Call-level Analysis of a Two-Link Multirate Loss Model based on a Convolution Algorithm

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Abstract—We consider a two-link system that accommodates Poisson arriving calls from different service-classes and propose a multirate teletraffic loss model for its analysis. Each link has two thresholds, which refer to the number of in-service calls in the link. The lowest threshold, named support threshold, defines up to which point the link can support calls offloaded from the other link. The highest threshold, named offloading threshold, defines the point where the link starts offloading calls to the other link. The adopted bandwidth sharing policy is the complete sharing policy, in which a call can be accepted in a link if there exist enough available bandwidth units. The model does not have a product form solution for the steady state probabilities. However, we propose approximate formulas, based on a convolution algorithm, for the calculation of call blocking probabilities. The accuracy of the formulas is verified through simulation and found to be quite satisfactory.

Keywords—*call blocking, convolution, loss model, offloading, product form.*

1. Introduction

Quality of service (QoS) mechanisms are necessary in contemporary communication networks in order to provide the required bandwidth needed by calls. In the case of call-level traffic in a single link, modeled as a loss system, such a QoS mechanism is a bandwidth sharing policy [1]. The simplest bandwidth sharing policy is the complete sharing (CS) policy, where a new call is accepted in the system if there exist enough available bandwidth units (b.u.). Otherwise, call blocking occurs. The simplest teletraffic loss model that adopts the CS policy is the classic Erlang model [1]. In this model, the call arrival process is Poisson, while each call requires one b.u. to be accepted in the system. An accepted call has a generally distributed service time. The fact that call blocking probabilities (CBP) are calculated via the classic Erlang B formula has led to numerous extensions of Erlang's model for the call-level analysis of wired (e.g. [2]–[16]), wireless (e.g. [17]–[28]), satellite (e.g. [29]–[31]) and optical networks (e.g. [32]–[37]).
In the work of [25], the Erlang B formula has been adopted for the determination of CBP in a two access link sys-

tem that accommodates Poisson arriving calls of a single service-class. Each access link is modelled as a loss system (i.e. no queueing is permitted) and has two thresholds, which refer to the number of in-service calls in the link. The lowest threshold, named support threshold, defines up to which point the access link can support calls offloaded from the other access link. The highest threshold, named offloading threshold, defines the point where the access link starts offloading calls to the other access link. By the term offloaded call, we refer to a call that initially arrived in a link, but is served by the other link, if there exist available b.u. The model of [25] does not have a product form solution (PFS) for the steady state probabilities. This is due to the fact that the offloading mechanism destroys local balance (LB) between adjacent states (states that differ only by one call) of the system. To calculate the various performance measures of the system, e.g. CBP or link utilization, either a linear system of global balance (GB) equations should be solved or an approximate method that relies on the independence between the links and the classic Erlang B formula can be adopted. The system of GB equations leads to an accurate calculation of the performance measures but it requires the knowledge of the state space of the two-link system. Such a state space may consist of millions of states if the capacity of the links is high. Thus, the method of solving the GB equations can only be applied in small (tutorial) systems [38]. On the other hand, the link independence assumption and the Erlang B formula facilitate the necessary calculations.

A potential application of the offloading scheme of [25] is in the area of mobile/Wi-Fi networks. To manage the increasing traffic in mobile networks, traffic can be offloaded to Wi-Fi networks [39], [40]. To further increase the available bandwidth of Wi-Fi access links, recent research focuses on the aggregation of backhaul access link capacities and on the bandwidth sharing policies that should be adopted (see e.g. the BeWi-Fi concept that enables users in proximity to share their Internet access if their link utilization is below a threshold) [41]. The impact of such an aggregation to CBP in the case of a single service-class can be well studied by the offloading scheme of [25].

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In this paper, we extend the model of [25] to include the important case of multirate traffic, i.e. we consider a two access link system that accommodates Poisson arriving calls of different service-classes and different bandwidth-per-call requirements. The CBP calculation in the proposed two-link model under the CS policy is based on the classic Erlang Multirate Loss Model (EMLM) [42], [43], which refers to a single link. To differentiate, we name the proposed model 2EMLM. In the 2EMLM model, the determination of CBP can be done via a 3-step convolution algorithm. The latter exploits the PFS of the EMLM and the principle of independency among service-classes and, therefore, the link occupancy distribution can be determined by successively convolving the link occupancy distributions obtained for each service-class. Contrary to macro-state recursive formulas (such as the classic Kaufman-Roberts formula used for CBP calculation in EMLM [42], [43]), a convolution algorithm keeps the micro-state information of the number of in-service calls in a link. Such information is necessary when studying more complicated (than the CS policy) call admission policies (e.g. [44]–[53]).

The remainder of this paper is organized as follows: in Section 2, we review the system of [25]. In Section 3, we propose the 2EMLM and provide a convolution algorithm for CBP determination. In Section 4, we provide analytical and simulation CBP results for the proposed model. We conclude in Section 5. In the Appendix, we provide a tutorial example of the system of [25].

2. The Two-Link System with Single-Rate Traffic

We consider a system of two links with capacities C_1 and C_2 b.u., respectively. Each link accommodates Poisson arriving calls of a single service-class which require one b.u. in order to be connected in a link. Let λ_1 and λ_2 be the arrival rates in the 1st and the 2nd link, respectively. We also denote by j_1 and j_2 the occupied b.u. in the 1st and the 2nd link, respectively. Then, $0 \leq j_1 \leq C_1$ and $0 \leq j_2 \leq C_2$. Since calls require one b.u., the values of j_1, j_2 also represent the number of in-service calls in the 1st and the 2nd link, respectively.

Each link l ($l = 1,2$) has two different thresholds: the support threshold th_{1l} and the offloading threshold th_{2l}, with $th_{1l} < th_{2l}$ and $0 \leq th_{1l}, th_{2l} \leq 1$. Assuming that $\lfloor x \rfloor$ is the largest integer not exceeding x, the role of these thresholds, in the l-th link, is the following (see Fig. 1):

- If $0 \leq j_l < \lfloor th_{1l}C_l \rfloor$ then the l-th link is in a *support mode* of operation, i.e. it accepts and serves not only new calls that initially arrive in the l-th link, but also new calls offloaded from the m-th link ($m = 1,2$, $m \neq l$).

- If $\lfloor th_{1l}C_l \rfloor \leq j_l < \lfloor th_{2l}C_l \rfloor$ then the l-th link is in a *normal mode* of operation, i.e. it does not accept

calls offloaded from the m-th link. It only accepts calls that initially arrive in the l-th link.

- If $\lfloor th_{2l}C_l \rfloor \leq j_l$ then the l-th link is in an *offloading mode* of operation, i.e. a new call that initially arrives in the l-th link will be offloaded to the m-th link. If the m-th link is in *support mode* (i.e. $0 \leq j_m < \lfloor th_{1m}C_m \rfloor$) then the call will be accepted in the m-th link. If the m-th link is not in *support mode* and $j_l \leq C_l - 1$, the call will be accepted in the l-th link. Otherwise the call will be blocked and lost.
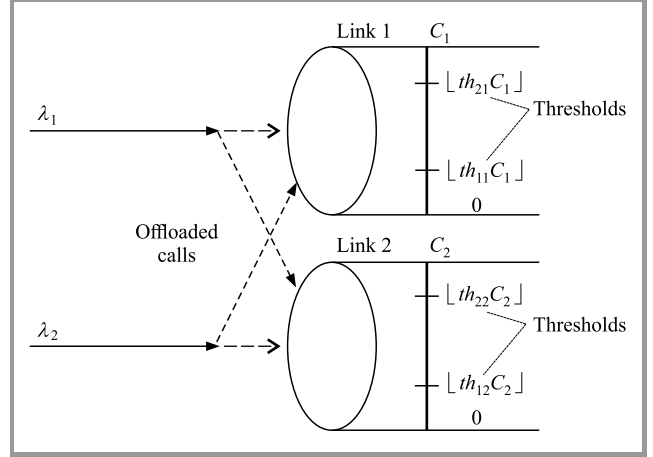

Fig. 1. The system of the two links.

Based on the above, the admission of a new call that initially arrives in the l-th link ($l = 1,2$) is summarized in the following steps:

1) If $\left(0 \leq j_l < \lfloor th_{2l}C_l \rfloor\right)$ then the call is accepted by the l-th link and remains for a generally distributed service-time with mean μ^{-1}.

2) If $\lfloor th_{2l}C_l \rfloor \leq j_l$ then:

   2a) if $0 \leq j_m < \lfloor th_{1m}C_m \rfloor$ the call is offloaded to the m-th link and remains for a generally distributed service-time with mean μ^{-1};

   2b) if $\lfloor th_{1m}C_m \rfloor \leq j_m$, the m-th link is in a *normal mode* of operation and does not support offloaded calls from the l-th link. In that case, the call will try to be accepted in the l-th link. If $j_l \leq C_l - 1$, then the call is accepted in the l-th link and remains for a generally distributed service-time. Otherwise, the call is blocked and lost without further affecting the system of the two links.

A tutorial example in the Appendix, presents in detail the call admission mechanism and the required calculations for CBP determination.

Due to the *support* and *offloading modes* of operation of the two links, the 2-D Markov chain of the system is not reversible and, therefore, LB between adjacent states (states that differ only by one call) is destroyed. Thus, the steady

state distribution, $P(j) = P(j_1, j_2)$, of this system cannot be described by a PFS. To determine the values of $P(j_1, j_2)$ (and consequently CBP) there exist two different methods. The first method provides accurate results (compared to simulation) but requires the knowledge of the state space of the system and the solution of the set of linear GB equations for each state $j = (j_1, j_2)$ expressed as *rate into state* j = *rate out of state* j:

$$\lambda_1(j_1-1, j_2)P(j_1-1, j_2) + \lambda_2(j_1, j_2-1)P(j_1, j_2-1) +$$
$$+ (j_1+1)\mu P(j_1+1, j_2) + (j_2+1)\mu P(j_1, j_2+1) =$$
$$= \lambda_1(j_1, j_2)P(j_1, j_2) + \lambda_2(j_1, j_2)P(j_1, j_2) +$$
$$+ (j_1\mu + j_2\mu)P(j_1, j_2) , \qquad (1)$$

where:

$$\lambda_1(j_1, j_2) \overset{l=1,2,\, m\neq l}{=} \begin{cases} \lambda_l + \lambda_m, & \text{if } (j_l < \lfloor th_{1l}C_l \rfloor) \\ & \cap (j_m \geq \lfloor th_{2m}C_m \rfloor) \\ 0, & \text{if } (j_l \geq \lfloor th_{2l}C_l \rfloor) \\ & \cap (j_m < \lfloor th_{1m}C_m \rfloor) . \\ 0, & \text{if } (j_1, j_2) \text{ is} \\ & \text{a boundary state} \\ \lambda_l, & \text{otherwise} \end{cases} \qquad (2)$$

Having obtained the values of $P(j_1, j_2)$, we can determine the CBP in the 1st and the 2nd link, P'_{b_1} and P'_{b_2} via Eqs. (3) and (4), respectively [25]:

$$P'_{b_1} = \sum_{j_2 = \lfloor th_{12}C_2 \rfloor}^{C_2} P(C_1, j_2), \qquad (3)$$

$$P'_{b_2} = \sum_{j_1 = \lfloor th_{11}C_1 \rfloor}^{C_2} P(j_1, C_2). \qquad (4)$$

In addition, we can calculate the total blocking probability in the system via the following weighted summation:

$$P'_b = \frac{\lambda_1}{\lambda_1 + \lambda_2}P'_{b_1} + \frac{\lambda_2}{\lambda_1 + \lambda_2}P'_{b_2}. \qquad (5)$$

Before we proceed with the second method, we emphasize that the state space determination and the solution of the set of GB equations can be quite complex even for systems of moderate size and, therefore, is only practically used for small tutorial examples (see Appendix).

The second method provides approximate CBP results by assuming that the two links operate independently from one another. Such an assumption simplifies the necessary CBP calculations. Since each independent link behaves as an Erlang loss system, the CBP in the 1st and the 2nd link can be approximated by Eqs. (6) and (7), respectively:

$$P_{b_1} = P_1(C_1)P_2(j_2 \geq \lfloor th_{12}C_2 \rfloor), \qquad (6)$$

$$P_{b_2} = P_2(C_2)P_1(j_1 \geq \lfloor th_{11}C_1 \rfloor), \qquad (7)$$

where $P_l(C_l)$ refers to the CBP in the l-th link ($l = 1, 2$) which can be determined by the Erlang B formula:

$$P_l(C_l) = \frac{\dfrac{\alpha_l^{C_l}}{C_l!}}{\displaystyle\sum_{i=0}^{C_l} \dfrac{\alpha_l^i}{i!}}, \qquad \alpha_l = \frac{\lambda_l}{\mu}. \qquad (8)$$

As far as the values of $P_l(j_l \geq \lfloor th_{1l}C_l \rfloor)$ are concerned they are given by:

$$P_l(j_l \geq \lfloor th_{1l}C_l \rfloor) = \sum_{j_l = \lfloor th_{1l}C_l \rfloor}^{C_l} P_l(j_l), \qquad (9)$$

where $P_l(j_l)$ is determined by the truncated Poisson distribution:

$$P_l(j_l) = \frac{\dfrac{\alpha_l^{j_l}}{j_l!}}{\displaystyle\sum_{i=0}^{C_l} \dfrac{\alpha_l^i}{i!}}, \qquad \alpha_l = \frac{\lambda_l}{\mu}. \qquad (10)$$

The rationale behind Eqs. (6) and (7) is that a call that initially arrives in the l-th link will be blocked if there are no available b.u. in that link and the m-th link is not in *support mode* of operation.

Finally, the total blocking probability can be determined via the following formula:

$$P_b = \frac{\lambda_1}{\lambda_1 + \lambda_2}P_{b_1} + \frac{\lambda_2}{\lambda_1 + \lambda_2}P_{b_2}. \qquad (11)$$

3. The Proposed 2EMLM

In the proposed 2EMLM, we consider again the system of the two links. Each link accommodates Poisson arriving calls of K service-classes. Calls of service-class k ($k = 1, ..., K$) require b_k b.u. in order to be connected in a link. Let λ_{1k} and λ_{2k} be the arrival rates in the 1st and the 2nd link of service-class k calls, respectively. We also denote by j_1 and j_2 the occupied b.u. in the 1st and the 2nd, respectively. Then, $0 \leq j_1 \leq C_1$ and $0 \leq j_2 \leq C_2$. Similar to Section 2, each link l ($l = 1, 2$) has a support threshold th_{1l} and an offloading threshold th_{2l}, with $th_{1l} < th_{2l}$ and $0 \leq th_{1l}, th_{2l} \leq 1$.

The call admission of a new service-class k call that initially arrives in the l-th link ($l = 1, 2$) is summarized in the following steps:

1) If $(0 \leq j_l < \lfloor th_{2l}C_l \rfloor) \cap (j_l + b_k \leq C_l)$ then the call is accepted by the l-th link and remains for a generally distributed service-time with mean μ_k^{-1}.

2) If $\lfloor th_{2l}C_l \rfloor \leq j_l$ then:

   2a) if $(0 \leq j_m < \lfloor th_{1m}C_m \rfloor) \cap (j_m + b_k \leq C_m)$ the call is offloaded to the m-th link and remains for a generally distributed service-time with mean μ_k^{-1};

   2b) if $\lfloor th_{1m}C_m \rfloor \leq j_m$, the m-th link is in *normal mode* of operation and does not support offloaded calls from the l-th link. In that case, the call will try to

be accepted in the l-th link. If $j_l + b_k \leq C_l$, then the call is accepted in the l-th link and remains for a generally distributed service-time with mean μ_k^{-1}. Otherwise, the call is blocked and lost.

To determine in an approximate but efficient way the CBP of service-class k calls we assume that the two links operate independently from one another. In that case, each independent link behaves as an EMLM system, and therefore the CBP of service-class k calls in the 1st and the 2nd link can be approximated by Eqs. (12) and (13), respectively:

$$P_{b_{1k}} = P_{1k}(C_1)P_2(j_2 \geq \lfloor th_{12}C_2 \rfloor), \tag{12}$$

$$P_{b_{2k}} = P_{2k}(C_2)P_1(j_1 \geq \lfloor th_{11}C_1 \rfloor), \tag{13}$$

where $P_{lk}(C_l)$ refers to the CBP of service-class k calls in the l-th link ($l = 1, 2$).
The values of $P_{lk}(C_l)$ in Eqs. (12) and (13) are determined by:

$$P_{lk}(C_l) = \sum_{j_l = C_l - b_k + 1}^{C_l} G_l^{-1} q(j_l), \tag{14}$$

where $q(j_l)$ refers to the unnormalized values of the link occupancy distribution of link l ($l = 1, 2$) while $G_l = \sum_{j_l=0}^{C_l} q(j_l)$ is the normalization constant.

In Eq. (14), the values of $q(j_l)$ can be recursively determined via a 3-step convolution algorithm. To describe it, let $q_{l,k}(j)$ ($k = 1, \ldots, K$) be the link occupancy distribution assuming that only service-class k exists in the link l. Then, the 2EMLM convolution algorithm is as follows:

Step 1. Determine $q_{l,k}(j)$ of each service-class k via:

$$q_{l,k}(j_l) = q_{l,k}(0)\frac{\alpha_{lk}^i}{i!}, \text{ for } 1 \leq i \leq \left\lfloor \frac{C_l}{b_k} \right\rfloor \text{ and } j_l = i \times b_k, \tag{15}$$

where $\alpha_{lk} = \lambda_{lk}/\mu_k$ is the offered traffic-load (in Erl) of service-class k calls in link l.
Step 2. Determine the aggregated occupancy distribution $Q_{(-k)}$ based on the successive convolution of all service-classes (in link l) apart from service-class k:

$$Q_{(-k)} = q_{l,1} \cdot \ldots \cdot q_{l,k-1} \cdot q_{l,k+1} \cdot \ldots \cdot q_{l,K}. \tag{16}$$

The term "successive" means that initially we convolve $q_{l,1}$ and $q_{l,2}$ to obtain $q_{l,12}$. Then we convolve $q_{l,1}$ with $q_{l,3}$ to obtain $q_{l,123}$ etc. The convolution operation between service-classes k and r is as follows:

$$q_{l,k} \cdot q_{l,r} = \left\{ q_{l,k}(0)q_{l,r}(0), \sum_{x=0}^{1} q_{l,k}(x)q_{l,r}(1-x), \ldots, \right.$$
$$\left. \ldots, \sum_{x=0}^{C} q_{l,k}(x)q_{l,r}(C_l - x) \right\}. \tag{17}$$

Step 3. Determine the values of $q(j_l)$ based on the convolution operation of $Q_{l,(-k)}$ (step 2) and $q_{l,k}$ as follows:

$$Q_{l,(-k)} \cdot q_{l,k} = \left\{ Q_{l,(-k)}(0)q_{l,k}(0), \sum_{x=0}^{1} Q_{l,(-k)}(x) \right.$$
$$\left. q_{l,k}(1-x), \ldots, \sum_{x=0}^{C_l} Q_{l,(-k)}(x)q_{l,k}(C_l - x) \right\}. \tag{18}$$

Normalizing the values of (18), we obtain the occupancy distribution $q(j_l)$, $j = 0, 1, \ldots, C_l$ via the formulas:

$$q(0) = \frac{Q_{l,(-k)}(0)q_{l,k}(0)}{G_l}$$
$$q(j) = \frac{\sum_{x=0}^{j} Q_{l,(-k)}(x)q_{l,k}(j-x)}{G_l}, \; j = 1, \ldots, C_l \tag{19}$$

As far as the values of $P_l(j_l \geq \lfloor th_{1l}C_l \rfloor)$, in Eqs. (12) and (13), are concerned, they are calculated by:

$$P_l(j_l \geq \lfloor th_{1l}C_l \rfloor) = \sum_{j_l = \lfloor th_{1l}C_l \rfloor}^{C_l} G_l^{-1} q(j_l), \tag{20}$$

where $q(j_l)$ is determined via (19).
Finally, we propose the following formula for the total blocking probability of service-class k calls in the system of the two links:

$$P_{b_k} = \frac{\lambda_{1k}}{\lambda_{1k} + \lambda_{2k}}P_{b_{1k}} + \frac{\lambda_{2k}}{\lambda_{1k} + \lambda_{2k}}P_{b_{2k}}. \tag{21}$$

4. Numerical Examples – Evaluation

In this section, we present an application example and provide analytical and simulation results of the total CBP of the proposed model. Simulation results are derived via the Simscript III simulation language [54] and are mean values of 7 runs. As far as the reliability ranges are concerned, they are less than two orders of magnitude, and therefore are not presented in the following figures. All simulation runs are based on the generation of eight million calls per
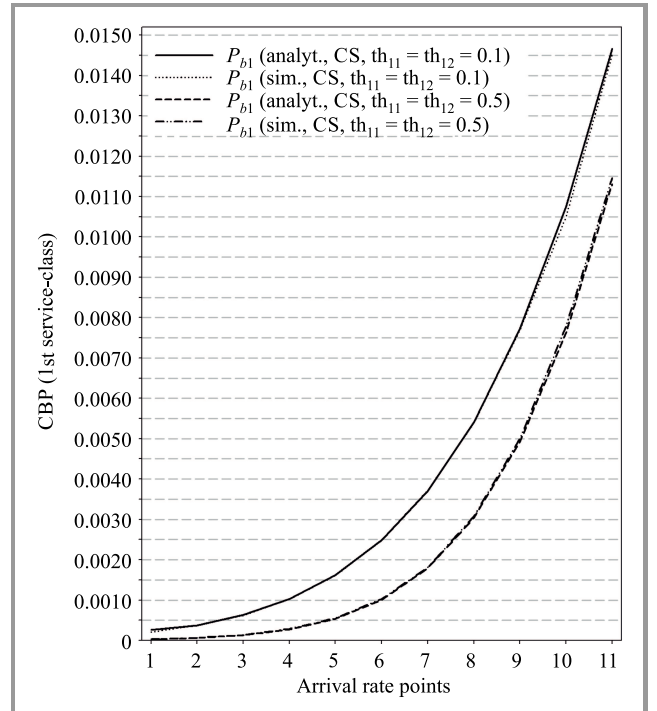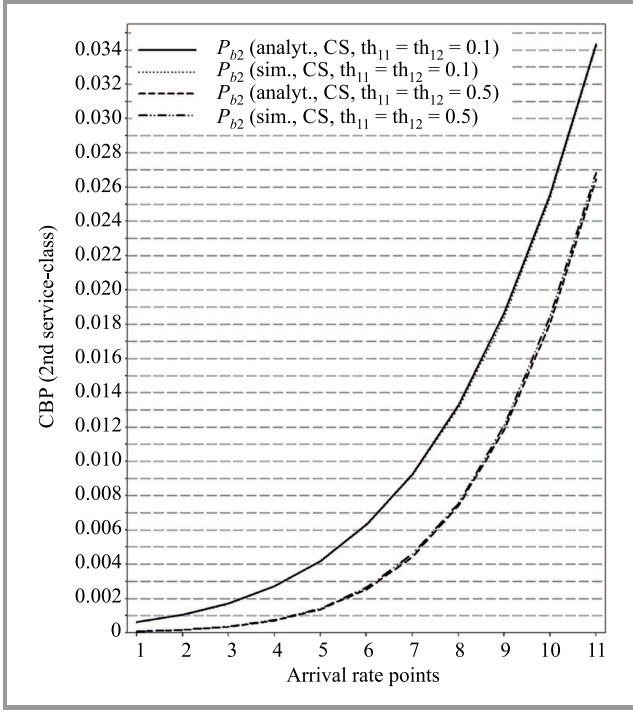

Fig. 2. CBP under the CS policy – 1st service-class.

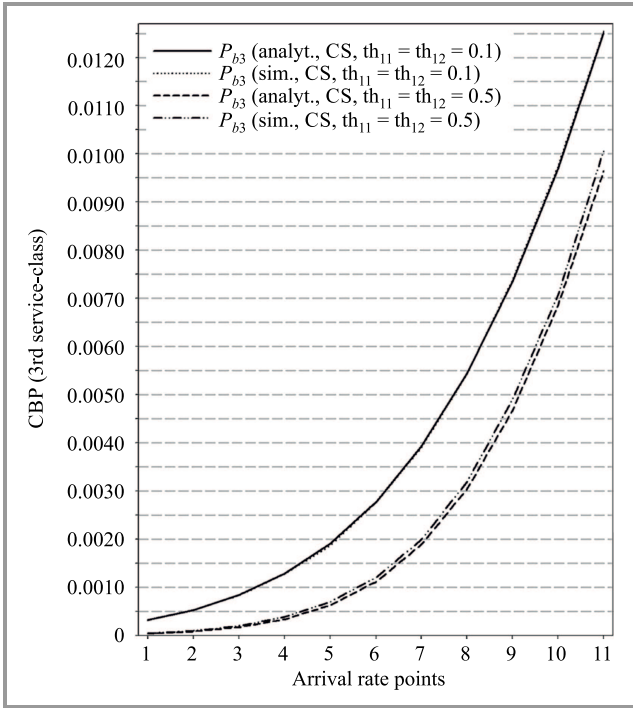**Fig. 3.** CBP under the CS policy – 2nd service-class.

Fig. 4. CBP under the CS policy – 3rd service-class.

run. To account for a warm-up period, the first 5% of these generated calls are not considered in the CBP results.

As an application example, consider a system of two links of capacities $C_1 = 30$ and $C_2 = 25$ b.u., that accommodates $K = 3$ service-classes whose calls require $b_1 = 1$, $b_2 = 2$ and $b_3 = 5$ b.u., respectively. For the 1st link, let: $\lambda_{11} = 4.0$, $\lambda_{12} = 1.0$, $\lambda_{13} = 0.5$. Similarly, for the 2nd link, let:

$\lambda_{21} = 2.0$, $\lambda_{22} = 1.0$, $\lambda_{23} = 0.5$. We also assume that $\mu_1^{-1} = \mu_2^{-1} = \mu_3^{-1} = 1.0$.

We consider two different support thresholds: 1) $th_{11} = th_{12} = 0.1$ and 2) $th_{11} = th_{12} = 0.5$. In both cases, we assume that the offloading thresholds do not alter and are equal to: $th_{21} = th_{22} = 0.7$.

In the x-axis of Figs. 2–4, λ_{11} and λ_{21} increase in steps of 1.0 and 1.0, respectively. So, point 1 is: ($\lambda_{11} = 4.0$, $\lambda_{12} = 1.0$, $\lambda_{13} = 0.5$, $\lambda_{21} = 2.0$, $\lambda_{22} = 1.0$, $\lambda_{23} = 0.5$) while point 11 is: ($\lambda_{11} = 14.0$, $\lambda_{12} = 1.0$, $\lambda_{13} = 0.5$, $\lambda_{21} = 12.0$, $\lambda_{22} = 1.0$, $\lambda_{23} = 0.5$).

In Figs. 2–4, we present CBP in the 2EMLM for the three service-classes, respectively. Figures 2–4 show that the analytical CBP results: a) are close to the simulation results and b) decrease as the support thresholds increase, an intuitively expected fact since both links cooperate with each other. Similar conclusions have been observed for systems of more than three service-classes but are not presented herein.

5. Conclusion

In this paper we propose a multirate loss model for a two-link loss system that accommodates Poisson arriving calls. A link can share a part of its capacity in order to support calls from the other link and vice versa. The proposed model does not have a PFS for the steady state distribution due to the existence of the offloading mechanism. However, we show that an approximate method does exist (based on a convolution algorithm) that provides quite satisfactory CBP results compared to simulation. As a future work, we intend to study this two-link system under the assumption that it serves different service-classes whose calls follow a quasi-random process, i.e. calls that are generated by a finite number of sources.

Appendix – Tutorial Example

Consider a system of two links with $C_1 = 6$ and $C_2 = 5$ b.u., that accommodates calls of a single service-class. Let $\lambda_1 = 4$ calls/min, $\lambda_2 = 2$ calls/min and $\mu^{-1} = 1$ min. The thresholds for this system are the following:

1st link ($l = 1$): $th_{11} = 0.2, th_{21} = 0.7$,
2nd link ($l = 2$): $th_{12} = 0.2, th_{22} = 0.7$.
Based on the thresholds' values we have:

First link
a) If $0 \leq j_1 < \lfloor th_{11}C_1 \rfloor \Rightarrow 0 \leq j_1 < 1$ then the 1st link is in a *support mode* of operation.
b) If $\lfloor th_{11}C_1 \rfloor \leq j_1 < \lfloor th_{12}C_1 \rfloor \Rightarrow 1 \leq j_1 < 4$ then the 1st link is in a *normal mode* of operation.
c) If $\lfloor th_{21}C_1 \rfloor \leq j_1 \Rightarrow 4 \leq j_1$ then the 1st link is in an *offloading mode* of operation.

Second link
a) If $0 \leq j_2 < \lfloor th_{12}C_2 \rfloor \Rightarrow 0 \leq j_2 < 1$ then the 2nd link is in a *support mode* of operation.
b) If $\lfloor th_{12}C_2 \rfloor \leq j_2 < \lfloor th_{22}C_2 \rfloor \Rightarrow 1 \leq j_2 < 3$ then the 2nd link is in a *normal mode* of operation.

c) If $\lfloor th_{22}C_2 \rfloor \leq j_2 \Rightarrow 3 \leq j_2$ then the 2nd link is in an *offloading mode* of operation.
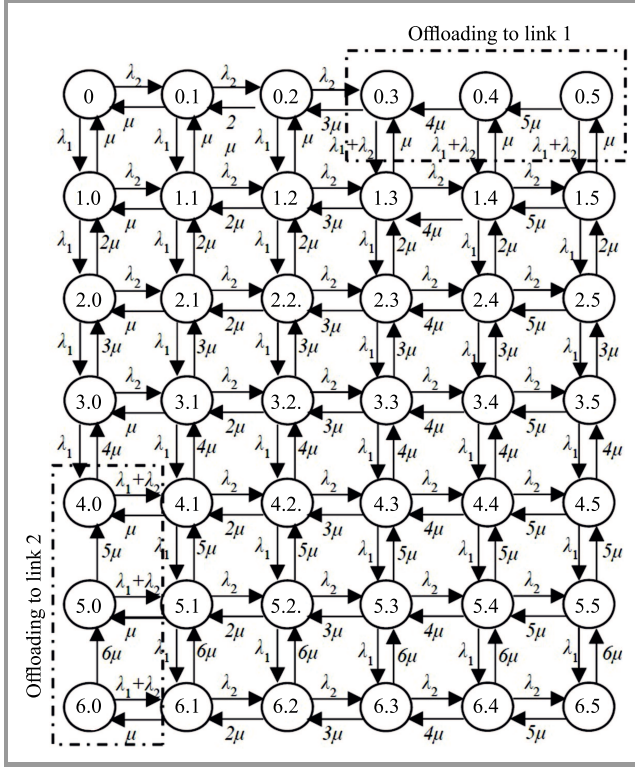

Fig. 5. State transition diagram of the tutorial example.

The state space of the system consists of 42 states of the form (j_1, j_2), depicted in Fig. 5 together with the corresponding transition rates. To help a reader understand the state transition diagram of Fig. 5 and the offloading mechanism, assume that the system is in state $(0, 2)$ when a new call arrives in the 2nd link. Then, the call will be accepted in the 2nd link and the new state will be $(0, 3)$. If another new call arrives in the 2nd link then the call will be offloaded to the 1st link (and served by that link) and the new state will be $(1, 3)$. If now, another call arrives in the 2nd link, then this call cannot be offloaded to the 1st link (since $j_1 = 1$) but it can be served by the 2nd link due to bandwidth availability. In that case the new state will be $(1, 4)$. A similar rationale exists when we consider call arrivals in the 1st link and the states $(3, 0)$, $(4,0)$, $(4,1)$ and $(5,1)$.

Based on the solution of the 42 GB equations of Fig. 5, the CBP in the 1st and 2nd link is given by:

$$P_{b_1}' = \sum_{j_2 = \lfloor th_{12}C_2 \rfloor} P(C_1, j_2) = \sum_{j_2=1}^{5} P(6, j_2) = 0.10370,$$

$$P_{b_2}' = \sum_{j_1 = \lfloor th_{11}C_1 \rfloor} P(j_1, C_2) = \sum_{j_1=1}^{6} P(j_1, 5) = 0.03758.$$

On the same hand, the total blocking probability in the two-link system is determined by:

$$P_b' = \frac{\lambda_1}{\lambda_1 + \lambda_2} P_{b_1}' + \frac{\lambda_2}{\lambda_1 + \lambda_2} P_{b_2}' \stackrel{\lambda_1=4, \lambda_2=2}{=} 0.08166.$$

Based on the approximate method of link independence and Eqs. (6), (7), we have:
$P_{b_1} = P_1(C_1)P_2(j_2 \geq \lfloor th_{12}C_2 \rfloor) = P_1(6)P_2(j_2 \geq 1) =$
$0.11716 \times 0.862386 \Rightarrow P_b' = 0.10104$.
$P_{b_2} = P_2(C_2)P_1(j_1 \geq \lfloor th_{11}C_1 \rfloor) = P_2(5)P_1(j_1 \geq 1) =$
$0.03670 \times 0.979405 \Rightarrow P_b' = 0.03594$.
The total blocking probability in the two-link system is determined by:

$$P_b = \frac{\lambda_1}{\lambda_1 + \lambda_2} P_{b_1} + \frac{\lambda_2}{\lambda_1 + \lambda_2} P_{b_2} \stackrel{\lambda_1=4, \lambda_2=2}{=} 0.07934.$$

The previous results reveal that the approximate method provides quite satisfactory results compared to the exact values, even in small tutorial examples.

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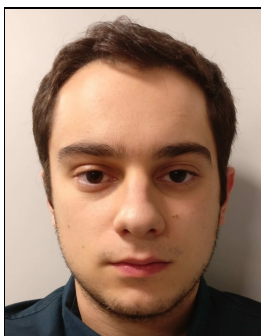
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