Focusing Operators
and Tracking Moving Wideband Sources

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Abstract—In this paper, the localization of wideband source with an algorithm to track a moving source is investigated. To locate the wideband source, the estimation of two directions of arrival (DOA) of this source from two different arrays of sensors is used, and then, a recursive algorithm is applied to predict the moving source's position. The DOA is estimated by coherent subspace methods, which use the focusing operators. Practical methods of the estimation of the coherent signal subspace are given and compared. Once the initial position is estimated, an algorithm of tracking the moving source is presented to predict its trajectory.

Keywords—DoA, localization, tracking, wideband signal.

1. Introduction

In many problems in signal processing, the received data can be considered as a superposition of a finite number of elementary source signals and an additive noise. Generally, in a multi-sensor environment application, such as sonar, radar, and underwater acoustics, the objective is the estimation of the number and the direction of arrival (DOA) or radiating sources bearing. In the 1990s, an eigenstructure based methods [1]–[5] yield resolution have been proposed to the problem of wideband sources bearing estimation. Over the last years other approaches were published [6]–[14], some of these propositions are popular, such as Test of Orthogonality of Projected Subspaces (TOPS) [8]. TOPS is the most recent wideband DOA method and estimates DOAs by measuring the orthogonal relation between the signal and noise subspaces of multiple frequency components of the sources. However, TOPS is different from coherent methods that form a general coherent correlation matrix using focusing angles and it is different from usual incoherent methods since it takes advantage of subspaces from multiple frequencies simultaneously. Another well-known approach, the Weighted Average of Signal Subspaces (WAVES) [13] combines a robust near-optimal data-adaptive statistic. This method is used with an enhanced design of focusing matrices to ensure statistically robust preprocessing of wideband data. The DOA estimation is used to locate a mobile systems, Hence it is great deal of interest in wireless communication systems [15]–[17]. Most conventional location techniques, based on the angle, or the time of signal arrival, use the signal being transmitted by a mobile to determine its location [15]. For practical reasons, the reception points

are usually existing base stations [15], [16] using array of sensors. However, to estimate the direction of arrival of a mobile, the line of sight (LOS) is required. In the case of non-line of sight (NLOS), the accuracy drastically decreases.

In the classical array processing (narrow-band signals), the parameter of interest is the direction of arrival of the radiating sources from the recorded data [1]. However, all the wideband array processing methods for DOA are based on the well-studied algorithms for narrow band sources. Indeed, two approaches were developed in the 1990s: some methods sample the frequency spectrum to create narrow band signals [2], then at each frequency bin a narrow band signal methods are used to estimate the DOA – that is the incoherent method. The other approach is the coherent signal subspace method. The cross-spectral matrices at different frequency bins are combined to form an average cross-spectral matrix. Then, the high-resolution algorithm, such as Music [3]–[5], is used to estimate the DOA. In the coherent signal subspace method, the combination of the narrow-band samples is done through the observation vector or the cross-spectral matrices, this is called focusing. The focusing operator is a matrix that transforms the location matrix at a sampling frequency to the location matrix at the focusing frequency. An improved version of the coherent signal subspace method is also reported in the [5] that uses unitary focusing matrices. A two-sided transformation is applied on the data spectral matrices.

To locate mobile station, the DOA geolocation method [17] uses simple geometrical principle to solve transmitter position. The receiver measures the direction of received wideband signals from the target transmitter using antenna arrays. DOA measurements at two receivers will provide a position fix but the accuracy of the position estimation depends on the transmitter location with respect of the two receivers, multi-path propagation etc. As a result, more than two receivers are normally needed to improve the position accuracy.

In this paper authors propose an algorithm to perform the prediction of the trajectory of a moving source. This approach will use the ARMA modeling movement of the object. Its trajectory is described by a series of coordinates calculated, previously, from two DOAs and from two arrays. The convergence of this algorithm is given.

This rest of this paper is organized as follows. In Section 2 the problem formulation is given. In Section 3, some fo-

cusing operators for wideband localization are presented. Localization of DOA and mobile location is given in the Section 4. The simulation performance is presented in the Section 5.

2. Problem Formulation

The estimation of the angle of arrival for a known signal is the main function for location systems. The conventional approach for estimating the DOA in a wireless communication system is based on transmitting a known signal, i.e. pulse (wide-band signal), and performing correlation or parametric estimations of DOA.

Let us consider an array of N sensors, which receives the waveforms generated by P wide-band sources, with M frequency bins, in the presence of an additive noise. The received signal vector in the frequency domain is given by:

$$r(f_n) = A(f_n)s(f_n) + n(f_n), \quad n = 1, \ldots, M, \qquad (1)$$

where r_n is the Fourier transform of the array output vector, s_n is the $P \times 1$ vector of complex signals of P wavefronts: $s_n = [s_1(f_n), s_2(f_n), \ldots, s_P(f_n)]^T$. n_n is the $N \times 1$ vector of additive noise $n_n = [n_1(f_n), n_2(f_n) \ldots, n_N(f_n)]^T$, and $A_n(\theta)$ is the $N \times P$ transfer matrix of the source-sensor array systems with respect to some chosen reference point, given by:

$$A_n(\theta) = [A_n(\theta_1), A_n(\theta_2), \ldots, A_n(\theta_P)],$$

where $A_n(\theta_i)$ is the steering vector of the array toward the direction θ_i at frequency f_n.

For simplification reason, r_n is used instead of $r(f_n)$, the same for $A(f_n)$, $s(f_n)$ and $n(f_n)$: A_n, s_n, n_n, respectively. For example, the steering vector of a linear uniform array with N sensors is:

$$A(f_n, \theta_i) = \begin{bmatrix} 1 & e^{j\varphi_i} & e^{2j\varphi_i} \ldots & e^{(N-1)j\varphi_i} \end{bmatrix}^T,$$

where: $\varphi_i = 2\pi f_n \frac{d}{c} \sin(\theta_i)$, d – sensor spacing, θ_i – direction of arrival (DOA) of the i-th source as measured from the array broadside, c – velocity wave propagation, f_n – analysis frequency.

Assume that the signals and the additive noise are stationary and ergodic zero mean complex valued random processes. In addition, the noise is assumed to be uncorrelated between sensors, and have different variances $\sigma_i^2(f_n)$ at each sensor. It follows from these assumptions that the spatial $(N \times N)$ cross-spectral matrix of the observation vector at frequency f_n is:

$$\begin{cases} \Gamma_n = E[r_n r_n^+] \\ \Gamma_n = A_n \Gamma_n^s A_n^+ + \Gamma_n^n \\ \Gamma_n^n = \sigma_n^2 I \end{cases},$$

where $E[.]$ denotes the expectation operator, the superscript $^+$ represents conjugate transpose, $\Gamma_n^s = E[s_n s_n^+]$ is the $P \times P$ sources cross-spectral matrix, $\Gamma_n^n = E[n_n n^+(f_n)]$ is

the $P \times P$ noise cross-spectral matrix, and σ_i^2 are the noise variances at sensor i. Authors assume that the number of the sources P is supposed known. For locating the wideband sources several solutions have been proposed in the literature and are summarized as:

- the incoherent subspace methods – the analysis bandwidth is divided into several frequency bins and then at each frequency the treatment is applied and obtained results are combined to obtain the final result,

- the coherent subspace methods – the different subspaces are transformed in a predefined subspace using the focusing matrices [2]–[5].

3. Focusing Operators

The focusing matrices F_n^o's are the solution of the equations: $F_n^o A_n = A_o$, $\forall f_n \in L$, where f_o is the focusing frequency and A_o is the focusing location matrix.

The matrices A_o and A_n are function of the DOA's θ. An ordinary beamforming pre-process gives an estimate of the angles-of-arrival that can be used to form A_o. Using the focusing matrices F_n^o, the observation vectors at different frequency bins are transformed into the focusing subspace.

3.1. Coherent Signal Subspace Method

Hung and Kaveh [2] have shown that the focusing is lossless if F_n^o's are unitary transformations and proposed use of the transformation matrices obtained by the constrained minimization problem:

$$\begin{cases} \min_{F_n^o} \| A_o - F_n^o A_n \| \\ F_n^{+o} F_n^o = I \end{cases}. \qquad (2)$$

The focusing matrix F_n^o that solves Eq. (2) is $F_n^{Hung} = V_n W_n^+$, where the singular value decomposition of $A_o A_n^+$ is represented by $V_n \Sigma_n W_n^+$ [2].

3.2. Adaptive Focusing Operator

The focusing matrices [3] are based on signal subspace rotation at each frequency to the signal subspace at focusing frequency. The focusing matrix presented in [3] is $F_{Adaptive}^o = V_o V_n^+$, where V_o and V_n are the eigenvector matrices of Γ_o and Γ_n, respectively: $\Gamma_n = V_n \Delta_n V_n^+$, with Δ_n is the diagonal eigenvalue matrix of Γ_n.

3.3. TCT Method

In [5], the Two-sided Correlation Transformation (TCT) approach is based on transformation of the matrices $P_n = A_n \Gamma_{Sn} A_n^+$, where P_n is the cross-spectral matrix of the received data at the n-th frequency bin in a noise-free environment. Let P_o be the focusing noise-free cross-spectral

matrix. The TCT focusing matrices can be found by minimizing:

$$\begin{cases} \min_{F_n^o} \| P(f_o) - F_n^o P_n F_n^{o+} \| \\ F_n^{o+} F_n^o = I \end{cases}. \qquad (3)$$

It is shown [5] that the optimal solution of (3) is given by the eigenvectors of the cross-spectral matrix at the frequencies f_o and f_n. The solution of the equation system (3) is [5]:

$$F_{TCTn}^o = X_o X_n^+, \qquad (4)$$

where X_o and X_n are the eigenvector matrices of P_o and P_n, respectively, $P_n = X_n \Pi_n X_n^+$, with Π_n is the eigenvalue diagonal matrix.

3.4. Fast TCT Method

In this section, the focusing operator [4] based in the rotation of the source subspace is presented (only at the frequency f_n to the source subspace at the focusing frequency f_o). This limitation to the transformation of the signal subspace reduces the computational load, and has, almost, the same performance than the TCT method.
Let the partition of the eigenvector matrix $X_n = \left[X_n^S \mid X_n^B \right]$, where X_n^S is $(N \times P)$ of P largest eigenvectors, and X_n^B is $N \times (N-P)$ of $(N-P)$ smallest eigenvectors of the cross-spectral matrix P_n. The eigenvalues of the cross-spectral of the received data P_n is:

$$\Pi_n^S = \begin{bmatrix} \Pi_n^S & 0 \\ 0 & \Pi_n^B \end{bmatrix}, \qquad (5)$$

where Π_n^S is $P \times P$ of P largest eigenvalues, and Π_n^B is $(N-P) \times (N-P)$ of $(N-P)$ smallest eigenvalues of P_n. The proposed focusing operator is [4]:

$$F_{FTCTn}^o = X_o^S X_n^{S+}. \qquad (6)$$

Then average cross-spectral matrix is:

$$\widetilde{P}_o = \frac{1}{M} \sum_{n=1}^{M} F_n^{So} P_n F_n^{So+}.$$

It is shown in [1] that the noise and signal subspaces are orthogonal, hence $X_n^{S+} X_n^B = X_n^{B+} X_n^S = 0$.
The P eigenvectors corresponding to the P largest eigenvalues of the cross-spectral matrix of the observation are orthonormalized [1], so we have $X_n^{S+} X_n^S = I$. Using the above properties:

$$\widetilde{P}_o = X_o^S \left[\frac{1}{M} \sum_{n=1}^{M} \Pi_n^S \right] X_o^{S+}.$$

This formula shows that the proposed operator focuses the signal subspace into the focusing frequency f_o, all the power of the different signal subspaces of the analysis band. The eigendecomposition of $\widetilde{P}^o$ is $\widetilde{P}_o = \widetilde{X}_o \widetilde{\Pi}_o \widetilde{X}_o^+$, and the partition of the eigenvector matrix $\widetilde{X}_o$: $\widetilde{X}_o = \left[\widetilde{X}_o^S \mid \widetilde{X}_o^B \right]$.

Because $\widetilde{X}_o^S$ and $\widetilde{X}_o^B$ are orthogonal, this property is used to estimate the DOA [1].
The focusing matrices could be extracted from the received cross-spectral matrix Γ_n [4]. The partition of the eigenvector matrix V is $V_n = \left[V_n^S \mid V_n^B \right]$, where V_n^S is $(N \times P)$ of P first eigenvectors, and V_n^B is $N \times (N-P)$ of $(N-P)$ last eigenvectors.
Therefore, the proposed focusing operator is then: $F_{FTCTn}^o = V_o^S V_n^{S+}$

4. TOPS Method

The test of orthogonality of projected subspaces (TOPS) [8] estimates DOAs by measuring the orthogonal relation between the signal and the noise subspaces of multiple frequency components of the sources. TOPS can be used with arbitrary shaped one-dimensional (1D) or two-dimensional (2D) arrays. Unlike other coherent wideband methods, such as the coherent signal subspace method and WAVES, this method doesn't require any preprocessing for initial values. This algorithm is summarized as follows:

1. Divide the sensor output into M identical sized blocks.

2. Compute the temporal Discrete Fourier Transform (DFT) of the M blocks.

3. For the m-th block, select $x_{m,k}$, at preselected ω_k, where $k = 0, 1, \ldots, K-1$ and $m = 0, 1, \ldots, M-1$.

4. Compute the signal subspace $V_{s,1}$ and the noise subspace $V_{n,k}$ for $k = 1, \ldots, K-1$ by Singular Value Decomposition (SVD) of estimated cross spectral matrices $\Gamma_{n,k}$.

5. Generate for each hypothesized DOA φ:
   $\Pi_i(\varphi) = I - (a_i(\varphi)^+ a_i(\varphi))^{-1} a_i(\varphi) a_i(\varphi)^+$,
   $V_{s,i}(\varphi)^\ddagger = \Pi_i(\varphi) V_{s,i}(\varphi)$,
   $D(\varphi) = [V_{s,1}^{+\ddagger} V_{n,1} \mid V_{s,2} + \ddagger V_{n,2} \mid, \ldots, \mid V_{s,K-1} + \ddagger V_{n,K-1}]$.

6. Estimate $\theta = \arg\max_\varphi \dfrac{1}{\sigma(\varphi)}$, where $\sigma(\varphi)$ is the smallest singular value of $D(\varphi)$; the estimation is now to find P local maxima by doing a one-dimensional search.

TOPS method is different from coherent methods that form a general coherent correlation matrix using focusing angles. It is also different from usual incoherent methods since it takes advantage of subspaces from multiple frequencies simultaneously.

5. Mobile Localization

The FTCT algorithm is summarized as follows. First, an ordinary estimator is used to scan the space and find an initial estimate of the DOA.

1. Form $\widehat{A}_n$ and estimate

$$\widehat{\Gamma}_n^S = \left(\widehat{A}_n^+\widehat{A}_n\right)^{-1}\widehat{A}_n^+\left[\widehat{\Gamma}_n^{NS}\right] \times \widehat{A}_n\left(\widehat{A}_n^+\widehat{A}_n\right)^{-1},$$

where $\widehat{\Gamma}_n^{NS}$ is the noiseless received data.

2. Obtain the average the source cross-spectral matrices:

$$\widetilde{\Gamma}_o^S = \frac{1}{M}\sum_{n=1}^{M}\widehat{\Gamma}_n^S.$$

3. Estimate the cross-spectral matrix of the received data: $\widetilde{\Gamma}_o = \widehat{A}_o\widehat{\Gamma}_o^S\widehat{A}_o + \widehat{\sigma}_o^2 I$.

4. Find $\widetilde{P}_o = \widehat{A}_o\widetilde{\Gamma}_o^S\widehat{A}_o^+$, and $\widehat{P}_n = \widehat{\Gamma}_n - \widehat{\sigma}_n^2 I, n = 1,\ldots,M$.

5. Determine the focusing operator. Multiply these matrices by the sample cross-spectral matrices, and average the results: $\widetilde{P}_o = X_o^S\left[\frac{1}{M}\sum_{n=1}^{M}\Pi_n^S\right]X_o^{S+}$.

6. Apply a localization method e.g. MUSIC [1] to find the DOA of the source.
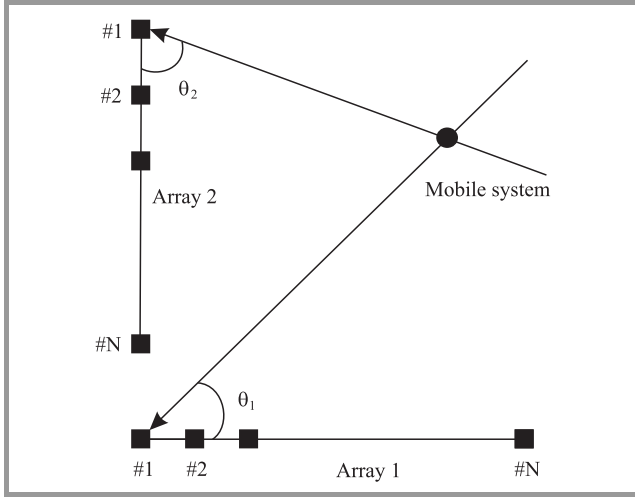

Fig. 1. Geolocation by angulation.

Finally, once these DOAs are estimated from two arrays of sensors, as shown in Fig. 1, the angulation technique is then used to estimate the moving source position.

6. Tracking a Moving Source

It is desired to perform the trajectory prediction of a moving source on a map. This approach will use the ARMA modeling object movement. Moving source on a map is observed. Its trajectory is described by a series of coordinates stored in a file containing the following information. These coordinates were calculated from two DOAs estimated from two arrays. The goal here is to anticipate the best path of the source position and to predict the source location at to time $t+1$ using known values. We denote $\xi(t+1)$ the x and y coordinates of the source. The coordi-

nates are estimated from two DOAs estimated below, and $\widehat{\xi}(t+1|t)$ of the prediction $\xi(t+1)$ knowing the position of the object until a time t. This prediction can be considered as a filtering performed by:

$$\widehat{\xi}(t+1|t) = H(z^{-1})\xi(t),$$

with

$$H(z^{-1}) = H_0 + H_1.z^{-1} + \cdots + H_n.z^{-L},$$

where L is the filter order and H_i is the dimension of 2×2. This trajectory modeling is a modeling type Auto Regressive (AR). This prediction $\widehat{\xi}(t+1|t)$ can be rewritten as least squares formalism as:

$$\widehat{\xi}(t+1|t) = \widehat{\Omega}_t^T \varphi(t), \tag{7}$$

where $\varphi(t)$ is a vector of dimension $2(L+1)$ and $\widehat{\Omega}_t$ is a matrix $2 \times 2(L+1)$, respectively, and given by:

$$\varphi(t) = \begin{pmatrix} \xi(t) \\ \xi(t-1) \\ \ldots \\ z(t-L) \end{pmatrix} \quad \text{and} \quad \widehat{\Omega}_t = \begin{pmatrix} H_0 \\ H_1 \\ \ldots \\ H_L \end{pmatrix}.$$

Let

$$\begin{cases} \widehat{\omega}_t = col(\widehat{\Omega}_t^T) \\ \phi(t) = \varphi(t) \otimes I_2 \end{cases},$$

where $\widehat{\omega}_t = col(\widehat{\Omega}_t^T)$ is a column vector of dimension $4(L++1)$, $\phi(t) = \varphi(t) \otimes I_2$ a matrix of dimension $4(L+1) \times 2$, and $\otimes$ is the Kronecker product. The equation (7) can also be put in a more general form as:

$$\widehat{\xi}(t+1|t) = \phi^T(t)\widehat{\omega}_t. \tag{8}$$

To enable a recursive estimation of $\widehat{\omega}_t$ vector parameter and therefore a correct prediction $\xi(t+1)$ is proposed to use the least-squares algorithm with recursive forgetting factor. The adapted version of the algorithm of multivariable problem (the output is a vector with 2 dimensions) has the following form:

1. $\widehat{\xi}(t+1|t) = \phi(t)^T\widehat{\omega}_t,$

2. $\varepsilon(t+1|t) = \xi(t+1) - \widehat{\xi}(t+1|t),$

3. $K_t = P_t\phi(t)(\phi(t)^T P_t\phi(t) + \lambda I_2)^{-1},$

4. $\widehat{\omega}_{t+1} = \widehat{\omega}_t + K_t\varepsilon(t+1|t),$

5. $P_{t+1} = \frac{1}{\lambda}(P_t - K_t\phi(t)^T P_t).$

where $0 < \lambda < 1$ is the forgetting factor. The algorithm converges rapidly for high value of λ but with an important variance. However, for a very low λ one can notice the weak variance with the very slow convergence.

7. Simulation Results

To analyze the performance of the presented focusing operators, the normalized root mean-square error (NRMSE) is employed as a performance tool of the input estimates, it is given by:

$$\text{NRMSE} = \frac{1}{\|\boldsymbol{\theta}\|}\left(\frac{1}{K}\sum_{i=1}^{K}\|\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}\|^2\right)^{\frac{1}{2}}, \qquad (9)$$

where K is the number of the trials and $\hat{\boldsymbol{\theta}}_i$ is the estimated DOA from the i-th trial.

A linear array of $N = 5$ omnidirectional sensors at the base station in order to have a spatial diversity with equal inter-element spacing $d = \frac{c}{2f_o}$ is used, where f_o is the center frequency and c is the propagation velocity. The source signals are temporally stationary zero-mean bandpass white Gaus-sian processes with the center frequency $f_o = 902.5$ MHz and the same bandwidth $B_w = 25$ MHz. The distance d between two consecutive sensors at the base station is 16.62 cm, hence the total distance of the array is 66.48 cm. The noise is stationary zero-mean bandpass (the same pass-band as that the signals) white gaussian process, independent of the signals, and statistically independent and identical. One moving punctual source imping-ing from the angle $-55°$ at the array 1 and from $10°$ at the array 2, with a SNR of 3 dB is used for this simulation. Figures 2 and 3 represent the results of localization functions $f(\theta)$ of the FTCT method by two arrays.

In the second part of the simulation, the performance of TCT, FTCT, adaptive and Hung methods was compared, where the SNR varies from 0 dB to 35 dB, the results of the NRMSE are presented in Fig. 4.
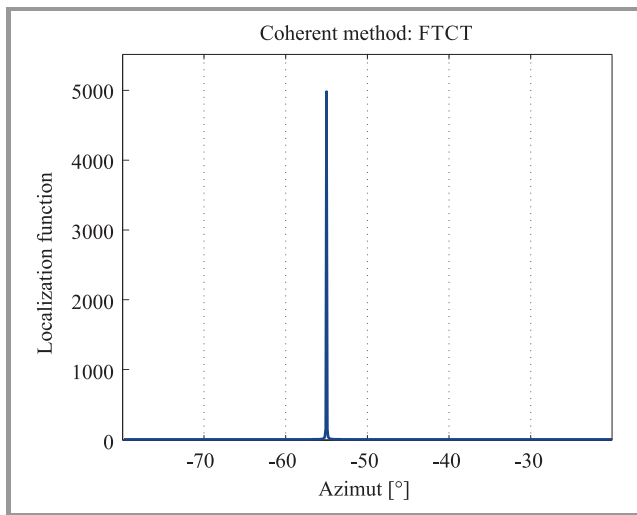

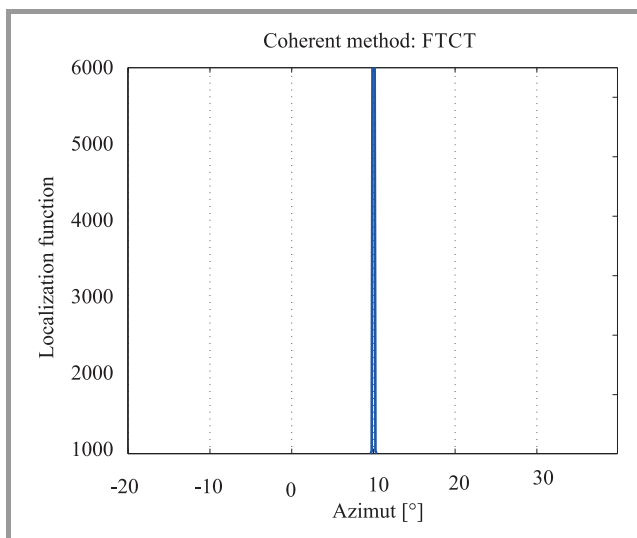*Fig. 2.* Estimation of the DOA of the source from Array 1, SNR = 3 dB.

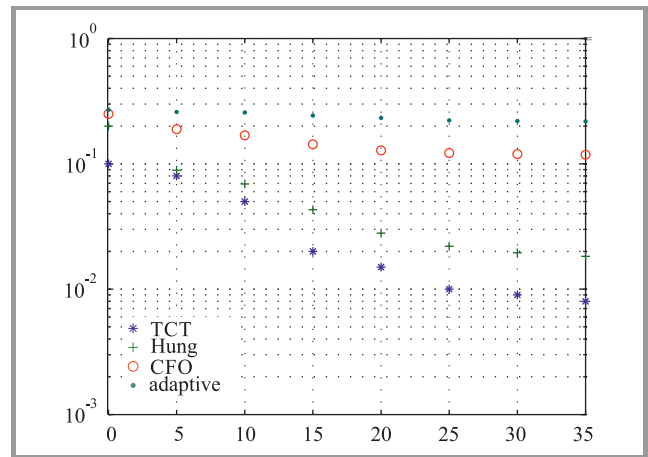*Fig. 3.* Estimation of the DOA of the source from Array 2, SNR = 3 dB.

Fig. 4. NRMSE of one mobile location (y axis) vs. SNR (x axis) for different focusing operators.
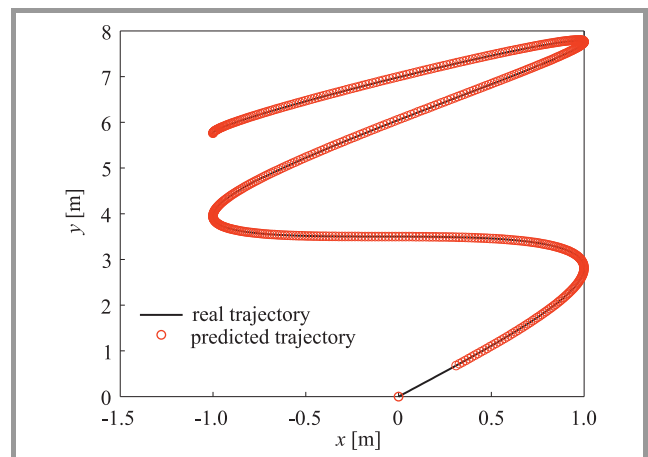

Fig. 5. Estimation of the trajectory of moving punctual source.

The TCT and Hung's operator achieve a higher perfor-mance compared to the adaptive and FTCT methods. How-ever, this last method is very interesting in term of compu-tational load because only one part of the eigenvectors are used to built the focusing operators.

In the case of moving punctual radiating source, such as a robot with single transmitting antenna, the arrays system and the moving object are in the same horizontal plane. The following parametric model for the trajectory is proposed:

$$\begin{cases} \vec{r}(t) = x(t)\vec{e}_x + y(t)\vec{e}_y \\ x(t) = \sin(\omega_1 t) \\ y(t) = x(t) + 7\sin(\omega_2 t) \end{cases}, \qquad (10)$$

where $\vec{r}(t)$ is the position of the punctual source at instant t, $\omega_1 = \pi$ rad/s and $\omega_2 = 0.5234$ rad/s, the initial position $r(t = 0) = 0$ m, corresponds to the solution of the angulation technique from the DOAs $\theta_1 = -55°$ and $\theta_2 = 10°$. The least mean square algorithm is used to predict the trajectory of the moving source with forgetting factor $\lambda = 0.99$. The results of the prediction proces, for a period $T = 3.5$ s, are given in Fig. 5 and Fig. 6. Finally, in Fig. 7, the convergence rate of the first parameter $\omega_t(1)$ is presented.
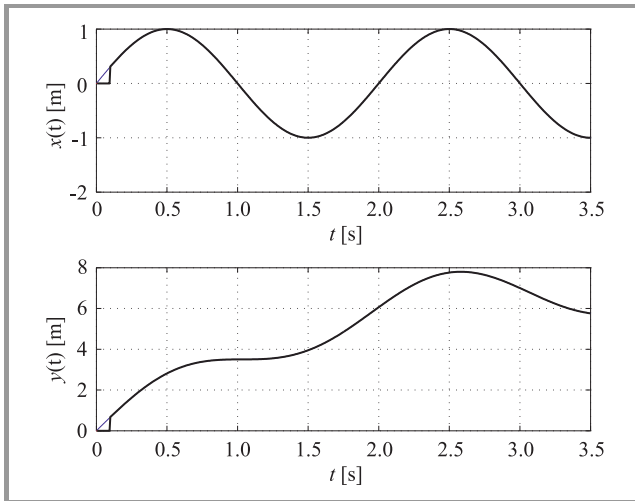

Fig. 6. *x-y* coordinates of moving punctual source.
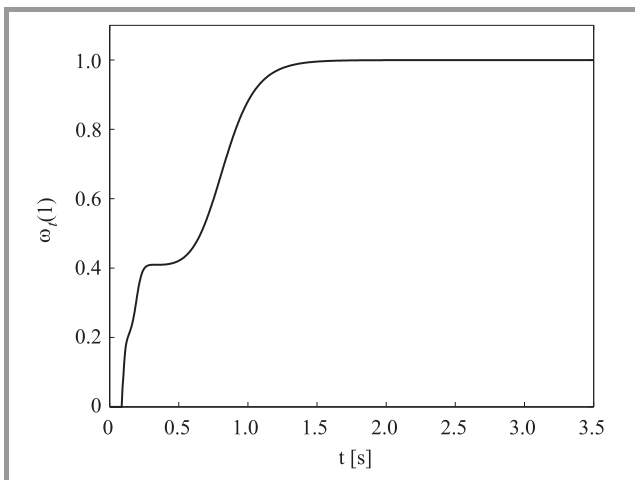

Fig. 7. Convergence of the 1st parameter $\omega_t(1)$.

8. Conclusion

In this paper a class of focusing operators were studied, and their numerical performances were evaluated. The obtained results show the efficiency of the eigenvectors focusing operators in term of accurate angles. These operators were used to estimate the DOAs of a known pulse and to locate mobile station. Once two directions of arrival of wideband source is estimated at instant t, an algorithm of LMS is used to track this moving source.

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