Paper

Designing Auctions: A Historical Perspective

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Abstract—Auction is a form of organization of competition that leads to the assignment and valuation of resources based on the information obtained from the competing agents. From the perspective of systems science it is a distributed resource allocation algorithm applied in the environment with information asymmetry, i.e., where the interconnected and interacting subsystems have different information about the system as a whole. This paper presents an overview of the historical development of mathematical theory underlying modern approach to auction design. Selected practical applications of the theory are also discussed.

Keywords—auctions, game theory, mechanism design.

1. Introduction

Auctions are used to buy and sell almost anything one can imagine. Number of categories of items being up for the Internet auctions at Allegro, Amazon or eBay web sites is truly astonishing. Auction houses, such as Sothebys, sell art, antiques, books, jewelry, toys, dolls, and other collectible memorabilia. Securities worth billions of dollars are regularly auctioned worldwide by the Departments of Treasury. Directives of the European Parliament recommend application of auctions in awarding of public contracts and coordinating the procurement procedures. Auctions are also widely used to regulate markets of strategic resources such as electric power or radio spectrum. Recently there have also been many attempts to apply auction mechanisms to allocate bandwidth in communication networks, improve industrial supply chain management and efficiency of allocation of landing and take-off time slots in air traffic flow management.

One of the reasons for the popularity of auction is that it provides a convenient way of assigning goods to those who value them the most. The common auction formats used in practice to allocate a single object are the English auction, the Dutch auction, the first-price and the second-price sealed-bid auction. The most popular variant is the English auction in which the auctioneer calls ascending prices until there is only one bidder willing to pay. In the Dutch auction the auctioneer also calls prices, however, he initially starts from the high level and successively lowers the price until there is someone willing to pay. In contrast with the dynamic open bidding formats of the English and Dutch auctions, the sealed-bid auctions are conducted in a single step. The auctioneer determines the outcomes based on the sealed offers submitted by the bidders'. Both in the first-price and the second-price auction the winner is the bidder with the highest bid. The difference is in the amount of money the winner is obliged to pay. In the first-price auction the winner pays his bid. In the second-price auction the winner pays the second highest bid.

Multiple objects can be sold in a sequence of single-object auctions or simultaneously. There are three traditional formats of simultaneous multi-unit auctions: discriminatory (pay-as-bid), uniform and Vickrey auction. In each case bidders submit to the auctioneer a vector of nonincreasing bids (marginal values), which indicate each bidder's willingness to pay for each additional item. In a discriminatory auction a bidder pays the amount of money equal to the sum of his winning bids, i.e., the sum of those bids that belong to the set of K highest bids, where K is the number of goods. In a uniform-price auction all goods are sold at a *market-clearing price*, i.e., a maximal price at which the total amount demanded is greater or equal to the total amount supplied. In a Vickrey auction, each bidder pays an amount equal to the externality exerted on others. If a bidder wins k units of resource, the his payment is equal to the sum of k highest bids of other bidders (defeated by his bids) [1], [2].

The choice of a particular auction format has been a vital problem. On one hand auction may serve as a solution to many problems of decentralized resource allocation. On the other, each format suffers from drawbacks that may negatively influence both efficiency of the outcomes and auctioneer's revenue. In this paper a historical overview of selected aspects of auction design is presented. The key issues that are raised concern contributions of the related game-theoretic analysis.

2. The Systems Science Perspective

Auction is a form of organization of the competition that leads to the assignment and valuation of resources based on the information obtained from the competing agents. From viewpoint of the systems science auction is a distributed resource allocation algorithm applied in the environments with information asymmetry, i.e., where the interconnected and interacting subsystems have different information about the system as a whole. This perspective is taken in the discussion below. First, a survey of results concerning theory of competitive equilibrium is presented. This is justified by the role it plays in the design of resource allocation mechanism, even though its assumptions hardly ever correspond to the reality. Second, we refer to the historical development of the theory of incentives underlying modern approach to auction design. The theory emerged from the game theoretic analysis of choices made in distributed systems under information asymmetry. As a consequence its models are much more realistic than those derived from competitive equilibrium theory.

2.1. Competitive Equilibrium Theory

The goal of auction design is to take the advantages of competition to solve the problem of resource allocation. In this context the competitive (Walrasian) equilibrium serves as an aspiration point for the auction design. It is defined as a solution of the system of interaction balancing (market-clearing) equations according to which preference maximizing demand equals preference maximizing supply. Static properties of competitive equilibria and conditions that guarantee their existence are described by the fundamental theorems of welfare economics; see Walras [3], Wald [4], [5], Lange [6], Arrow and Debreu [7]. Traditionally, they are viewed as formalization of the Adam Smith's famous conjecture regarding the *invisible hand* of market competition [8]. In essence:

- Pareto-efficiency is consistent with individual self-interest since *price-taking* behavior is reasonable in competitive market, especially if the number of decision makers is large [9].

Stability of competitive equilibrium was first investigated by Samuelson. In [10], [11], [12] he surveyed dynamic models of market-clearing process and examined the relationship between the conditions for stability of competitive equilibrium given by Hicks [13] and general conditions for stability of dynamical systems. Hicks described equilibrium as perfectly stable if an increased demand for a good raises its price even when any subset of other prices is arbitrarily held constant. Samuelson showed that this condition is neither necessary nor sufficient for dynamic stability in Lyapunov sense, except in the case of symmetric matrix of the partial derivatives of *excess demand* – a difference between the value of demand and supply. An extensive exploration of dynamic stability of price adjustment process in perfectly competitive market was later given by Arrow and Hurwicz [14], [15]. The market price adjustment process, described by the system of differential equations defined by continuous and sign-preserving functions of aggregate excess demand, is globally stable if the following assumptions are satisfied:

– agents maximize rational, continuous, monotone and strictly convex preferences,

– agents' preferences are commonly known,

– agents are price-takers (do not anticipate equilibrium prices),

– aggregate demand satisfies the *(weak) axiom of revealed preferences* and has the property of *gross substitution*.

General treatment of the sufficient conditions for the stability of competitive equilibria was also given by Uzawa in [16], [17], [18]. Extensive study of price-based hierarchical control methods was given by Findeisen *et al*. in [19], as well. Saari and Simon [20], [21], on the other hand, investigated local stability of competitive equilibria. They noticed that there is a tradeoff between global stability conditions and information required by the price adjustment procedure to converge to local equilibrium. In particular, they considered Newton algorithm as a price adjustment process and studied the information content it requires for convergence.

Applications in telecommunication. Perhaps the most impressive recent application of competitive equilibrium theory is the design of telecommunication protocols for congestion control. As an illustration of the general approach one can consider the uniform-price auction mechanism proposed by Kelly [22]. Transmission rates of the traffic sources in the computer network are gradually adjusted until their willingness to pay for the introduced congestion equals the corresponding congestion cost. In this model each link in the network acts as an auctioneer, it adjusts its individual congestion price until the demand for the link resources equals the supply. See Srikant [23] and Low [24], [25], [26] for details.

Attractiveness of this approach relates to the common sense of competitiveness of the network environment. Indeed, telecommunication networks consist of a large number of similar traffic sources (characterized by similar preferences) controlled by the same telecommunication protocols. Therefore, the assumptions of competitive equilibrium theory may be regarded as a reasonable description of the traffic exchange process. If all traffic sources calculate transmission rates taking the congestion signals (link prices) as given, then the fixed point of the traffic exchange process can be established in competitive equilibrium maximizing the effectiveness of network utilization. This observation has served as a justification for the design of several recent TCP congestion control algorithms.

2.2. Theory of Incentives

Clearly, assumptions of the competitive equilibrium model are not satisfied in most real-life settings. Namley, economy is rarly a complete system of markets (in which every agent is able to exchange every good with every other agent), externalities are present (prices do not reflect the full costs or benefits), common property resources exist in economy (consumption of such good by one individual does not reduce availability of the good for consumption by others, no one can be effectively excluded from using the good), decision makers anticipate prices, information is imperfect and time delays cannot be ignored, *etc*. From the engineering point of view model inadequacies of this sort can be recognized as a potential source of system distress; see e.g. Stiglitz [27] and Mas-Colell [28].

The shortcomings and failures of competitive equilibrium theory inspired the search for a much more sophisticated models. General solutions to the resource allocation problems arising in the systems with information asymmetry emerged from the investigations of incentives motivating individuals in decision making. Historically they related to the three streams of thought: theory of market socialism, social choice theory and theory of competitive markets; see e.g. Green, Laffont and Tirole [29], [30]. Currently, the obtained results are included in the theory of incentives (principal-agent models) and mechanism (or game) design theory.

Theory of market socialism, co-founded by Polish economist Oskar Lange in 1930's [31], postulated centralized control in order to reach predefined goals of the economy. The responsibility assigned to the central planner was to determine the values of coordination variables: prices, production inputs and outputs. These were then applied to control performance of local industrial organizations [32]:

> *(...) a market mechanism could be established in a socialist economy which would lead to the solution of the simultaneous equations by means of an empirical procedure of trial and error. Starting with an arbitrary set of prices, the price is raised whenever demand exceeds supply and lowered whenever the opposite is the case. Through such a process of tatonnements, first described by Walras, the final equilibrium prices are gradually reached. These are the prices satisfying the system of simultaneous equations. It was assumed without question that the tatonnement process in fact converges to the system of equilibrium prices. (...) Let us put the simultaneous equations on an electronic computer and we shall obtain the solution in less than a second. The market process with its cumbersome tatonnements appears old-fashioned. Indeed, it may be considered as a computing device of the preelectronic age.*

It seems evident that not only the problem of incentives was ignored but also there was a belief that a government agency could glean and process all the relevant information required to make an economy function well. In practice, on one hand the constraints were imposed on production outputs, but, on the other, the government either provided insufficient inputs or provided more than it was necessary. As a result, with severe conflicts concerning personal freedom and civil rights in the background, the economy strode towards the state of constant struggle to realize production plans. Strategic manipulations to outwit the system, both in order to meet predefined goals of the economy of public goods and to satisfy privately defined interests, arose naturally in effect of recurring coordination failures. Inefficiency of directly coordinated system was largely due to incompatibility of the private interests and goals of the central planner. To assure that they coincide proper incentives were required. However, as it quickly became apparent, without sufficient autonomy, private property or the profit motive, putting democratic procedures aside, incentives were lacking. System's collapse was inevitable. An interesting debate revealing important historical background of the discussed issues can be found in [33]. See also Stiglitz [27], [34].

Social choice theory is concerned with the problem of rational aggregation of preferences within the collective decision rules, including voting systems and competitive markets. Its central result, due to Arrow [35], shows that necessary conditions that preference aggregations should be expected to meet are inconsistent and cannot hold together:

> *If we exclude the possibility of interpersonal comparisons of utility, then the only methods of passing from individual tastes to social preferences which will be satisfactory [i.e. will not reflect individuals' desires negatively and the resultant social tastes will be represented by an ordering having the properties of rationality ascribed to individual orderings] and which will be defined for a wide range of sets of individual orderings are either imposed or dictatorial.*

As it can be noticed, the key concern that motivated the related work grew out of the observation that the concept of preference aggregation, by its very nature, deals with the problem of *interpersonal comparisons* and *measurability* of preferences' intensity. The focus on ordinal preferences, which was largely due to the influential arguments that *no common denominator of feelings is possible* [36], was an attempt to eschew the related controversies. Unfortunately, Arrow's impossibility theorem demonstrated that there are other substantial difficulties that arise as an unavoidable trade off – the impossibility result is the price for the *incomparability* requirement. In an immediate response it was therefore proposed, mostly due to Sen [37], [38], [39], that informational constraints imposed on the collective choice rule should be modified. The line of argumentation was taken that the results of preference aggregation should be *invariant* with respect to the utility signals that provide the same information in terms of the applied notion of measurability and interpersonal comparisons. Consequently, the counterargument gained strong support that the notion of ordinal preferences is inadequate for representing conflicts of gains and losses. These conflicts, however, inevitably occur in many collective choice settings, especially when welfare judgments are involved and the resource constraints are present. When dealing with the considerable number of social choice situations, interpersonal comparisons of intensity of preferences, or weights of interests, provide desirable informational basis for the determination of decision. Conditions imposed on the social choice function by Arrow's theorem may be interpreted as necessary but not sufficient for collective choice. On the other hand, cardinality and full interpersonal comparability of individual welfare units are sufficient but not necessary for rational choice under aggregate welfare maximization. To generate a *complete* and *transitive* aggregation of orderings (preferences) their partial comparability is sufficient as well; see Arrow [35] and Sen [40] for details. In essence, social choice theory shows how to design a satisfactory procedure for preference aggregation. However, it is not concerned with the question if the aggregated preferences, revealed by the interacting agents, are true or not. This observation inspired investigations of the gaming aspect of collective decision-making, commonly observed in many votings and auctions. Finally, the concept of incentive-based regulation arose as a potential remedy to the wide scope of imperfections of the markets traditionally designed within the framework of fundamental theorems of welfare economics. Spectacular examples intensively discussed in the literature include the global depression of the 1930s, East Asia financial crisis in the late 1990s and California Power Exchange collapse in 2001. The following macroeconomical comment by Stiglitz [27] emphasizes significance of the related issues and places them in somewhat wider perspective of market design for developing economies:

> *even if Smith's theory were relevant for advanced industrialized countries, the required conditions are not satisfied in developing countries. The market system requires clearly established property rights and the courts to enforce them; but often these are absent in developing countries. The market system requires competition and perfect information. But competition is limited and information is far from perfect – a well-functioning competitive markets cannot be established overnight. The theory says that an efficient market economy requires that all of the assumptions be satisfied. In some cases reforms in one area, without accompanying reforms in others, make actually matters worse. (...) economic theory and history show how disastrous it can be to ignore sequencing.*

Inefficiencies arising under asymmetric and imperfect information were first studied by Stiglitz [41]–[43], Akerlof [44] and Spence [45]. For general results see [30].

The above considerations eventually gave rise to the theory of incentives and game design. Its contributions, and especially its rigorous game-theoretic analysis of the incentive compatibility concept introduced by Hurwicz [46], have deepened the knowledge regarding the possibility for achieving Pareto-optimal allocations in decentralized systems and designing efficient auctioning procedures. The following results are often viewed as the most influential [9]:

- When a delegation of tasks occurs within the firm, then because of asymmetric information the firm does not maximize its profit, i.e., allocative inefficiency occurs.
- In markets of private and public goods with a finite number of agents, there are no nonparametric mechanisms (which process only the information received from the agents) that simultaneously yield Pareto-efficient allocations and provide individual agents with incentives to report their true preferences honestly.
- In markets of private and public goods with a finite number of agents, there are nonparametric mechanisms that yield Pareto-efficient allocations when all agents follow their self-interest by playing a Nash-equilibrium strategy.
- In the bilateral trade problem, there is no mechanism that yields efficient allocations, provides individual agents with incentives to report their true preferences honestly, guarantees profitable participation and covers the costs of allocations.

Applications in telecommunication. If the assumption of price-taking behavior is dropped, then in most cases competitive equilibria cannot be reached by means of the decentralized price-based coordination methods, such as uniform-price auctions. This problem was recently investigated in the networking context by Johari [47]–[49]. The major result of his work, focused on the mechanisms of *price-anticipating* bidding, demonstrates that there exist implementations of the uniform-price auctions generating outcomes with bounded loss of efficiency. An interesting conclusion is also due to Roughgarden [50], [51]. Namely, the ratio of efficiency loss, arising in the networks as a consequence of the price-anticipating behavior, is independent of network topology. Following the similar line of argument, Yang and Hajek [52], [53] analyzed the undesirable performance of the algorithm proposed by Kelly [22]. In the settings with strategic bidders competition for network paths is dominated in terms of efficiency by competition for the network links (that form the paths). Finally, sufficient conditions for efficiency of auctions in the environments with price-anticipating agents has been given by Karpowicz in [54].

Anticipation of price effects has been recognized in the literature as an urgent problem of dynamic interconnection management in communication networks. Consider a group of interconnected network service providers (ISPs) exchanging IP traffic between their autonomous systems. The basic observation that one can make about this resource allocation setting suggests that local decisions concerning bandwidth allocations can have a non-negligible influence on the overall network performance. As a result, ISPs may anticipate the effects of their actions on interconnection prices and view these prices as functions of the actions of all interrelated providers. Clearly, in such an environment routing and congestion control protocols applied locally by ISPs can be subject to strategic manipulations. Records of such strategic interactions can be found in the archives of the Polish Office of Electronic Communications (www.uke.gov.pl). For more general treatment of problems related to competition in telecommunications, especially from the viewpoint of interconnection agreements, such as peering and transit, and interconnection pricing, see Laffont and Tirole [55], Laskowski [56], Norton [57], Baake and Wichmann [58].

3. Auction Design and Game Theory

In order to benefit from allocating resources by means of an auction it is necessary that its rules be designed and tailored to the particular allocation setting. To cope with the complexity of this multistage design process it is therefore reasonable to apply convenient modeling tools. Game theory, a branch of applied mathematics, plays an important role in this context. It is a study of mathematical models of interaction (competition or cooperation) of intelligent and rational (in a specified sense) agents making interrelated choices under incomplete (asymmetric) information [30], [59], [60]. On one hand, it aims at providing answers to some of the essential questions regarding properties of different auction formats. On the other, it provides recommendations for the design of resource allocation and pricing rules defining games that are characterized by the desired features.

Properties of outcomes generated by auctions were first identified by means of game-theoretic analysis in the seminal work of Vickrey [61]. Its major conclusions were based on the following observation: information about demand and supply, revealed by the competing agents and used to determine the outcomes, influences the market clearing price, thus encouraging agents to submit price-anticipating bids. As a consequence, investigation of the incentives that agents may have to submit

> *an unbiased report of the marginal-cost (competitive supply) curves (...) and of the marginal-value (competitive demand) curves (...), or at least of the portions of these curves covering a range of prices that will be sure to contain the equilibrium price,*

became the main theme of the auction (game or mechanism) design theory [1], [2], [60]. Major contributions in this field are due to Hurwicz [46], [62]–[66], Myerson [59], [67], [68] and Maskin [69]–[73]. An overview of selected historical attempts to apply the theory in practice is given below.

3.1. Treasury Bill Auctions

An influential investigation of the adverse effects of strategic bidding in auctions of shares was presented in the paper by Wilson [74]. It demonstrated existence of bidding strategies that may lead to the reduction of sale price, which in effect reduces revenue of the resource manager, even with the increasing number of agents placing their bids. The result given by Wilson was next generalized by Back and Zender [75] in the context of the auction of U.S. Treasury bills. Conclusions presented in their paper served as an argument in the debate regarding the merits of different formats of multi-unit auctions that the Treasury could apply for the sale of securities.

Traditionally the discriminatory (pay-as-bid) auction was used, according to which all bidders whose offers exceed the market-clearing price (determined by the auctioneer) are obliged to pay their bids. However, in 1960s suggestion came from Milton Friedman that in order to improve revenues the Treasury should consider switching to the uniform-price format. Back's and Zender's paper supported the resulting debate with the formal arguments against any unconditional and simplified recommendations. In particular, it warned against extrapolation of properties of auctions with single unit demand to the more general situations of multi-unit demand. Interestingly, it was not until recently that the equilibrium properties of the multi-object uniform-price auctions have been thoroughly investigated. The general result was obtained by Ausubel and Cramton [76]. It relates potential inefficiency of the uniform-pricing scheme outcomes to the fact that the scheme creates strong incentives for *demand reduction*: each agent's optimal strategy is to shade bids for units of resource other than the first one; since bids placed on the other units determine the final (clearing) price with positive probability, agents may increase their profits by submitting lowered marginal values. (From the viewpoint of the supply side of the system, the result implies increased marginal production costs revealed to the auctioneer.) The similar result is also given in [77].

Indeed, revenue implications of the potential underpricing has become the subject of intensive studies in the context of Treasury auctions. From 1992 to 1998 the U.S. Treasury, motivated by various academic conjectures and market manipulation scandals (in 1991 a major trader in the U.S. Treasury securities admitted that it had violated auction rules by submitting fraudulent bids [78]), experimented with the sealed-bid uniform-price auctions for selling two-year and five-year notes. Eventually it switched entirely to the uniform price format in the end of 1998. The goal was to verify whether incentives to shade bids would be reduced with uniform pricing rule, which in turn would improve revenues to the Treasury. The experiment did not provide strong support for this conjecture. The impact on revenues of the two pricing formats was demonstrated by Malvey and Archibald [79] to be statistically insignificant. Umlauf [80] and Tenario [81], on the other hand, slightly favor the uniform pricing scheme using data from the Mexican Treasury auctions and Zambian foreign exchange auctions, respectively. One can view this conclusions as consistent with the results of Ausubel and Cramton [76], and Back and Zender [75], which state that the ranking of the two formats is inherently ambiguous. There are cases which show that uniform-price format outperforms in both efficiency and revenue the pay-as-bid format in the particular auction setting, and results which show the reverse. This also seems to correspond to the well known result of the theory of single-object auctions; for models that include both affiliation (log-supermodularity of densities) of bidders' valuations and risk aversion, the first- and second-price auctions of single-objects cannot be generally ranked by their expected prices [1], [82].

Another important result is due to Keloharju, Nyborg and Rydqvist [83] who give an extensive exploration of historical data from the Finnish Treasury auctions. On one hand, their finding is that individual bidders' demand increases with number of bidders, which is consistent with the argument that bidders exercise market power. On the other hand, however, statistical data show that equilibria with extremely low prices, e.g. predicted by Wilson [74], usually do not occur in practice. The similar conclusion was given by Nyborg and Sundaresan [84] and Goldreich [85]. The practical reason why bidders do not coordinate on the revenue reducing low price equilibria is the strategic behavior of the auctioneer himself. By determining the amount of securities sold in response to the submitted collection of bids, imposing restrictions on the bidding procedures and revealing sufficient amount of information, the Treasury effectively protects itself against revenue reduction. This advocates the important result of auction design theory – games induced by the rules of allocation mechanisms are played not only between the agents but between the agents and the mechanism designer as well.

3.2. Electric Power Auctions

Both discriminatory and uniform-price auctions have also been used in the electricity markets. Interesting examples come from Scandinavia, UK and France. Norway, Sweden, Finland and Denmark buy and sell electricity on the Nordic Power Exchange, Nord Pool, which has been the world's only multinational exchange for trading electric power since 1990s [86], [87]. Since 2001 in the UK electricity generators sell their output on daily basis in the discriminatory auctions, after the switch from the uniform-price format originally adopted in 1990 [88]. The uniform pricing scheme had also been used in the California Power Exchange before its collapse in 2001. Electricité de France (EDF) gave an undertaking to the European Commission in early 2001 to give access to generation capacities in France in the form of contracts conveying the right to purchase energy. Currently contracts with durations between 3 and 48 months are being sold at pay-as-bid auctions conducted approximately every 3 months; see www.edf.com.

Bidding behavior in the electric power auctions has been a growing concern, as it may be related to prices being increased above competitive levels [89]. An intensively studied real-life example that servers as a support of this argument relates to the collapse of the electric power market in California where the uniform-price auctions were used to buy electricity on the power exchange. It is believed that the strategic bidding of the suppliers, extracting the highest possible electricity prices, was among the causative factors of the crisis in the summer of 2000 [90], [91]. Indeed, many mathematical models have been developed to explain and prevent events of this sort. Research that are of great interest in this context concerns especially the ways in which suppliers' bidding manipulations aimed at improving profits may influence allocations of energy production. Indeed, knowledge of the related threats has been playing a role in adjusting regulatory policy around the world. For example, Green and Newbery [92], [93] applied the *supply function equilibrium* approach, originally introduced by Klemperer and Meyer [94], to show that markups on marginal costs may be constituted by the Nash equilibrium of the game induced by the British electricity spot market. Von der Fehr and Harbord [95] reached the similar conclusion with the sealed-bid auction model. However, they also showed that if supply signals are step functions, as it usually is in practice, pure-strategy equilibria do not exist for a wide range of demand distributions. Other results along this line include works of Cramton [96], [97], Engelbrecht-Wiggans and Kahn [98], [99], Baldick and Hogan [100], Day and Hobbs [101], just to name a few examples.

3.3. Spectrum Auctions

The design of spectrum auctions for the Federal Communications Commission (FCC) in the United States[1] is often regarded in the literature as one of the most successful applications of game theory. In fact, Milgrom [2] argues that it was the design that started the era of *putting the theory to work*.

The primary goal of the FCC was the maximization of economic efficiency of spectrum allocation – licenses were to be assigned to those who are capable of providing better services at lower costs. Designers confronted with the regulatory goals turned to game theory for methodological support. Its recommendations narrowed the set of admissible solutions by pointing out the threats related to the expected bidding strategies [2], [102], [103]. Theoretical models also guided the development of experimentation scenarios testing the applicability of the key design judgments [104], [105]. The following conclusions determined the final auction format:

Open bidding is better than a single sealed bid.

Open bidding process reveals information about valuations of goods and provides feedback increasing auction revenues [1], [106].

Simultaneous open bidding is better than sequential auctions.

Sequential auctions of goods requires agents to condition their decisions on the future actions of others. This guesswork is in practice very likely to reduce efficiency of the auction. With simultaneous bidding much of the guesswork is eliminated [1].

Package bids (combinatorial auctions) are too complex.

Once bidding for a combination of goods is admitted, inefficiencies are likely to arise due to threshold problem, a variant of the free-rider problem. The transparency of auction is weakened as well [102], [103], [107].

As a result the simultaneous multiple-round ascending-bid auction was proposed, a multi-object version of the English auction. According to its rules, a number of licenses is auctioned simultaneously in discrete, successive rounds. In every round, a bidder can bid (offering a buy price)

[1] This application of mechanism design theory was indicated by Prof. Eric Maskin in the telephone interview following the announcement of the 2007 Nobel Prize in Economics.

on any license subject to constraints given by the activity rules and bidder's eligibility defined by the upfront payment. Open bidding format gives each bidder information about the highest bids, identities of bidders, their upfront payments and handicaps. The auction stops if a single round passes in which no new bids are placed on any license. Licenses are sold for the price equal to their highest standing bid [2], [102], [103].

Auction rules implemented by the FCC proved its efficiency in series of spectrum auctions and became a worldwide standard. It should be noticed, though, that they did not eliminate the incentives for strategic bidding, potentially decreasing efficiency of allocations. Cramton and Schwartz [108], [109] described several cases of bid signaling that occurred in FCC auctions and identify it as an example of profitable *collusive* behavior – incentives for tacit collusion were especially strong among incumbents and large bidders capable of exerting their market power. Consequently, the experiences gained in practice have guided evolution of the auction. In response to the observed problems many design recommendations have been given to reduce the effectiveness of signaling and collusion, e.g., by concealing bidders identities, offering preferences (handicaps) for small businesses and new entrants, increasing reserve prices, bounding supply by offering licenses that are harder to split up, allowing package bidding. Again, game-theoretic considerations have been often applied in the examination of the refinements.

4. Final Remarks

Game theory has been helpful in explaining bidding behavior in different auction settings. In some cases its qualitative predictions have turned out to be influential enough to affect the regulation policies. The examples presented above may serve as an evidence of its contributions. On the other hand, however, the very same studies unveil its weak points. Clearly, relevance of its recommendations depends on the particular decision setting.

In reality efficiency of auction outcomes depends on many factors that often dominate any influence that a particular allocation or pricing rule may have. Issues that are faced by the auction designer in practice, often playing more important role than the rules of an auction, are listed below.

Auction items. One of the key design problems is related to the choice of an object to be put up on auction. Whether it is divisible or indivisible, homogeneous or heterogeneous may have a decisive influence on the allocation process. This stems from the fact that a particular definition of an allocation determines preference profile of the competing decision makers. Empirical and theoretical evidence show that rules of auction may be irrelevant under particular allocation definitions.

Auction participants. It is essential to define who is eligible to participate in an auction and what approvals are required. As pointed out by Milgrom [2], marketing a sale is often the biggest factor in its success. Announcement of an auction or definition of a resource allocation procedure must provide information targeted to potential participants enabling them to study the opportunity.

Flexible goals. Auctions are conducted to achieve specific economic goals – typically, maximization of efficiency of allocations or maximization of auctioneer's revenue. However, because of the complexity of the auctioning process adjustments are often required. In fact, in many cases it may be reasonable not to conduct an auction, e.g., because of the overall performance of the economy or insufficient legislative support.

Interactions. What to allocate to agents depends on their demand, which depends on who agents are, which in turn may depend on the way the auction is conducted. Decisions made by auction designer are not independent [2]. Interactions occur between agents as well. There are many occasions for them to cooperate before, during and after the auction. Collusion and mergers clearly have a significant influence on the outcomes, as well as possibility of reallocation after the auction.

Information. What information is required to determine allocations and final payments, and what information is revealed to agents may play a decisive role. One of the key motivations for pricing resources by means of an auction is gaining information about agents' privately known valuations and preferences. Under auction bidding process it is not only the resource manager but also agents themselves that are responsible for the final price and allocation of the resource. Related guesswork is therefore distributed between auction designer and auction participants. On the other hand, the responsibility for resource allocation outcomes inevitably creates incentives for agents to manipulate the process. Information about reserve prices, bidding increments, agents' eligibility revealed before the auction, as well as information about submitted bids revealed during the auction may significantly influence competition, bidding behavior and efficiency of outcomes, especially if agents' valuations are interdependent.

To solve at least some of the problems of auction design one may settle the judgments on game-theoretic models *approximating* the auction outcomes. However, any reasoning should be extremely careful and substantiated by experimental verifications of the dominating factors, since arbitrary estimations and behavioral assumptions are inevitable in this context.

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