who selects action a_j. If player i severs all his links in C_1, takes the same action as a_j, and offers to form a new link with player j in C_2, then player j accepts the offer because of $u(a_j,a_j) > c$, and therefore any player can move another component.

For the case of $c < d(2)$, because any component is completely connected, the condition that player i does not have any incentive to move from C_1 to C_2 is as follows.

$$(|C_1|-1)(u(a_i,a_i)-c) \geq |C_2|(u(a_j,a_j)-c)\,. \quad (24)$$

For player j, the following similar condition is obtained.

$$(|C_2|-1)(u(a_j,a_j)-c) \geq |C_1|(u(a_i,a_i)-c)\,. \quad (25)$$

Thus, if the following inequality condition is satisfied, both players do not move.

$$0 \geq (u(a_i,a_i)-c) + (u(a_j,a_j)-c)$$

However, because $\min\{u(a_i,a_i), u(a_j,a_j)\} > c$, the above inequality does not hold, and therefore in the case of $c < d(2)$ a network with two or more x-components is not an equilibrium network.

For the case of $c \geq d(2)$, assume that player i forms a new link with player k in C_2 who has a link with player j. At this time, the utility of player i is represented by

$$\pi_i = |C_2|u(a_j,a_j) - (D_j + d(2))\,,$$

where D_j denotes the total cost of player j. If the following inequality condition is satisfied, player i does not have any incentive to move from C_1 to C_2.

$$(|C_1|-1)u(a_i,a_i) - D_i \geq |C_2|u(a_j,a_j) - (D_j + d(2))\,. \quad (26)$$

For player j, the following similar condition is obtained.

$$(|C_2|-1)u(a_j,a_j) - D_j \geq |C_1|u(a_i,a_i) - (D_i + d(2))\,. \quad (27)$$

Thus, if the following inequality condition is satisfied, both players do not move.

$$0 \geq (u(a_i,a_i) - d(2)) + (u(a_j,a_j) - d(2))\,.$$

However, this inequality is inconsistent with $\min\{u(a_i,a_i), u(a_j,a_j)\} > c \geq d(2)$, and therefore even in the case of $c \geq d(2)$, a network with two or more x-components is not an equilibrium network.

Finally, consider the case where there exists an isolated x-player. Assume that player j, $j \neq i$, is in some component C. Similarly to the cases of $c < d(2)$ and $c \geq d(2)$, the condition that player i is not willing to form any link is expressed as follows.

$$0 \geq |C|(u(a_j,a_j) - c), \quad \text{if } c < d(2)\,, \quad (28)$$

$$0 \geq |C|u(a_j,a_j) - (D_j + d(2)), \quad \text{if } c \geq d(2)\,. \quad (29)$$

If the component C is an equilibrium component, the utility π_j of player j is nonnegative, i.e.,

$$\pi_j = (|C|-1)(u(a_j,a_j) - c) \geq 0, \quad \text{if } c < d(2)\,, \quad (30)$$

$$\pi_j = (|C|-1)u(a_j,a_j) - D_j \geq 0, \quad \text{if } c \geq d(2)\,. \quad (31)$$

Because (28) is inconsistent with (30), and (29) is inconsistent with (31), a network with an x-component and an isolated x-player is not an equilibrium. Moreover, because $u(a_j,a_j) > c$, the empty network is also not an equilibrium. Thus, by these facts, the lemma is proven. ■

Proof of Lemma 4

If player i is directly connected with a leaf player j, the utility of player i increases by severing link ij because $c > u(x,x)$. Therefore, equilibrium x-components are leafless, and it must include one or more loops.

When player i interacts with a player of distance L, the condition that link ij is maintained is that $c + \sum_{k=1}^{L} d(k) \leq Lu(x,x)$ holds. This condition is rewritten as

$$c - u(x,x) \leq (L-1)u(x,x) - \sum_{k=1}^{L} d(k),$$

which is the same as condition (6) given in the lemma. Condition (6) means that if the gain arising from the interaction with distant players through a path is larger than the loss, the link cost minus the payoff of the game, of the interaction with an adjacent player directly connected by a link, the link is maintained. Therefore, maintaining an x-component requires all the players in the component to satisfy condition (6). Because the length of a path in a component is at most $\lfloor n/2 \rfloor$, if condition (6) is satisfied for the length $L \in (1, n/2]$ of a loop in an x-component, the x-component can be equilibrium component.

Next, we show that there can exist multiple x-components in an equilibrium network. For sake of simplicity, consider two ring shaped components with $L+1$ players, and they are denoted by C_1 and C_2. Let i and j be players in C_1 and C_2, respectively. After link ij is formed, the variation of the utility of player i is

$$(2L+1)u(x,x) - c - 2\sum_{k=1}^{L+1} d(k). \tag{32}$$

From condition (6), $(2L+1)u(x,x) - c - 2\sum_{k=1}^{L} d(k)$ is positive. However, if $d(L+1)$ is sufficiently large, (32) can be negative and then forming link ij results in decrease of the utility of player i. Then, a link between the two components C_1 and C_2 is not formed.

In the other cases such as the case where player i offers to form two or more links with players in C_2, or the case where player i offers to form two links with players in C_2 and one link with another player in C_1, from a similar discussion, one finds that it is possible that the utility of player i decreases, and therefore, in an equilibrium network, there can exist multiple leafless components.

Finally, we consider the condition with respect to the link cost c such that there exists an integer l satisfying condition (6). Because $d(\cdot)$ is a strictly monotone increasing function, if the gain arising from the interaction with distant players through a path is positive, that is, the right hand side of (6) is positive, it is necessary that $u(x,x) > d(2)$. If $c < d(2)$, then from $u(x,x) < c$, one finds $u(x,x) < d(2)$, and there dose not exist an integer l satisfying condition (6). Therefore, $d(2) \leq c$ must be satisfied. When an integer l satisfies condition (6), there exists a loop with $2l+1$ players choosing action x. In the loop, the cost of any player i is $2(c + \sum_{k=1}^{l} d(k))$, and when player i severs a link, the cost changes to $c + \sum_{k=1}^{2l} d(k)$. Because severing a link increases the cost of a player, the following inequality must be satisfied:

$$c \leq \sum_{k=l+1}^{2l} d(k) - \sum_{k=1}^{l} d(k) = RC(2l). \tag{33}$$

Namely, to exist an x-component in an equilibrium network, $d(2) \leq c \leq RC(2l)$ must be satisfied. ■

Proof of Theorem 1

Proof of (1): From Lemma 1, all the players in an equilibrium component choose the same action: α or β. From $c < b$ and Lemma 3, there exists only one α-component or one β-component. For both of the α-component and the β-component, by Lemma 2, the structure of the equilibrium network is determined as described in the theorem.

Proof of (2): Consider the empty β-network. Because there exists no link between players, switching an action from β to α does not change the payoff of a player. Moreover, because forming a link results in decrease of the payoff of a player by $b - c$, such a link is not formed. Thus, any player does not have any incentive to change his strategy, and then the β-network can be an equilibrium network.

As for nonempty networks, from Lemma 1, all the players in an equilibrium component choose the same action: α or β. From Lemma 2 and Lemma 3, by setting $x = \alpha$, one draws the conclusions of (a), (b), and (c) for α-networks, and from Lemma 4, by setting $x = \beta$, one draws the conclusion of (b) for β-networks. In the following, we will prove that a network with both of an α-component and a β-component can be an equilibrium in (b).

Because $d(2) \leq c \leq RC(n-1)$ in (b), from Lemma 4, a β-component can be an equilibrium component. Consider the condition that in an equilibrium network, an α-component with m (≥ 2) players coexists with a ring shaped β-component with $2L+1$ players. If no link between the α-component and the β-component is formed or an α-player and a β-player do not move to the β-component and the α-component, respectively, the network with both of the α-component and the β-component can be an equilibrium.

When a link between an α-player and a β-player is formed, the upper limit of the utility variation of the β-player is

$$mu(\beta,\alpha) - c - (m-1)d(2), \tag{34}$$

and it occurs when the α-component is star shaped. Because (34) is negative if $u(\beta,\alpha) = e < d(2)$, then such a link is not formed. When a β-player changes her action to α, the upper limit of the utility of the β-player is

$$2Lu(\alpha,\beta) - 2c - 2\sum_{k=1}^{L} d(k) + mu(\alpha,\alpha) - c - (m-1)d(2). \tag{35}$$

From $u(\alpha,\beta) = f < e$, the sum of the first, the second, and the third terms of (35) is negative if $e < d(2)$, and then if L is sufficiently large compared with m, (35) can be negative. Therefore, under the above situation, no link between an α-component and a β-component is formed.

Next, consider the case where an α-player moves into the β-component after the α-player changes his action to β, or a β-player moves into the α-component after the β-player changes her action to α. In the former case, it results in decrease of the utility of a β-player originally included in the β-component. Conversely in the latter case, the utility of an α-player originally included in the α-component increases. After the β-player changes her action from β to α, the utility variation of the β-player is

$$2\left(Lb-\left(c+\sum_{k=1}^{L}d(k)\right)\right)-(ma-c-(m-1)d(2)), \quad (36)$$

and if (36) is nonnegative, the β-player does not have any incentive to migrate to the α-component. In order for (36) to be nonnegative, $a \le Lb-(c+\sum_{k=3}^{L}d(k))$ must be satisfied, and it holds if L is sufficiently large and $b > d(L)$. The condition of ring shaped components in Lemma 2 is compatible with $b > d(L)$, and therefore if L is sufficiently large, (36) can be nonnegative.

From the facts shown above, the network with both of the α-component and the β-component can be an equilibrium.

Proof of (3): From $a < c$, forming a new link results in decrease of the utility of a player, i.e., $u(a_i,a_j)-c<0$ and $u(a_j,a_i)-c<0$, and therefore an empty network is an equilibrium.

As for nonempty networks, from Lemma 1, all the players in an equilibrium component choose the same action: α or β. For $x=\alpha$, from Lemma 4, if there exists an integer $l=l_a$ satisfying condition (6), there exists an equilibrium network with an α-component. Similarly, for $x=\beta$, if there exists an integer $l=l_b$ satisfying condition (6), there exists an equilibrium network with a β-component.

When $c<d(2)$, the right hand side of condition (6) is negative, and then an equilibrium network does not include any component. Namely, an equilibrium network is an empty network, because there does not exist an integer l satisfying condition (6). If there exists an integer l satisfying condition (6), $d(2)\le c\le RC(2l)$ must hold. However, because of $2l+1\le n$, $d(2)\le c\le RC(2l)$ implies $d(2)\le c\le RC(n-1)$, and therefore when $d(2)\le c$ and $c>RC(n-1)$, there does not exist such an integer l. Then, an equilibrium network does not include any component. On the contrary, when $d(2)\le c\le RC(n-1)$, because this condition is compatible with $d(2)\le c\le RC(2l)$, it is possible that there exists a nonempty equilibrium network.

For the coexistence of an α-component and a β-component in the case of $d(2)\le c\le RC(2l)$, in a way similar to that of (2), it is shown that a link between the α-component and the β-component is not formed if $u(\beta,\alpha)<d(2)$. Because of $b<a<c$, migration from one component to the other one results in decrease of the utility of a player as shown in the proof of (2). Thus, the network with the α-component and the β-component can be an equilibrium. ■

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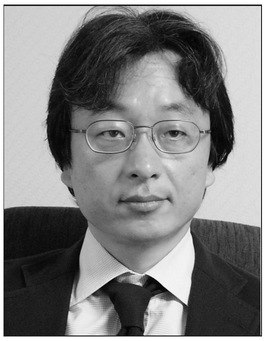

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