Paper

Subscriber location in radio communication nets

Piotr Gajewski, Jan Marcin Kelner, and Cezary Ziółkowski

Abstract— Our aim is to develop the method of location subscriber to be used in military communication nets. This method is based on frequency offset measurement. Analytic description of the Doppler effect creates one of such possibilities. In this paper, an original location subscriber method is presented. We showed the Maxwell equations solution in the case of a transmitter continually changing location in relation to a receiver. It makes possible to calculate exactly the value of the received signal parameters especially frequency offset. The methodology described in this paper shows that the Doppler frequency offset value can be used for radio signal sources location.

Keywords— mobile radio communication, Doppler effect, location of radio waves sources.

1. Introduction

Over that last decade, the emerging location service of electromagnetic waves sources has found numerous applications in the commercial as well as the military radio systems. The rapid technological advances have made it to implement radio navigation, radio communication nets and military radio electronic recognition. In this paper, we concentrate on location service of a subscriber in radio communication nets.

Several methods for subscriber location in radio communication nets have been already presented [1–3]. Many researches were focused on development of methods that allow the application of well-known time-spectrum signal structure. Some methods do not require the system information. Nevertheless, there are few efficient methods for location of a subscriber in radio communication nets:

– access station identification – so-called cell ID or cell of origin (CoO),
– angle of arrival (AoA),
– time of arrival (ToA),
– time difference of arrival (TDoA),
– received signal strength (RSS),
– global positioning system (GPS).

Each of foregoing methods have some advantages and some disadvantages too. In radio communication net, when the location signal source is an element of this system, few methods could be applied. If located source is an element of unknown communication nets, only AoA method could be used. In this case, direction determination of electromagnetic wave source requires complex receiving antenna system and equipment.

Disadvantages of foregoing methods make difficult for practical utilization. These factors have motivated the development of new subscriber location methods in communication nets. Analytic description of the Doppler effect creates one of such possibilities.

In [4] we showed the Maxwell equations solution in the case of a transmitter continually changing location in relation to a receiver. The obtained solution presents the exact description of the received signal parameters that result from the location change between a transmitter and a receiver. The analytic description of this problem makes it possible to calculate exactly the value of the received signal parameters especially frequency offset.

An original location method is derived using analytical expression for frequency offset. In this paper, our aim is to described method of subscriber location that can be used in military communication nets. This method bases on frequency offset measurement.

2. Analytic description of the Doppler effect

The analytic description of the Doppler effect results from solution of Maxwell equations for space stationary system included signal source and signal receiver. In classical approach to this problem the formula of the electric field strength for a far-field region presents the basis for the description of the object location change. In the case of the half wavelength dipole as an antenna, the temporary value of the electric field strength is given by (cf. [5]):

$$E(t) = i\mu_0 f_0 \frac{I_0}{\beta r} \frac{\cos\left(\left(\frac{\pi}{2}\right)\cos\theta\right)}{\sin\theta} \mathrm{e}^{\mathrm{i}(\omega_0 t - \beta r)} \;, \tag{1}$$

where: f_0 – the signal frequency, $\omega_0 = 2\pi f_0$, $\beta = 2\pi/\lambda = \omega_0/\mathrm{c}$ – the wave number (λ – wavelength, c – speed of light), μ_0 – the magnetic permeability of free-space, θ – the elevation angle, I_0 – the amplitude of the antenna current, and r – denotes distance (in general) between transmitter and receiver.

Therefore the phase angle $\Phi_0(t)$ is following:

$$\Phi_0(t) = 2\pi f_0 t - \beta r \;. \tag{2}$$

When the transmitter is moving and it is covering a distance δr then the phase is changing due to change of the value r, namely:

$$\Phi(t) = 2\pi f_0 t - \beta(r \mp \delta r) \;, \tag{3}$$

(when the signal source is moving from the receiver the sign is "+", whereas when transmitter is moving toward the receiver the sign is "−").
In the case of constant velocity of motion v the increase of distance amounts $\delta r = vt$, thus the change of signal frequency is described by the following dependence (cf. [6]):

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\Phi(t) = \frac{d}{dt}\left(f_0 t - \frac{\beta r}{2\pi} \pm \frac{v}{\lambda}t\right) = f_0 \pm \frac{v}{\lambda} \ . \quad (4)$$

If the signal source moves towards the position of the receiver at an angle φ then:

$$f(t) = f_0 + \frac{v}{\lambda}\cos\varphi \ . \quad (5)$$

The frequency $f_D = (v/\lambda)\cos\varphi$ is called the Doppler frequency.
In the case of free space, the Faraday and the Ampere equations as well as the property of vector field double rotation are the basis for the following wave equation describing the vector of the electric field strength:

$$\mathbf{E}(\mathbf{x},t) = \Big[E_x(\mathbf{x},t), E_y(\mathbf{x},t), E_z(\mathbf{x},t)\Big] \ ,$$

$$\frac{1}{\mathrm{c}^2}\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{x},t) - \Delta\mathbf{E}(\mathbf{x},t) = -\mu_0\frac{\partial}{\partial t}\mathbf{i}_0(\mathbf{x},t) \ , \quad (6)$$

where: $\mathbf{x} = (x,y,z)$ is the space coordinate, $\mathbf{i}_0(\mathbf{x},t) = \Big(i_x(\mathbf{x},t), i_y(\mathbf{x},t), i_z(\mathbf{x},t)\Big)$ is the vector of current density (distribution of current in antenna − source of the electromagnetic field), $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ − Laplacian.

In [4] we have concentrated on two considerations:

– the linear antenna system, i.e., we assume that the current density vector has the form

$$\mathbf{i}_0(\mathbf{x},t) = (0, 0, i_z(\mathbf{x},t) = i_0(t)\cdot I(z)\cdot\delta(x)\cdot\delta(y)) \ , \quad (7)$$

– the motion of signal source model in x coordinate direction with v velocity.

Thus the problem has been reduced to solve the following second orders partial differential equation:

$$\begin{aligned}&\frac{1}{\mathrm{c}^2}\frac{\partial^2}{\partial t^2}E(\mathbf{x},t) + \Delta E(\mathbf{x},t)\\ &= -\mu_0\frac{\partial}{\partial t}[i_0(t)\cdot I(z)\cdot\delta(x-vt)\cdot\delta(y)] \ .\end{aligned} \quad (8)$$

The solution of the above equation has been carried out in two stages. Firstly, we have found the fundamental solution of the Eq. (8). To solve the distribution wave equation we used the integral transformations, i.e., the Laplace transform (in relation to normalized time variable) and the Fourier transform (in relation to space variables). The original form of the analysis problem solution has been calculated by applying the Cagniard-deHoop method. The solution of analysis problem is the convolution of the fundamental solution with the function describing current space distribution.
The analytic form of the phase of electric field generated by moving transmitter is ([4]):

$$\Phi(\mathbf{x},t) = \omega_1 t - \beta_1 kx - \beta_1 R_0(\mathbf{x},t) - \frac{\pi}{2} \ , \quad (9)$$

where: $k = v/\mathrm{c}$, $\beta_1 = \omega_1/\mathrm{c} = \beta/(1-k^2)$,

$\omega_1 = \omega_0/(1-k^2) = 2\pi f_0/(1-k^2)$,

$R_0(\mathbf{x},t) = \sqrt{(x-vt)^2 + (1-k^2)\cdot(y^2+z^2)}$.

Hence, the instantaneous frequency $f(\mathbf{x},t)$ is expressed as follows:

$$\begin{aligned}f(\mathbf{x},t) &= \frac{1}{2\pi}\frac{d}{dt}\Phi(\mathbf{x},t)\\ &= \frac{f_0}{1-k^2} - \frac{f_0}{\mathrm{c}(1-k^2)}\frac{d}{dt}R_0(\mathbf{x},t) \ .\end{aligned} \quad (10)$$

So, the Doppler frequency expresses the following dependence:

$$f_D(\mathbf{x},t) = f(\mathbf{x},t) - f_0 = \frac{k}{1-k^2}\left[k + \frac{x-vt}{R_0(\mathbf{x},t)}\right] f_0 \ . \quad (11)$$

Then, we notice that the value of the Doppler frequency $f_D(\mathbf{x},t)$ depends not only on signal source velocity and carrier frequency but also on mutual space location of a signal source and a receiver. Notice the $f_D(\mathbf{x},t)$ linear dependence on the frequency carrier, whereas the dependence on velocity and space coordinate has a more complex character. Location calculation are made on the basis of the above formula and the described value of Doppler frequency shifts as a function of movement and coordinates of signal source parameters. The frequency temporary value $f(\mathbf{x},t) = f_0 + f_D(\mathbf{x},t)$ measurement over mobile station is the basis of the new method of the subscriber location.

3. New location method of radio signals sources

The illustration of the subscriber location methodology is shown in Fig. 1. Measurement of the Doppler frequency offset is base of this method.
The Doppler curves for five different locations of subscriber (station) are presented in Fig. 2. The diverse courses of the Doppler curves (Fig. 2) are characteristic for each subscriber location. It determinates methodology of the frequency offset value using to three-dimensional location.

Basing on Eq. (11) formulas of the subscriber location individual x, y, z coordinates relative to initial mobile station location can be obtained. In case of the mobile station is moving at fixed altitude ($y =$ const), then it should mark

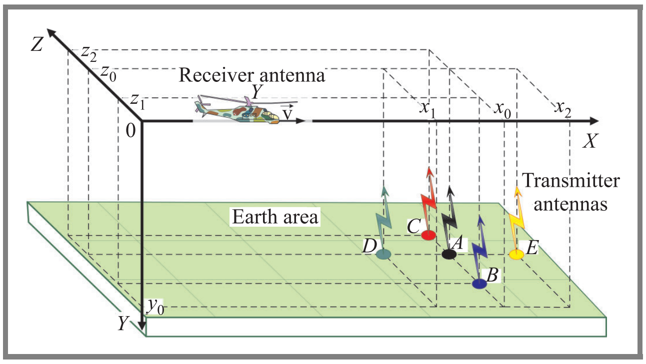

Fig. 1. Space structure of the mutually mobile station and five station locations.

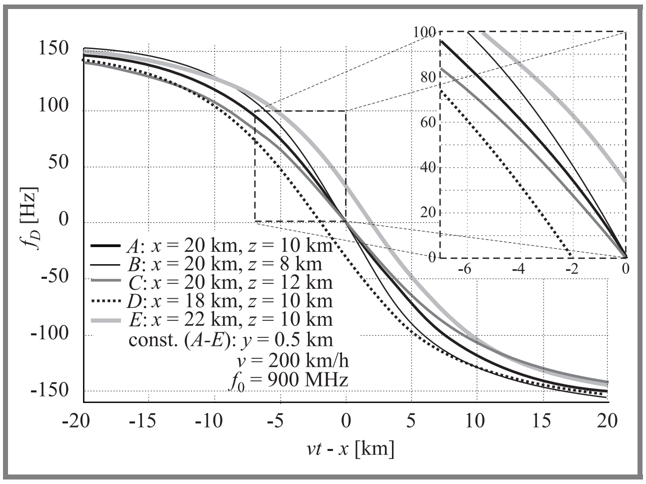

Fig. 2. Diverse courses of the Doppler curves as function of the mobile station to subscriber $vt-x$ distance for five different subscriber locations.

only two x and z coordinates. After elementary transformation of the expression (11) for two moments t_1 and t_2 the formulas describing x and z coordinates are following:

$$\begin{cases} x = v\dfrac{t_1 A(t_1) - t_2 A(t_2)}{A(t_1) - A(t_2)}, \\ z = \pm\sqrt{\dfrac{\left[\dfrac{v(t_1-t_2)A(t_1)A(t_2)}{A(t_1)-A(t_2)}\right]^2}{1-k^2} - y^2}\ , \end{cases} \tag{12}$$

where:

$$A(t) = \frac{\sqrt{1-F^2(t)}}{F(t)}, \qquad F(t) = \frac{f_D(t)}{f_0}\frac{1-k^2}{k} - k. \tag{13}$$

This methodology and also method of bearing and three-dimensional location has been described in patent application [7].

4. Results and discussion

Noise and limited precision of the parameters measurement limit a precision of this method. Analysis of the measurement errors influence is conducted by following assumptions: $\mathbf{x} = (x,y,z) = (20,0.5,10)$ km – located station position relative mobile station, v = 200 km/h – mobile station velocity, f_0 = 900 MHz – transmitted signal frequency, Δf =1 Hz – frequency absolute error.
The Doppler curves for foregoing foundations are presented in Fig. 3.

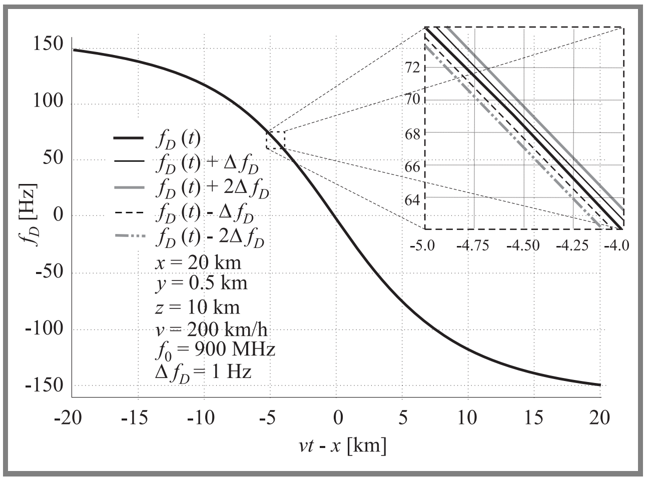

Fig. 3. Courses of the Doppler curves as function of the $vt-x$.

In order to perform precision evaluation of this location methodology, following new quality measure was introduced:

$$\begin{aligned} \Delta r &= \sqrt{(\Delta x)^2 + (\Delta z)^2} \\ &= \sqrt{|x-\tilde{x}|^2 + |z-\tilde{z}|^2}\ , \end{aligned} \tag{14}$$

where: $\tilde{x}$, $\tilde{z}$ – station position coordinates calculated from formulas (12), x, z, – real coordinates.

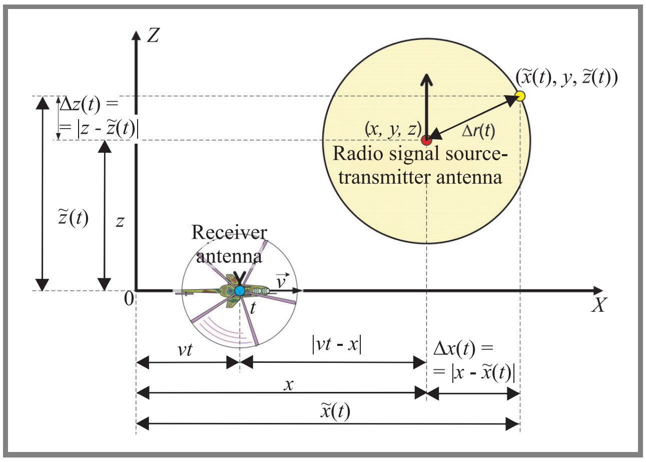

Fig. 4. The script of numeric calculations – the orthographical projection of the movement measuring receiver set trajectory in relation to radio signal source.

Figure 4 shows graphic explanation of introduced quantities.

Figures 5–9 present measures $\tilde{x}$, $\tilde{z}$, Δx, Δz and Δr change as function of the $vt - x$ mobile station to subscriber distance.

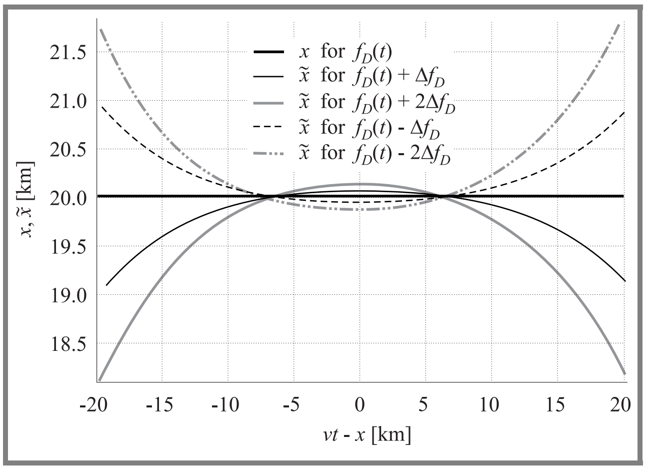

Fig. 5. Courses of changes $\tilde{x}$ as function of the $vt - x$.

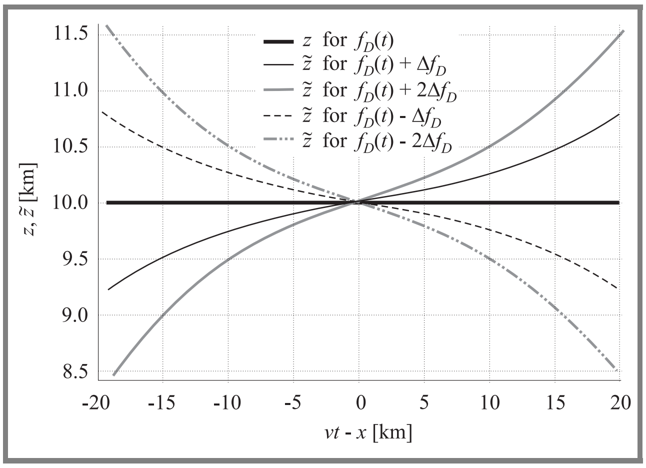

Fig. 6. Courses of changes $\tilde{z}$ as function of the $vt - x$.

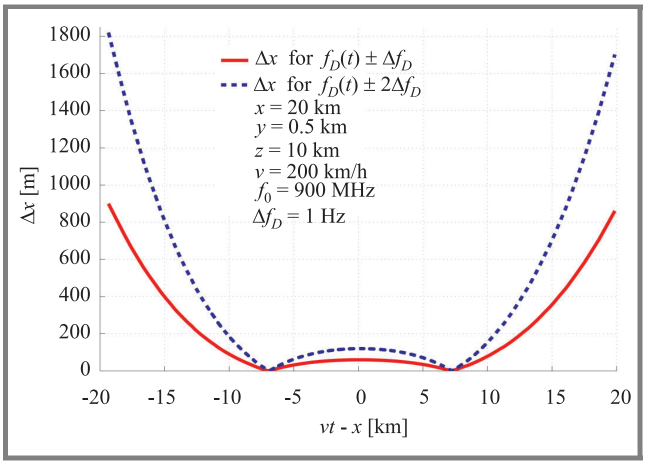

Fig. 7. Courses of changes Δx as function of the $vt - x$.

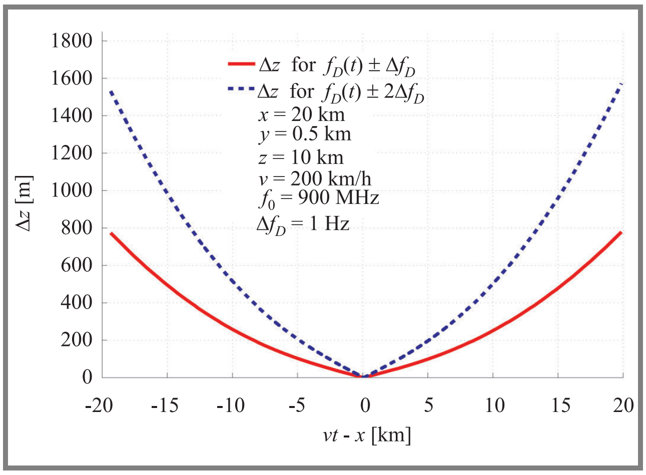

Fig. 8. Courses of changes Δz as function of the $vt - x$.

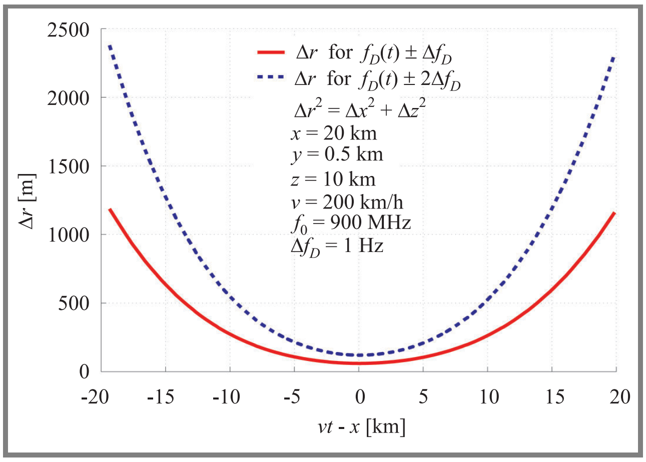

Fig. 9. Courses of changes Δr as function of the $vt - x$.

5. Conclusions

The methodology introduced in this paper shows that the Doppler frequency offset value could be used for radio signal sources location. Coordinates of these sources can be calculated basing on the frequency offset of the received signal at two different measurement moments. The limited area of effective measurement of frequency, where source coordinates can be determined, results in the Doppler curves course character. These limitations impose measurements exercise methodology in practice. The location range area is determined by change of measurement parameters, e.g., velocity value and movement trajectory of receiver.

Fundamentally, location precision is conditioned by: precision of the frequency offset value and the Doppler frequency range as well as the distance from transmitter to receiver at a moment of the measurement. In practice, presented method is determined by foregoing factors.

References

[1] *An Introduction to Mobile Positioning.* Mobile Lifestreams Ltd., 1999.

[2] Y. Zhao, "Standardization of mobile phone positioning for 3G systems", *IEEE Commun. Mag.*, vol. 40, iss. 7, pp. 108–116, 2002.

[3] M. Vossiek, L. Wiebking, P. Gulden, J. Weighardt, and C. Hoffmann, "Wireless local positioning – concepts, solutions, applications", in *IEEE Radio Wirel. Conf. RAWCON'03*, Boston, USA, 2003, pp. 219–224 (or *IEEE Microw. Mag.*, vol. 5, iss. 4, pp. 77–86, 2003).

[4] C. Ziółkowski and J. Rafa, "Influence of transmitter motion on received signal parameters – analysis of the Doppler effect", *Wave Mot.*, vol. 45, iss. 3, pp. 178–190, 2008.

[5] C. A. Balanis, *Antenna Theory: Analysis and Design.* New York: Wiley, 1997.

[6] M. Kayton and W. R. Fried, *Avionics Navigation Systems.* New York: Wiley, 1969.

[7] C. Ziółkowski, J. Rafa, and J. M. Kelner, "Sposób namiaru i lokalizacji źródeł przestrzennych fal radiowych z wykorzystaniem efektu Dopplera" (Method of bearing and location of the space radio wave sources using Doppler effect), Polish Patent Office, no. P381154, Warsaw, Poland, 27th Nov. 2006 (in Polish).

Piotr Gajewski was born in Poland in 1946. He received M.Sc., Ph.D. and D.Sc. degrees from Military University of Technology, Warsaw, Poland, in 1970, 1978 and 2002, respectively, all in radio communications. He was engaged in many research projects, especially in the fields of wireless and mobile communications, satellite communications, antennas and propagation, electromagnetic compatibility, communications and information systems engineering, communications and information systems modeling and simulations, interoperability, communications intelligence and electronics warfare. He is an author or co-author of over 180 scientific papers and research reports.

e-mail: pgajewski@wel.wat.edu.pl
Telecommunications Institute
Faculty of Electronics
Military University of Technology
Gen. S. Kaliskiego st 2
00-908 Warsaw, Poland

Jan Marcin Kelner was born in Poland in 1977. He received M.Sc. degree from the Military University of Technology, Warsaw, Poland, in 2001, in physical phenomena computer modeling. He was engaged in several research projects, especially in the fields of radio communications systems engineering, mobile communications and electromagnetic compatibility. He is an author or co-author of over 15 scientific papers and research reports.

e-mail: jkelner@wel.wat.edu.pl
Telecommunications Institute
Faculty of Electronics
Military University of Technology
Gen. S. Kaliskiego st 2
00-908 Warsaw, Poland

Cezary Ziółkowski was born in Poland in 1954. He received M.Sc. and Ph.D. degrees from the Military University of Technology, Warsaw, Poland, in 1978 and 1993, respectively, all in telecommunication engineering. In 1989 he received M.Sc. degree from the University of Warsaw in mathematics, speciality – analysis mathematics applications. He was engaged in many research projects, especially in the fields of radio communications systems engineering, radio waves propagations, radio communication network resources management and electromagnetic compatibility in radio communication systems. He is an author or co-author of over 100 scientific papers and research reports.

e-mail: cziolkowski@wel.wat.edu.pl
Telecommunications Institute
Faculty of Electronics
Military University of Technology
Gen. S. Kaliskiego st 2
00-908 Warsaw, Poland