

Analytical Investigation on a New Approach for Achieving Deep Penetration in a Lossy Medium: The Lossy Prism

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Abstract—Recent studies highlighted deep-penetration properties of inhomogeneous waves at the interface between a lossless and a lossy medium. Such waves can be generated by means of radiating structures known as Leaky-Wave Antennas (LWAs). Here, a different approach is proposed based on the use of a lossy prism capable to generate an inhomogeneous wave when illuminated by a homogeneous wave. The lossy prism is conceived and designed thinking of Ground-Penetrating Radar (GPR). The results achieved by the lossy prism will be compared with those obtained by means of a previously designed LWA that was created with the identical objective. The approach of this paper is purely theoretical, and it aims at providing basic ideas and preliminary results useful for an innovative LWA design.

Keywords—deep-penetration, Ground Penetrating Radar, leaky-wave antennas.

1. Introduction

The penetration depth is a very important parameter in many fields of applied electromagnetism, and particularly in Ground Penetrating Radar (GPR) applications where highly lossy media are often encountered. A typical technique to increase penetration employed in GPR surveys is the reduction of the operating frequency: this implies lower resolution [1]. An alternative approach, where applicable, requires the employment of ground-coupled antennas [2]. Those antennas increase the coupling with the soil avoiding the first reflection at the interface between air and soil and potentially reducing the speed of a survey. Ground-coupled antennas do not resolve the issue of sub-soil lossy materials.

In this paper we propose a particular configuration of a structure, named lossy prism, as an alternative technique for increasing the penetration. The lossy prism is a two-dimensional structure with two non-parallel, planar and infinite interfaces proposed in [3] that allows the generation of inhomogeneous waves. The lossy prism, designed here, radiates an inhomogeneous wave in air to meet the deep-penetration theory requirements. The proposed structure is therefore a preliminary input for the development of

an air-coupled antenna, which increases the penetration depth without the need of reducing the operating frequency. The deep-penetration condition was first defined in [4] for a plane wave incoming from a lossless medium and impinging on a separation surface with a lossy medium. This condition occurs when the attenuation vector of the transmitted wave in the lossy medium is parallel to the separation surface between lossless and lossy media. Mandatory requirements for the deep-penetration condition are: inhomogeneous incident wave incoming from a lossless medium, oblique incidence, and an amplitude of the incident phase vector (or attenuation vector) greater than or equal to a given minimum value.

The article is divided into three main sections. In Section 1 a literature background on the deep-penetration effect is illustrated. Section 2 describes the lossy prism design proposed here and illustrates its potential in terms of penetration increase. Finally, conclusions are given in Section 3.

2. Overview on Previous Research Activities

2.1. Deep-penetration Condition and Large Penetration

In a lossless medium two plane-wave solutions are possible: a conventional homogeneous wave, with a real wave vector \underline{k} , and an inhomogeneous wave, where the wave vector \underline{k} is complex and can be expressed as a superposition of two real vectors, the phase vector $\underline{\beta}$ and the attenuation vector $\underline{\alpha}$; those vectors must be orthogonal to each other [5].

Let us take an inhomogeneous wave incoming from a lossless medium, said medium 1, and impinging on a lossy medium said medium 2, as illustrated in Fig. 1. The wave vector of the incident wave is $\underline{k}_1 = \underline{\beta}_1 - j\underline{\alpha}_1$ and the wave vector of the transmitted wave is $\underline{k}_2 = \underline{\beta}_2 - j\underline{\alpha}_2$. Let us now define with ξ_1 the angle that the phase vector of the incident wave $\underline{\beta}_1$ forms with the normal to the interface between air and lossy medium. The theory developed in [4], [6] demonstrates that, when ξ_1 is smaller than or equal to 45° it is

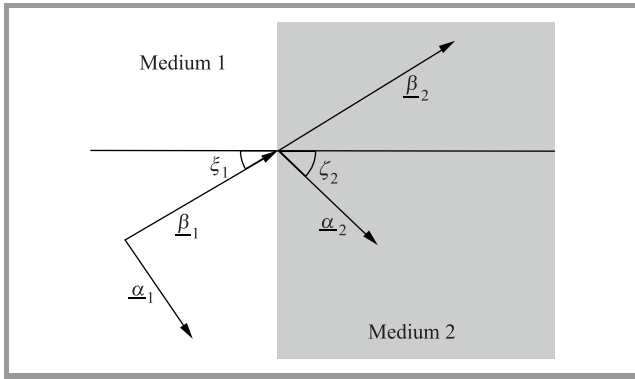


Fig. 1. Deep-penetration problem definition: inhomogeneous plane wave incoming from a lossless medium (medium 1) and impinging on the infinite and planar separation surface with a lossy medium (medium 2).

possible to find a critical value β_{1c} for the amplitude of $\underline{\beta}_1$ such that the angle ζ_2 formed by $\underline{\alpha}_2$ with the normal to the separation surface is 90° (deep-penetration condition), this happens for a critical incident angle $\xi_1 = \xi_{1c}$. For a given β_{1c} , the smaller the angle $\xi_1 < \xi_{1c}$, the worse the penetration. The deep-penetration condition can be met for smaller angles by increasing the amplitude of $\underline{\beta}_1$ (and consequently of $\underline{\alpha}_1$), anyway this condition can never be satisfied for normal incidence. Even when deep-penetration condition cannot be met, large penetration through inhomogeneous waves is always possible, because, if a homogeneous wave impinges on a separation surface with a lossy medium, the attenuation vector of the transmitted wave is necessarily orthogonal to the separation surface ($\zeta_2 = 0$). However, if the incident wave is inhomogeneous, the attenuation vector cannot be orthogonal because there is a tangential component to the interface with the lossy medium that has to be necessarily conserved ($\zeta_2 > 0$). This is due to the well-known conservation of the tangential component of the electromagnetic field at a planar boundary between two media [5].

2.2. Large EM Penetration Employing Leaky-wave Antennas

Possible physical solutions for inhomogeneous waves at infinite planar boundaries between lossless and lossy media are represented by lateral waves, surface waves and leaky waves. The last ones represent the only suitable solutions for deep penetration because they can effectively radiate in the lossy medium [7]. Leaky waves can be artificially generated by structures called leaky-wave antennas (LWAs) [8].

The large penetration achievable employing LWAs is a subject well known in the literature. In [9], [10] researchers designed LWA applicators, which guaranteed large penetration, and in [11], a wave was generated by means of a periodical, bi-dimensional, LWA structure operating at microwave frequencies (X band) to prove that the existence of an attenuation vector of the incident wave leads to large penetration also in practical applications.

In [11], the antenna proposed in [12] was designed on CST software to radiate at broadside, and to impinge on a lossy medium represented by a prism having one face parallel to the antenna aperture.

The amplitude of the electric field in the lossy medium was then compared against the E field transmitted into the same medium by a customary horn antenna.

The results of this preliminary investigation were interesting. The antenna presented a slightly larger penetration than the one produced by the horn antenna, but the deep-penetration condition could not be achieved because of the normal incidence. Moreover, even increasing the incidence angle, the deep-penetration condition could not be met. This is mainly due to the very low amplitude of the attenuation vector for the antenna chosen [12].

An alternative antenna design was proposed in [13] to allow the deep-penetration condition. The designed antenna was based on uniform and mono-dimensional LWAs, in particular, a microstrip LWA [14] derived from the Menzel antenna [15] was considered and designed using the method proposed in [16]. The antenna, originally designed by Menzel, had its maximum beam angle at 41° , with $\beta_{1n} = \frac{\beta_1}{k_0} = 1.0025$ and $\alpha_{1n} = \frac{\alpha_1}{k_0} = 0.0528$: such a value was considered very high, in comparison with values often used in LWA design, by Oliner and Lee in [14], but this value was not sufficient for deep-penetration. Therefore in [13] the design of the antenna was optimised for deep penetration, obtaining $\beta_{1n} = 1.0028$ at a maximum radiation angle of 45° . This value, according to Eqs. (12)–(13) of [4] allowed the deep-penetration condition on a medium with conductivity $\sigma_2 = 0.05$ S/m.

While we cannot exclude the possibility of realizing a conventional LWA [8], [17] that may provide even higher amplitudes of attenuation and phase vectors, the design proposed in [13] shows evident limits in the deep penetration achievable by means of a uniform LWA. Therefore, we propose a different, innovative approach that promises better results.

3. The Lossy Prism

Historically, to the best of our knowledge, leaky waves artificially generated were exclusively produced by means of leaky-wave antennas, but recent papers tried to exploit the inhomogeneous-wave generation that can be obtained by irradiating a two-dimensional lossy dielectric structure with a homogeneous wave. Such a two-dimensional structure was first called in [3] lossy prism, and presents two non-parallel, planar and infinite interfaces (see Fig. 2).

In [18], the prism was illuminated by a finite beam treated in the optical approximation in order to neglect the interaction with the wedge of the prism. Moreover, the impact of the first reflection was considered, and it was pointed out that while multiple reflections could be neglected for the lossy nature of the prism, at least the first reflection should be taken into account.

In the approximation that assumes the beam width negligible in comparison with the prism dimensions, which is the situation illustrated in [18], it is possible to find geometrical requirements that allow the avoidance of multiple reflections like the ones experienced in such a paper.

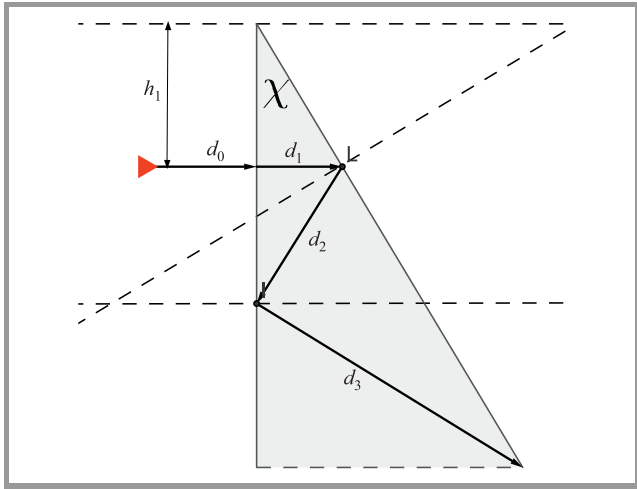


Fig. 2. Lossy prism: paths followed by direct and reflected waves when $\chi < 90^\circ$ is the angle formed by the prism wedge.

In the case of normal incidence of the homogeneous wave upon the vertical side, the transmitted wave is always normal to the separation surface [5]. With reference to Fig. 2, where d_1 is the path followed by the direct-transmitted wave from the first to the second interface, and d_2 is the path followed by the first reflection, it comes out from simple trigonometrical relations [18]:

$$d_1 = h_1 \tan \chi, \quad (1)$$

$$d_2 = \frac{d_1}{\cos 2\chi}, \quad (2)$$

where h_1 indicates the distance between the wedge of the prism and the center of the incident beam.

When the amplitude of the wedge angle χ is greater than or equal to 45° , d_2 never returns back to the illuminated edge, but, if $\chi < 45^\circ$, this happens. Therefore also a second reflection, indicated with d_3 in Fig. 2, appears. Such a wave may reflect back to the oblique interface if the angle χ is smaller than 30° . The length of d_3 can easily be computed by observing the triangle formed by d_2 and d_3 :

$$d_3 = d_2 \frac{\cos \chi}{\cos 3\chi} = \frac{[h_1 + d_1 \tan(2\chi)] \sin \chi}{\cos 3\chi}. \quad (3)$$

In Fig. 3 the normalized quantities $\frac{d_1}{h_1}$, $\frac{d_2}{h_1}$, and $\frac{d_3}{h_1}$ are plotted as a function of χ for $\chi \in]0.25^\circ[$ and it is clear that the larger the angle χ , the bigger the distance where the reflected rays will impinge on the prism edges.

An angle $\chi \geq 45^\circ$ must be chosen in order to avoid reflections. Anyway, the larger the angle χ , the higher is $\frac{d_1}{h_1}$, therefore also the attenuation introduced on the illuminating beam increases when χ increases. The prism must be

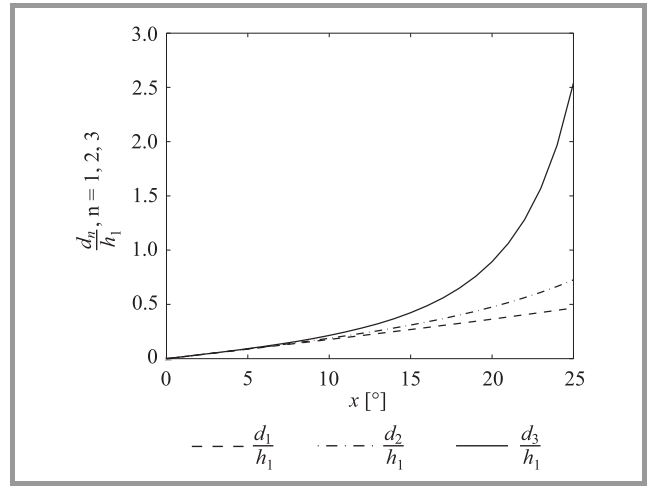


Fig. 3. Lossy prism: normalized direct wave path ($\frac{d_1}{h_1}$), normalized first-reflection path ($\frac{d_2}{h_1}$), and normalized second-reflection path ($\frac{d_3}{h_1}$) for $\chi \in [0.25^\circ]$.

built using a low-loss medium. But, if the beam is narrow enough, it is possible to accept angles even smaller than 30° . In this case the second reflection can be neglected, but the first one, which is directed toward the illuminator, remains critical (see Fig. 3), and to avoid it the beam illuminating the lossy prism must impinge obliquely on the first prism face.

The angle χ is not the only parameter that allows to control reflections. In numerical simulations reflections can be neglected choosing suitable values for the medium characteristics of the lossy prism. A similar approach was taken in [11], [13], where unitary values for both permittivity and permeability of the lossy medium were chosen, and the conductivity σ was assumed sufficiently low ($\sigma = 0.05$ S/m). This kind of medium adaptation is useful in numerical simulations, because it allows to isolate the transmission problem from the reflection problem, but such an approach may result in strong restrictions on the prototyping and manufacturing processes of the lossy prism.

In this study, we will impinge obliquely on the lossy prism and we will also choose a value for the χ angle that will allow to neglect internal reflections. The amplitudes of phase and attenuation vectors achievable employing this structure will then be compared against the corresponding values that can be obtained employing conventional LWAs. We know that, to obtain deep penetration, the amplitude of the attenuation vector needs to be greater than a minimum value [4], but LWAs are usually designed to produce efficient beams with a negligible amplitude of the attenuation vector ($\alpha_1 \ll k_0$ [8], [17]), this makes the design of a deeply penetrating antenna by means of commonly used LWAs very challenging, while the structure proposed here shows better results.

The first consideration that needs to be exposed when comparing the lossy prism and the LWA is that in the former losses are present in the material, while in the latter losses are mainly due to radiation (LWAs do not require a lossy

medium), and only a small quantity of energy is really dissipated (usually through a matched load posed at the end of the antenna).

In the following, we will neglect the effects of the prism wedge, this corresponds to assume that the lossy prism is larger than the antenna aperture. In the former assumption, also the hypothesis of infinite length for the lossy prism holds.

Equations (12)–(13) of [4] provide requirements in terms of amplitude of both phase vector and incident angle of an inhomogeneous wave incoming from a lossless medium to guarantee deep penetration on a lossy medium with given electromagnetic characteristics. An antenna designed for the deep-penetration condition needs to be able to radiate an inhomogeneous wave in a lossless medium, for instance a vacuum (air), such that the mentioned equations are satisfied. As a consequence, the structure proposed in this paper is designed to radiate in a vacuum. The amplitude of the phase vector generated by this structure is compared against the one obtained in [13] (that was also designed to radiate in a vacuum). A higher value of the phase vector, for a given incidence angle, implies higher penetration.

In the first part of this article the evaluation of the amplitude of phase and attenuation vectors shown in Fig. 4 will be studied considering exclusively the direct wave, while effects due to reflections will be analyzed at the end of this paper, in particular it will be shown that, in the proposed configuration, reflections can be neglected.

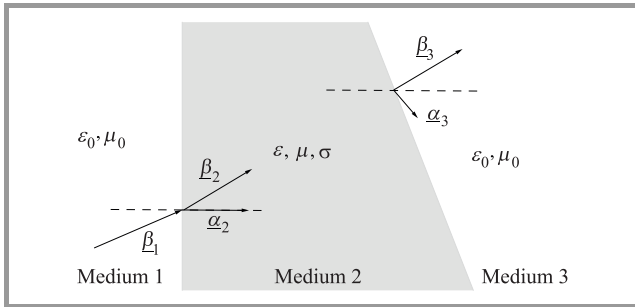


Fig. 4. Propagation in a stratified medium, in which medium 1 and medium 3 are vacuum and the inner medium (medium 2) is a lossy medium; the two interfaces are not parallel, planar and infinite.

The lossy prism structure work principle is the conservation of the tangential component. Let us consider a stratified medium as illustrated in Fig. 4, in which medium 1 and medium 3 are vacuum and medium 2 is a lossy region. The permittivity and permeability of the surrounding-vacuum region are indicated respectively with ϵ_0 and μ_0 . The complex permittivity of the lossy prism is indicated as:

$$\epsilon_{c2} = \epsilon_2 - j\sigma_2/\omega = \epsilon_0\epsilon_{r2} - \frac{j\sigma_2}{\omega},$$

where σ_2 is the conductivity, ω the angular frequency, ϵ_0 the vacuum permittivity, and ϵ_{r2} the relative permittivity. The permeability of medium 2 is $\mu_2 = \mu_0\mu_{r2}$, where

μ_0 is the vacuum permeability and μ_{r2} is the relative permeability.

Let us take a homogeneous wave incoming from medium 1 and impinging on an infinite planar interface with medium 2, with an incident angle ξ_1 . The incident wave, characterized by a phase vector $\underline{\beta}_1 = \underline{k}$ produces a transmitted wave in the medium 2 that must have attenuation vector $\underline{\alpha}_2$ that forms an angle $\zeta_2' = 0$ with the normal to the separation surface, because there is no tangential component of the incident wave that can be conserved. The transmitted wave is also characterized by a phase vector $\underline{\beta}_2$. The angle ξ_2' that this wave vector forms with the normal to the separation surface depends on the media involved and on ξ_1 (for angles definition see Fig. 5).

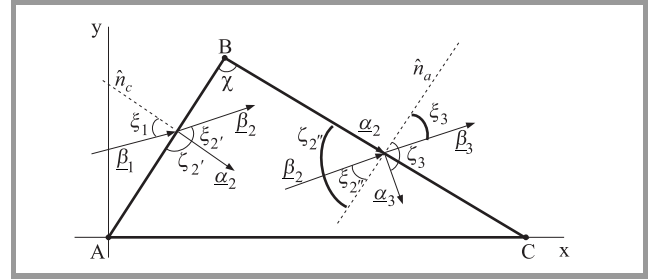


Fig. 5. Propagation in a prism in which $\chi = 90^\circ$: the wedge is shown here only to give evidence of the structure, anyway it is considered far from the source in the real scenario, BC and AB are assumed of infinite length (or sufficiently larger than the antenna aperture).

Let us now define with k_{1t} the amplitude of the tangential component to the first interface with the prism of the incident wave vector \underline{k}_1 . We can then evaluate the amplitudes of phase and attenuation vectors inside the prism employing Eqs. (13) and (14) of [19], that we report here:

$$\beta_2 = \sqrt{\frac{|k_{1t}|^2 + \text{Re}(k_2^2) + |k_{1t}^2 - k_2^2|}{2}}, \quad (4)$$

$$\alpha_2 = \sqrt{\frac{|k_{1t}|^2 - \text{Re}(k_2^2) + |k_{1t}^2 - k_2^2|}{2}}. \quad (5)$$

So, now there is a need to evaluate an expression of phase and attenuation vectors for the wave transmitted in the lossless medium 3.

Let us imagine that the inhomogeneous wave, described by Eqs. (4)–(5), reaches the other side of the lossy prism impinging on the separation surface with medium 3, which is assumed again a vacuum. This time, assuming that the separation interface is not parallel to the one between medium 1 and medium 2, the incident angle of $\underline{\beta}_2$ must be $\xi_2'' \neq \xi_2'$ and also the incident angle of $\underline{\alpha}_2$ needs to be $\zeta_2'' \neq 0$. Then a tangential component of the attenuation vector must exist at the interface and it needs to be conserved. As a result, the transmitted wave in the vacuum is, this time, inhomogeneous, and characterized by a phase vector $\underline{\beta}_3$ and an attenuation vector $\underline{\alpha}_3$ that form an angle $\theta_3 = 90^\circ$, i.e.

called ξ_3 and ζ_3 the angles that phase and attenuation vectors form with the normal to the separation surface, it needs to be $\xi_3 \pm 90^\circ = \zeta_3$.

Medium 3 does not introduce any losses, therefore the maximum value of the attenuation vector is obtained when $\underline{\alpha}_2$ is fully conserved. This happens when $\chi = 90^\circ$.

If the two faces of the lossy prism are parallel, then the incident wave in Medium 1 and the transmitted wave in medium 3 are both homogeneous. Therefore the attenuation vector $\underline{\alpha}_2$ created by the introduction of lossy medium 2, does not imply the existence of an attenuation vector $\underline{\alpha}_3$ in medium 3, and the wave produced in medium 3 is homogeneous. On the opposite, if the two faces form an angle of 90° , then $\underline{\alpha}_2$, being normal to the first face, needs to be parallel to the second one: therefore it is fully conserved. This is the case in which we fully exploit the $\underline{\alpha}_2$ vector generated in medium 2, and therefore this is the most interesting case for deep-penetration studies: the lossy prism needs to be modeled as in Fig. 5.

In Fig. 5, the wedge is drawn for clarity, but in a possible near-field simulation carried to verify the effect, the radiating aperture should not see the wedge, so that wedge effects can be avoided.

Let us apply the generalized Snell law:

$$\begin{cases} \beta_1 \sin \xi_1 = \beta_2 \sin \xi_2' \\ \beta_2 \sin \xi_2'' = \beta_3 \sin \xi_3 \\ \alpha_2 \sin \xi_2'' = \alpha_3 \sin \xi_3 \end{cases} \quad (6)$$

From the conservation of the tangential component, and from $\chi = 90^\circ$, it follows:

$$\begin{cases} \zeta_2' = 0 \\ \zeta_2'' = 90^\circ \\ \xi_2'' = 90^\circ - \xi_2' \\ \zeta_3 + \xi_3 = 90^\circ \end{cases} \quad (7)$$

Substituting the values of Eqs. (7) in Eqs. (6), we find:

$$\begin{cases} \beta_1 \sin \xi_1 = \beta_2 \sin \xi_2' \\ \beta_2 \cos \xi_2' = \beta_3 \sin \xi_3 \\ \alpha_2 = \alpha_3 \cos \xi_3 \end{cases} \quad (8)$$

Applying the dispersion relation to the media of interest [5]:

$$\begin{cases} \beta_1^2 = k_0^2 = \omega^2 \epsilon_0 \mu_0 \\ \beta_3^2 - \alpha_3^2 = \omega^2 \epsilon_0 \mu_0 \\ \beta_2^2 - \alpha_2^2 = \omega^2 \mu_2 \epsilon_2 \\ \beta_2 \alpha_2 \cos \xi_2' = \frac{\omega \mu \sigma_2}{2} \end{cases} \quad (9)$$

Now, we can put the second and the third of Eqs. (8) in the fourth of (9), obtaining:

$$\beta_3 \alpha_3 \sin(2\xi_3) = \omega \mu_2 \sigma_2 \quad (10)$$

The above equation allows to evaluate the amplitude of the ξ_3 angle, the expression of which was already found in

Eq. (12) of [4]. The amplitude of ξ_3 is reported here for completeness:

$$\xi_3 = \frac{1}{2} \arcsin \frac{\omega \mu_2 \sigma_2}{\alpha_3 \beta_3} \quad (11)$$

Putting the second of Eqs. (9) in Eq. (10), instead, values of β_3 and α_3 can be found as a function of ξ_3 and the conductivity of the medium 2. For β_3 it is:

$$\frac{\beta_3}{k_0} = \frac{1}{\sqrt{2}} \sqrt{1 + \sqrt{1 + \left[\frac{2\sigma_2}{\omega \epsilon_0 \sin(2\xi_3)} \right]^2}} \quad (12)$$

having assumed $\mu_2 = \mu_0$ (non-magnetic medium). These equations are the complementary of Eqs. (12)–(13) presented in [4]. In that paper the incidence from lossless to lossy media was presented, here the opposite.

The value for $\frac{\alpha_3}{k_0}$ simply follows from the second of Eqs. (9):

$$\frac{\alpha_3}{k_0} = \frac{1}{\sqrt{2}} \sqrt{\sqrt{1 + \left[\frac{2\sigma_2}{\omega \epsilon_0 \sin(2\xi_3)} \right]^2} - 1} \quad (13)$$

The inhomogeneous wave generated in medium 3 has larger β_3 and α_3 values for higher σ_2 values. Therefore, in principle, if there is enough power provided to the lossy prism, it is sufficient to increase the σ_2 value to obtain the wished β_3 amplitude.

In particular, we can compare the results obtained through this lossy prism configuration against the antenna presented in [13]. The minimum value of β that allows deep-penetration effect is found by imposing $\sin(2\xi_3) = 1$, i.e. $\xi_3 = 45^\circ$. In this condition, putting $\sigma_2 = 0.008$ S/m at a frequency of 12 GHz we obtain $\beta_{3n} = \frac{\beta_3}{k_0} \geq 1.00282$. Larger values can be obtained either increasing the angle ξ_3 (by means of suitably impinging with the angle ξ_1), or increasing the conductivity of the lossy prism.

The small amplitude of σ_2 obtained is sufficient to guarantee a larger phase vector amplitude than the one obtained with the microstrip leaky-wave antenna optimized for deep-penetration in [13].

Finally, the angle ξ_1 can be analytically determined as a function only of the quantities in the medium 3. Squaring the first and the second of Eqs. (8) and summing them together yields:

$$\beta_1^2 \sin^2 \xi_1 + \beta_3^2 \sin^2 \xi_3 = \beta_2^2,$$

but $\beta_1^2 = k_0^2$ and $\beta_2^2 = \omega^2 \mu_2 \epsilon_2 + \alpha_3^2 \cos^2 \xi_3$, so, in the case in which medium 3 is non magnetic:

$$\xi_1 = \arcsin \sqrt{\epsilon_{r2} + \beta_{3n}^2 \cos(2\xi_3) - \cos^2 \xi_3} \quad (14)$$

having introduced the normalized quantity β_{3n} defined as $\beta_{3n} = \frac{\beta_3}{k_3} = \frac{\beta_3}{k_0}$.

For the illustrated scenario, where $\xi_3 = 45^\circ$ is wished, it is:

$$\xi_1 = \arcsin \sqrt{\epsilon_{r2} - 0.5} \quad (15)$$

The above condition is valid for every ε_{r2} such that $0.5 \leq \varepsilon_{r2} \leq 1.5$. In particular, if $\varepsilon_{r2} = 1$, the incident angle is $\xi_1 = 45^\circ$ as it should be expected due to the continuity of the dielectric constant in this scenario.

In Fig. 6 it is illustrated how the radiation angle is dependent on the normalized amplitude of the attenuation vector $\beta_{3n} \in [1.1, 1.8]$ when $\varepsilon_{r2} = 1$. The curve $\xi_3 = \xi_3(\xi_1)$ tends to a constant value when β_{3n} increases (in the case considered, for which $\varepsilon_{r2} = 1$, this value corresponds to 45°).

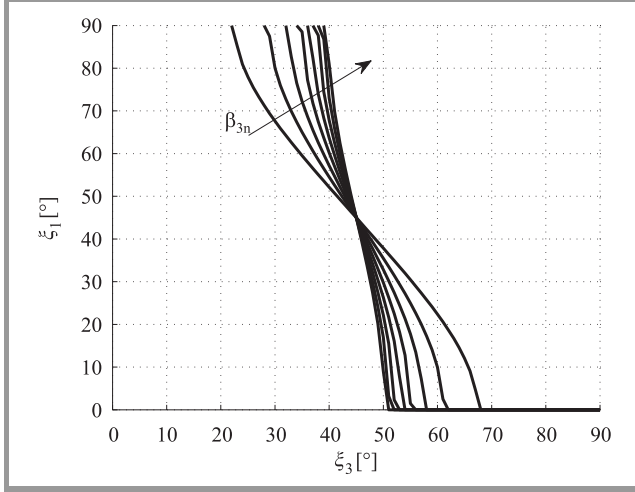


Fig. 6. Curves described by ξ_3 when ξ_1 varies from 0 to 90° and $\beta_{3n} \in [1.1; 1.8]$, being $\varepsilon_{r2} = 1$.

The condition $0.5 \leq \varepsilon_{r2} \leq 1.5$ can be satisfied properly choosing the material. We can conclude therefore that $\xi_3 = 45^\circ$ does not represent an issue, and can always be found. This condition represents, according to [4], the incident angle that guarantees maximum penetration in a lossy medium, therefore we can pose a lossy medium parallel to the separation surface between medium 2 and medium 3 and expect maximum-penetration condition, reproducing the configuration that we proposed through the microstrip leaky-wave antenna in [13]. This setup represents, in fact, the easiest scenario for experimenting the deep penetration through numerical simulations.

For some GPR applications, the condition $\xi_3 = 0$ is required. This requirement does not allow the deep-penetration condition, but, employing inhomogeneous waves, a larger penetration is still achieved. Imposing $\xi_3 = 0$ in Eq. (14), the following is obtained:

$$\begin{aligned} \xi_1 &= \arcsin \sqrt{\varepsilon_{r2} + \beta_{3n}^2 - 1} \\ &= \arcsin \sqrt{\varepsilon_{r2} + \alpha_{3n}^2}. \end{aligned} \quad (16)$$

From Eq. (16), we can see that the ξ_1 angle is real only for materials such that $-(1 - \alpha_{3n}^2) \leq \varepsilon_{r2} \leq 1 - \alpha_{3n}^2$, and achievable at the microwave frequencies through metamaterials because they can expose relative permittivities either negative or smaller than 1 [20]. Note that $\xi_3 \approx 0$ can also be guaranteed by $\varepsilon_{r2} = 1$, if α_{2n} is small enough.

It is worth mentioning that the former equation is valid only as a limit, and therefore $\xi_3 = 0$ does not strictly represent a valid solution for Eqs. (12)–(14) because such condition would come from a multiplication by zero in Eqs. (8) and (10). $\xi_3 = 0$ can be studied, instead, imposing $\alpha_3 = \alpha_2$ (for the conservation of the tangential component). From the second of Eqs. (8), it follows $\xi_2'' = 0$, and therefore, $\xi_2' = 90^\circ$, which is possible when ξ_1 is the critical angle for total reflection. Therefore such a wave does not penetrate the prism.

In Fig. 7, different curves are shown for $\varepsilon_{r2} \in [0.2, 2]$ and $\beta_{3n} = 1.2$. In particular, we can see that the smaller is ε_{r2} , the closer ξ_3 is to broadside radiation (note that $\xi_3 = 0$ for broadside radiation while $\xi_3 = 90^\circ$ for a generated surface wave).

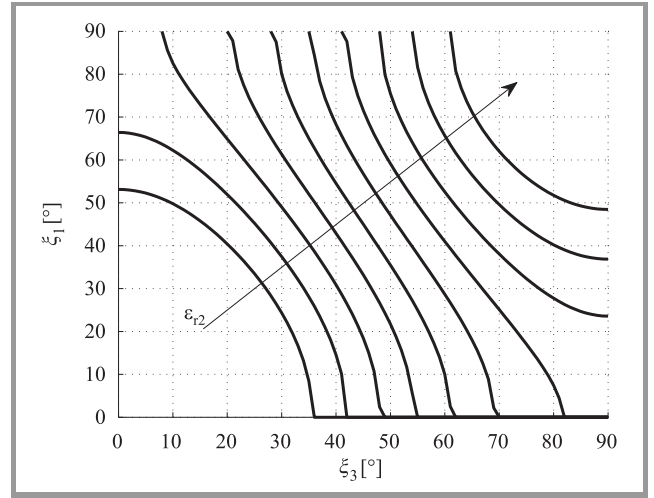


Fig. 7. Curves described by ξ_3 when ξ_1 varies from 0 to 90° and $\varepsilon_{r2} \in [0.2, 2]$, being $\beta_{3n} = 1.2$.

Let us finally study the effects of the reflections produced at the interface between medium 2 and medium 3: the incident angle at such interface is $\xi_2'' = 90^\circ - \xi_2'$; consequently, because of Snell law, the reflected angle is equal, again, to $90^\circ - \xi_2'$. It follows that the angle formed by the incident and reflected waves is $\theta = 180^\circ - 2\xi_2'$. The reflected wave hits again the interface between media 1 and 2 only when $\theta + \xi_2' < 90^\circ$, therefore $180^\circ - \xi_2' < 90^\circ$, i.e. $\xi_2' > 90^\circ$. Hence, it is possible to confirm that with this configuration the reflected wave does not return back to the illuminated interface. Moreover, in the assumption of infinite edges, the reflected wave never hits the edge \overline{AC} of Fig. 5, so the edge \overline{AB} of Fig. 5 experiences only the presence of the direct wave. In normal scenarios, the edges \overline{BC} and \overline{AB} will need to be chosen long enough to avoid that the reflection occurring on \overline{AC} would reach the \overline{AB} and \overline{BC} edges, again.

In the entire paper, we analyzed the prism from the point of view of the generated field. We optimized the prism structure to have negligible internal reflections, and to reduce the amplitude of multiple lobes of the radiated field. Doing so, we always neglected the wave eventually reflected by

the edge directly illuminated by the source. In applicative scenarios this first reflection cannot be neglected because it causes additional dispersion of power and, possibly, noise on the receiver, if this is in the path of the reflected wave. The noise in the receiver is avoided for $\xi_1 > 0$ if the receiver is small enough. The operative condition $\epsilon_{r2} < \frac{4}{3}$ was found, and it was also highlighted that a value of conductivity σ_2 of few mS/m is sufficient to guarantee higher penetration than the one obtained through a conventional LWA. The choice of a lossy prism medium such that $\epsilon_{r2} = 1$, and σ_2 of few thousands of S/m, can allow to neglect the reflection from the first side of the prism independently of the size of the transmitter and the incident angle ξ_1 .

4. Conclusions

An innovative approach, based on the use of a lossy prism for increasing the penetration of electromagnetic waves in lossy media, was presented. This approach promises to guarantee deeper penetration than the one achievable through conventional leaky-wave antennas. In particular, the amplitudes of phase and attenuation vectors can be controlled not only operating on the angle of the prism, but also on its conductivity. This allows to obtain good penetration conditions even for values close to the normal incidence (note, anyway, that the attenuation vector in the lossy medium to be penetrated will hardly be parallel to the separation surface in this case).

In this preliminary study, a finite beam behaving as a plane wave was assumed as excitation, while a realistic feeding or guiding structure should be considered.

The lossy prism also produces losses, the effects of which may conflict with the deep-penetration property of the generated wave.

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