

Product of Three Random Variables and its Application in Relay Telecommunication Systems in the Presence of Multipath Fading

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Abstract—In this paper, the product of three random variables (RVs) will be considered. Distribution of the product of independent random variables is very important in many applied problems, including wireless relay telecommunication systems. A few of such products of three random variables are observed in this work: the level crossing rate (LCR) of the product of a Nakagami- m random variable, a Rician random variable and a Rayleigh random variable, and of the products of two Rician RVs and one Nakagami- m RV is calculated in closed forms and presented graphically. The LCR formula may be later used for derivation of average fade duration (AFD) of a wireless relay communication radio system with three sections, working in the multipath fading channel. The impact of fading parameters and multipath fading power on the LCR is analyzed based on the graphs presented.

Keywords—level crossing rate, Nakagami- m fading, Rayleigh fading, relay telecommunication systems, Rician fading.

1. Introduction

Statistical characteristics of products and ratios of random variables (RVs) are essential in analyzing the performance of contemporary wireless telecommunications systems, as well as in solving numerous applied problems. The products of RVs are encountered naturally in such applications as: channel modeling, multihop wireless relaying systems, cascaded fading channels, MIMO keyhole systems [1], quantum physics, signal processing, tensor sensing problem, the rate offset of the hybrid automatic repeat request (H-ARQ) transmission, and even in biological and physical sciences, econometrics, classification, ranking and selection [2].

Because of that, in recent years, the products and ratios of random processes are investigated in the literature by many researchers [1]–[6]. In the 1960s, Donahue, Springer,

Thompson and Lomnicki started with derivations concerning the distributions of the products of two RVs [7]–[10]. At the beginning of this century, interest in this area has increased again [11]–[13]. The latest works, with significant application in wireless communications systems, are [14]–[17].

Computational algorithms for derivating the distribution of the product of two RVs are given in [9]. The product and the ratio of two independent, Student's t distributed RVs, are observed in [10]. The derivation of the probability density function (PDF) of the product of two independent, non-identical, and triangularly distributed RVs, by using integral calculation, is presented in [11].

The problem of characterizing products of independent RVs is investigated for normal RVs, products of their absolute values, and products of their squares in [13]. Power-log series expansions of cumulative distribution functions (CDF), based on the theory of Fox H functions, is computed. It is numerically shown that CDF is well approximated by the lowest orders of this expansion for small arguments. The moment generating functions (MGF) in terms of Meijer G functions are also computed for two non-negative RVs. In that paper, the fading amplitudes of cascaded fading channels have the distribution of the product of Nakagami- m RVs, as in [5] and [6], and of the product of Rician RVs, as in [18] and [19].

Signal envelope variations, called fading, are results of reflections, refractions, diffraction and scattering. They can be described by several distributions. So, Rayleigh [20] and Nakagami- m [21] distributions are used when no dominant component is present. Signal envelope variation is modeled by Rician distribution when a line-of-sight (LOS) dominant component exists in the channel [22].

Level crossing rate (LCR) of a product of two Nakagami- m random processes is analyzed in [6]. Then, the av-

erage fade duration (AFD) of a wireless relay communications system consisting of two sections in a Nakagami- m short term fading channel is obtained. The performance of the product of arbitrary and independent RVs with a general α - μ distribution is given in [15]. The closed-form expressions for PDF, CDF and moments are obtained and the calculation process used to obtain the amount of fading (AoF) and outage probability (OP) in cascaded channels is presented.

An analysis of the performance of the product of two independent and non-identically distributed κ - μ RVs is presented in [17], where analytical formulations for PDF, CDF and MGF are obtained. By using these formulations, closed-form expressions for higher order moments, AoF and channel quality estimation index are derived, as are analytical formulas for OP, average channel capacity, average symbol error probability (SEP) and average bit error probability. This applies to different fading scenarios, such as double Rayleigh, double Rician, double Nakagami- m , κ - μ /Nakagami- m , and Rician/Nakagami- m , identified as special cases.

The cascaded keyhole channels may be modeled using the product of individual channels [23]. Further, independent and identically distributed (i.i.d.) double fading channels in a line-of-sight (LOS) environment, typical for keyhole Multiple input multiple output (MIMO) systems, are analyzed in [24]. LOS double fading, i.e. double Rician fading in MIMO channels, is investigated and the density function of the signal-to-noise ratio (SNR) is discussed. After obtaining the exact expression in a form with infinite series, an approximation formula of SNR density is presented by using the Nakagami- m approximation of Rician distribution.

In [25], the product of Nakagami- m RV, Rician RV and Rayleigh RV is analyzed. LCR of this product is calculated. The results obtained may also be used for the evaluation the AFD of a relay wireless communications system with three sections in the presence of Nakagami- m fading in the first section, Rician fading in the second section and Rayleigh fading in the third section. LCR of the product of three independent Rician RVs is observed in [26].

In this article, except for the results from [25], the product of two Rician RVs and one Nakagami- m RV is processed. The result can be applied for derivation, in a closed form, of the second order performance of a wireless relay communications system with three sections operating in Rician and Nakagami- m fading environments. The formulas are validated by numerical results and impact of the individual parameters is analyzed.

This work is composed of four sections. Section 1 serves as an introduction and describes previous works in the area. In two next sections, the product of three random variables is given and an expression for LCR in a closed form is performed for two different sets of RVs. The influence of parameters is shown via several graphics for both sets. The work ends with conclusions presented in Section 4.

2. Derivation of LCR of Product of Nakagami- m , Rician and Rayleigh Random Variables

Here, we examine the scenario involving a wireless relay communications system with three sections. The signal envelope at the output of the relay communications system with three sections is the product of envelopes at the individual sections. In the first example observed, the Nakagami- m signal envelope is at the first section, the Rician signal envelope at the second section and the Rayleigh signal envelope at the third section.

As a rule, it is first necessary to calculate the probability density function. By using PDF, bit error probability (BEP) can be evaluated, and by using CDF, outage probability can be obtained [27], [28]. OP and BEP are the first order performance measure of the wireless communications system. OP may be calculated as probability that the signal envelope is below the threshold [27], [29]. Level crossing rate is the second order statistic measure of the wireless communications system and is associated with envelope fading, as well as with average fade duration. LCR shows how often the envelope crosses a specified level and may be calculated as the number of crossings at this defined level. AFD shows how long the envelope remains below a specified level and can be evaluated as the ratio of OP and LCR. These two quantities are second order statistics because they are affected not only by the scattering in environment, but also by the speed of mobile stations. Here, PDF of the product of Nakagami- m RV, Rician RV and Rayleigh RV will be obtained by using the transformation method. Further, using this PDF, CDF and moments can be evaluated, as can be the level crossing rate.

2.1. Distribution of Random Variables

Nakagami- m random variable x_1 follows the distribution defined in [21]:

$$p_{x_1}(x_1) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega_1}\right)^m x_1^{2m-1} e^{-\frac{m}{\Omega_1}x_1^2}, \quad m \geq \frac{1}{2}, \quad x_1 \geq 0, \quad (1)$$

where $\Gamma(\cdot)$ is a gamma function. This distribution has two parameters. The first parameter controls spread. Actually, $\Omega_1 = E[x_1^2]$ is the average power of the multipath scattering field. m is the fading depth parameter or the shape factor of the Nakagami distribution [30]. For RV x_1 the shape parameter is:

$$m = \frac{\Omega_1^2}{E\{[x_1^2 - \Omega_1^2]^2\}}.$$

It describes the fading degree of the propagation environment caused by the interference of scattering and multipath phenomena. So, the severity of fading is indicated by the Nakagami parameter m .

Random variable x_2 follows Rician distribution [22]:

$$p_{x_2}(x_2) = \frac{2(\kappa+1)}{\Omega_2} \sum_{i_1=0}^{\infty} \left(\frac{\kappa(\kappa+1)}{\Omega_2} \right)^{i_1} \times \frac{1}{(i_1!)^2} x_2^{2i_1+1} e^{-\frac{\kappa+1}{\Omega_2} x_2^2}, \quad x_2 \geq 0, \quad (2)$$

where Ω_2 is the average received power for x_2 and κ is the Rician factor. Rician factor κ is defined as a ratio of dominant component's power and the scattering components' powers. This type of fading, called Rician fading, is very often observed in microcellular and mobile satellite applications [27].

Rayleigh distribution can be easily derived from Rician distribution for Rician factor $\kappa = 0$. For $\kappa = \infty$ we have no fading, i.e. a channel with no multipath and only a LOS component. The fading parameter κ is therefore a measure of the severity of fading: a small κ implies severe fading, a large κ implies more mild fading [31].

Two-dimensional isotropic scattering, where the arriving waves arrive to the receiver from all directions, with equal probability, is a scattering model that is often used for the communication channel in a macrocellular system. For this type of scattering environment, the received envelope is Rayleigh distributed at any time, and is said to be Rayleigh fading.

Random variable x_3 follows Rayleigh distribution [31]:

$$p_{x_3}(x_3) = \frac{x_3}{\Omega_3} e^{-\frac{x_3^2}{\Omega_3}}, \quad x_3 \geq 0, \quad (3)$$

where Ω_3 is the average received signal power of signal x_3 , i.e. the received power based alone on path loss and shadowing alone [31].

2.2. Product of Three Random Variables

The product of three random variables x_1 , x_2 and x_3 is:

$$x = x_1 x_2 x_3. \quad (4)$$

Then, it is valid that:

$$x_1 = \frac{x}{x_2 x_3}. \quad (5)$$

The first time derivative of x is:

$$\dot{x} = \dot{x}_1 x_2 x_3 + x_1 \dot{x}_2 x_3 + x_1 x_2 \dot{x}_3. \quad (6)$$

The first time derivative of x_i has Gaussian distribution:

$$p_{\dot{x}_i}(\dot{x}_i) = \frac{1}{\sqrt{2\pi\hat{\sigma}_i^2}} e^{-\frac{\dot{x}_i^2}{2\hat{\sigma}_i^2}}, \quad -\infty < \dot{x}_i < \infty,$$

where $\hat{\sigma}_i^2 = \frac{\pi^2 f_m^2 \Omega_i}{m}$ and f_m being maximal Doppler frequency. The processes x_i and \dot{x}_i are considered to be independent, as Rice demonstrated in [32].

This is a very interesting result which shows that, in the Nakagami case and in the Rayleigh and Rice cases, x_i and \dot{x}_i are mutually independent random variables [33], i.e., it is valid that [32]:

$$p_{x_i \dot{x}_i}(x_i \dot{x}_i) = p_{x_i}(x_i) p_{\dot{x}_i}(\dot{x}_i).$$

Moreover, the probability density function of the time derivative of the Nakagami envelope is also Gaussian distributed as are the time derivatives of both Rayleigh and Rice envelopes [34].

So, all random variables \dot{x}_1 , \dot{x}_2 , and \dot{x}_3 have Gaussian distribution. A linear combination of Gaussian RVs is a Gaussian RV. The mean signal level of \dot{x} is:

$$\bar{\dot{x}} = \bar{\dot{x}}_1 x_2 x_3 + x_1 \bar{\dot{x}}_2 x_3 + x_1 x_2 \bar{\dot{x}}_3 = 0, \quad (7)$$

because:

$$\bar{\dot{x}}_1 = \bar{\dot{x}}_2 = \bar{\dot{x}}_3 = 0. \quad (8)$$

The variance of \dot{x} is:

$$\sigma_{\dot{x}}^2 = x_2^2 x_3^2 \sigma_{\dot{x}_1}^2 + x_1^2 x_3^2 \sigma_{\dot{x}_2}^2 + x_1^2 x_2^2 \sigma_{\dot{x}_3}^2, \quad (9)$$

where:

$$\begin{aligned} \sigma_{\dot{x}_1} &= \pi^2 f_m^2 \frac{\Omega_1}{m}, \\ \sigma_{\dot{x}_2} &= \pi^2 f_m^2 \frac{\Omega_2}{\kappa+1}, \\ \sigma_{\dot{x}_3} &= \pi^2 f_m^2 \Omega_3. \end{aligned} \quad (10)$$

After substituting, the expression for variance becomes:

$$\begin{aligned} \sigma_{\dot{x}}^2 &= \pi^2 f_m^2 \left(x_2^2 x_3^2 \frac{\Omega_1}{m} + x_1^2 x_3^2 \frac{\Omega_2}{\kappa+1} + x_1^2 x_2^2 \Omega_3 \right) \\ &= \pi^2 f_m^2 x_2^2 x_3^2 \frac{\Omega_1}{m} \left(1 + \frac{x^2}{x_2^4 x_3^2} \frac{\Omega_2}{\Omega_1} \frac{m}{\kappa+1} + \frac{x^2}{x_2^2 x_3^4} \frac{\Omega_3}{\Omega_1} m \right). \end{aligned} \quad (11)$$

The joint probability density function of x , \dot{x} , x_2 and x_3 is:

$$p_{x \dot{x} x_2 x_3}(x \dot{x} x_2 x_3) = p_{\dot{x}}(\dot{x}/x x_2 x_3) p_x(x/x_2 x_3) p_{x_2}(x_2) p_{x_3}(x_3), \quad (12)$$

where

$$p_x(x/x_2 x_3) = \left| \frac{dx_1}{dx} \right| p_{x_1} \left(\frac{x}{x_2 x_3} \right), \quad (13)$$

$$\frac{dx_1}{dx} = \frac{1}{x_2 x_3}. \quad (14)$$

The joint probability density function of x and \dot{x} is:

$$\begin{aligned} p_{x \dot{x}}(x \dot{x}) &= \int_0^{\infty} \int_0^{\infty} dx_2 dx_3 p_{\dot{x}}(\dot{x}/x x_2 x_3) \frac{1}{x_2 x_3} \\ &\quad \times p_{x_1} \left(\frac{x}{x_2 x_3} \right) p_{x_2}(x_2) p_{x_3}(x_3). \end{aligned} \quad (15)$$

2.3. LCR of Product of Three Random Variables

Level crossing rate of x in a fading environment is [35]:

$$N_x = \int_0^\infty d\dot{x} p_{x\dot{x}}(x\dot{x}) . \quad (16)$$

For our case LCR is:

$$\begin{aligned} N_x &= \int_0^\infty dx_2 \int_0^\infty dx_3 \frac{1}{x_2 x_3} p_{x_1} \left(\frac{x}{x_2 x_3} \right) p_{x_2}(x_2) p_{x_3}(x_3) \\ &\times \int_0^\infty d\dot{x} p_{\dot{x}}(\dot{x}/x x_2 x_3) = \int_0^\infty dx_2 \int_0^\infty dx_3 \frac{1}{x_2 x_3} p_{x_1} \left(\frac{x}{x_2 x_3} \right) p_{x_2}(x_2) \\ &\times p_{x_3}(x_3) \frac{1}{\sqrt{2\pi}} \sigma_{\dot{x}} = \int_0^\infty dx_2 \int_0^\infty dx_3 \frac{1}{x_2 x_3} p_{x_1} \left(\frac{x}{x_2 x_3} \right) p_{x_2}(x_2) \\ &\times p_{x_3}(x_3) \frac{1}{\sqrt{2\pi}} \pi f_m x_2 x_3 \frac{\Omega_2^{\frac{1}{2}}}{m^{\frac{1}{2}}} \left(1 + \frac{x^2}{x_2^4 x_3^2} \frac{\Omega_2}{\Omega_1} \frac{m}{\kappa + 1} \right. \\ &\left. + \frac{x^2}{x_2^2 x_3^4} \frac{\Omega_3}{\Omega_1} m \right)^{\frac{1}{2}} = \frac{1}{\sqrt{2\pi}} \pi f_m \frac{\Omega_1^{\frac{1}{2}}}{m^{\frac{1}{2}}} \int_0^\infty dx_2 \int_0^\infty dx_3 \\ &\times p_{x_1} \left(\frac{x}{x_2 x_3} \right) p_{x_2}(x_2) p_{x_3}(x_3) \left(1 + \frac{x^2}{x_2^4 x_3^2} \frac{\Omega_2}{\Omega_1} \frac{m}{\kappa + 1} \right. \\ &\left. + \frac{x^2}{x_2^2 x_3^4} \frac{\Omega_3}{\Omega_1} m \right)^{\frac{1}{2}} = \frac{1}{\sqrt{2\pi}} \pi f_m \frac{\Omega_1^{\frac{1}{2}}}{m^{\frac{1}{2}}} \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega_1} \right)^m \\ &\times x^{2m-1} \frac{2(\kappa+1)}{\Omega_2} \sum_{i_1=0}^{\infty} \left(\frac{\kappa(\kappa+1)}{\Omega_2} \right)^{i_1} \frac{1}{(i_1!)^2} \frac{2}{\Omega_3} \\ &\times \int_0^\infty dx_2 \int_0^\infty dx_3 x_2^{-2m+1+2i_1+1} x_3^{-2m+1+1} e^{-\frac{m}{\Omega_1} \frac{x^2}{x_2^4 x_3^2} - \frac{\kappa+1}{\Omega_2} x_2^2 - \frac{1}{\Omega_3} x_3^2} \\ &\times \left(1 + \frac{x^2}{x_2^4 x_3^2} \frac{\Omega_2}{\Omega_1} \frac{m}{\kappa + 1} + \frac{x^2}{x_2^2 x_3^4} \frac{\Omega_3}{\Omega_1} m \right)^{\frac{1}{2}} . \quad (17) \end{aligned}$$

The previous two-fold integral may be solved using the Laplace approximation theorem for the solution the two-fold integrals [36], [37]:

$$\int_0^\infty dx_2 \int_0^\infty dx_3 (x_2, x_3) e^{\lambda f(x_2, x_3)} = \frac{\pi g(x_{20}, x_{30})}{\lambda B(x_{20}, x_{30})} e^{\lambda f(x_{20}, x_{30})} , \quad (18)$$

where B is the matrix:

$$B(x_{20}, x_{30}) = \begin{vmatrix} \frac{\partial^2 f(x_{20}, x_{30})}{\partial x_{20}^2} & \frac{\partial^2 f(x_{20}, x_{30})}{\partial x_{20} \partial x_{30}} \\ \frac{\partial^2 f(x_{20}, x_{30})}{\partial x_{20} \partial x_{30}} & \frac{\partial^2 f(x_{20}, x_{30})}{\partial x_{30}^2} \end{vmatrix} , \quad (19)$$

and x_{20} and x_{30} are solution of the equations:

$$\frac{\partial f(x_{20}, x_{30})}{\partial x_{20}} = 0, \quad \frac{\partial f(x_{20}, x_{30})}{\partial x_{30}} = 0 . \quad (20)$$

For considered case, it is:

$$g(x_2, x_3) = x_2^{-2m+2i_1+2} x_3^{-2m+2} \times \left(1 + \frac{x^2}{x_2^4 x_3^2} \frac{\Omega_2}{\Omega_1} \frac{m}{\kappa + 1} + \frac{x^2}{x_2^2 x_3^4} \frac{\Omega_3}{\Omega_1} m \right)^{\frac{1}{2}} , \quad (21)$$

$$f(x_2, x_3) = -\frac{m}{\Omega_1} \frac{x^2}{x_2^2 x_3^2} - \frac{(\kappa+1)}{\Omega_2} x_2^2 - \frac{1}{\Omega_3} x_3^2 , \quad (22)$$

$$\frac{\partial f(x_2, x_3)}{\partial x_2} = \frac{2m}{\Omega_1} \frac{x^2}{x_2^3 x_3^2} - \frac{2(\kappa+1)}{\Omega_2} x_2 , \quad (23)$$

$$\frac{\partial f(x_2, x_3)}{\partial x_3} = \frac{2m}{\Omega_1} \frac{x^2}{x_2^2 x_3^3} - \frac{2}{\Omega_3} x_3 , \quad (24)$$

The solutions of the next two equations are x_{20} and x_{30} :

$$\frac{2m}{\Omega_1} \frac{x^2}{x_2^3 x_3^2} - \frac{2(\kappa+1)}{\Omega_2} x_2 = 0 , \quad (25)$$

$$\frac{2m}{\Omega_1} \frac{x^2}{x_2^2 x_3^3} - \frac{2}{\Omega_3} x_3 = 0 . \quad (26)$$

They should be introduced in Eq. (18) for solving two-fold integral from Eq. (17). In this manner LCR of the product of Nakagami- m , Rician and Rayleigh random variables will be obtained in a closed form.

2.4. Numerical Examples and Discussion

The level crossing rate of the product of Nakagami- m random variable, Rician RV and Rayleigh RV is shown in the next few figures versus resulting signal x for different values of fading parameters and signal powers.

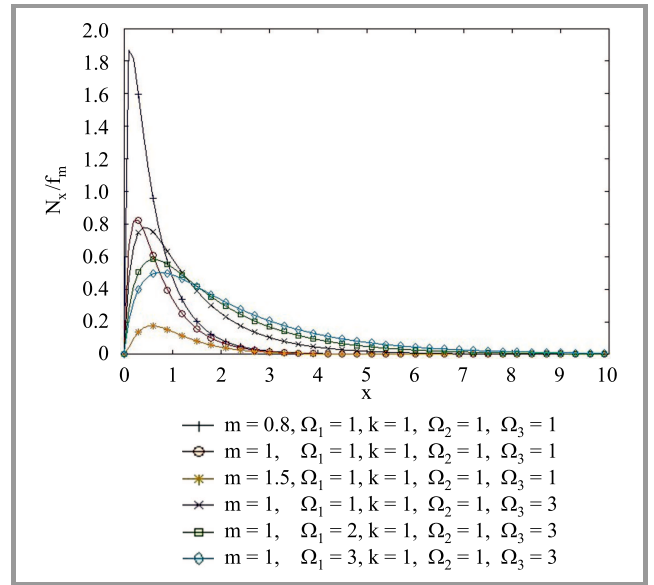


Fig. 1. LCR normalized by f_m depending on signal envelope x for various values of parameters m and Ω_1 .

Dependence of the LCR, normalized by f_m , on the resulting signal x , for various values of parameters m and Ω_1 is presented in Fig. 1. It is possible to notice that LCR increases for lower values of resulting signal and decreases for greater values of the resulting signal. All curves reach the maximum and start to decline. Lower values of the resulting signal have a greater impact on LCR. LCR increases for low values of Nakagami- m small scale fading parameter m . The impact of resulting x on LCR is larger for smaller magnitudes of parameter m . LCR is larger for smaller values of m .

From this picture, the influence of power Ω_1 can also be observed. For low values of x , LCR increases with the reduction of power Ω_1 , but for bigger values of x , LCR increases along with the growth of power Ω_1 .

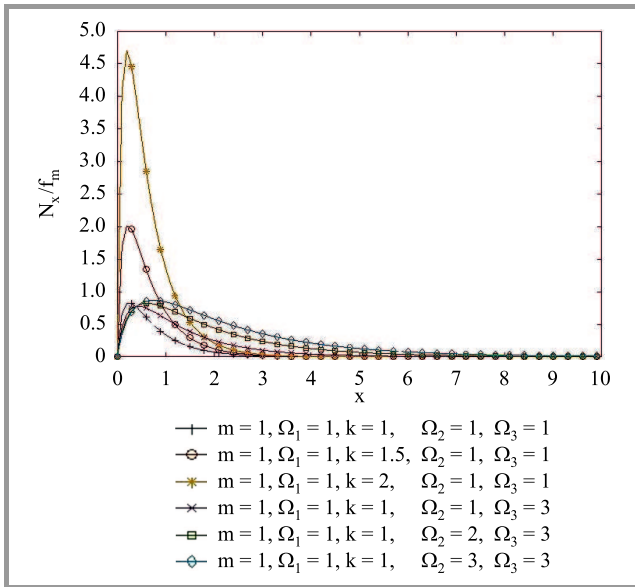


Fig. 2. LCR normalized by f_m for different parameters κ and Ω_2 .

Figure 2 shows the influence of the other two parameters: Rician factor κ and signal power Ω_2 . LCR becomes bigger as the Rician factor κ grows. The influence of x on LCR is greater for lower values of Rician factor κ . The impact of Nakagami- m fading parameter m on LCR is higher for bigger values of Rician factor κ . From this figure, one can also see that LCR is larger for greater values of power Ω_2 . In Fig. 3 the impact of power Ω_3 is shown. Based on the image, one may remark that LCR is higher for bigger values of Ω_3 and low values of x . For higher values of x , LCR is greater for smaller Ω_3 . The small resulting signal x exerts a greater impact on LCR.

The results obtained may be used to evaluate LCR of the product of Nakagami- m and two Rayleigh RVs, LCR of Rician and two Rayleigh RVs, and LCR of the product of three Rayleigh (3* Rayleigh) RVs. This can be achieved because Nakagami- m and Rician distributions are of the general variety. For the same reason, LCR of the product of three independent Rician RVs from [26] can be used for determination of LCR of the product of three Rayleigh RVs, or the LCR of the product of two Rician RVs and

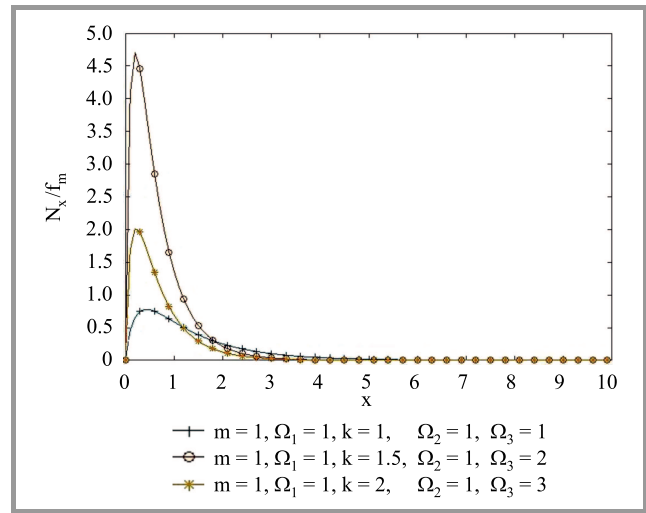


Fig. 3. LCR normalized by f_m for various values of Ω_3 .

Rayleigh RV, or LCR of the product of Rician RV and two Rayleigh RVs, because Rayleigh distribution may be easily derived from Rician distribution for Rician factor $\kappa = 0$. If Nakagami fading severity parameter $m = \frac{1}{2}$, Nakagami distribution is reduced to unilateral (one-sided) Gaussian distribution. For $m = 1$, Nakagami distribution reduces to Rayleigh distribution, and for $m > 1$, Nakagami distribution is reduced to Rician distribution. The ratio between Rician factor κ and parameter m is [27], [38]:

$$\kappa = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}}, \quad m > 1.$$

On the other hand, for [31]:

$$m = \frac{(\kappa + 1)^2}{2(\kappa + 1)},$$

the distribution in Eq. (1) is approximately Rician fading with parameter κ . For $m = \infty$ we get an additive white Gaussian noise (AWGN) channel without fading. We see that as m increases, fading decreases.

As the Nakagami distribution does not contain a Bessel function, it can get the close form solution more convenient than Rician distribution [30].

Thus, the Nakagami distribution may model Rayleigh distribution and Rician distribution, with certain restrictions [21]. Note that some empirical measurements support values of the m parameter being equal to less than one, in which case the Nakagami fading causes a more severe performance degradation than Rayleigh fading.

3. LCR of Product of Two Rician and Nakagami- m Random Variables

In the second example, presented in this section, Rician fading exists in the first two sections and Nakagami- m fading is present in the third section. These results are applicable

in analyzing the performance of multi-hop relay wireless telecommunications systems when the signal level is much higher than the noise level. In such a case, the noise level can be ignored. For that matter, the output signal is a product of as many random variables as there are sections in the relay system [38].

3.1. Distribution of the Second Set of Random Variables

Let random variables x_4 and x_5 have Rician distribution [22]:

$$p_{x_4}(x_4) = \frac{2(\kappa_1 + 1)}{\Omega_1} \sum_{i_2=0}^{\infty} \left(\frac{\kappa_1(\kappa_1 + 1)}{\Omega_1} \right)^{i_2} \times \frac{1}{(i_2!)^2} x_4^{2i_2+1} e^{-\frac{\kappa_1+1}{\Omega_1} x_4^2}, \quad x_4 \geq 0, \quad (27)$$

$$p_{x_5}(x_5) = \frac{2(\kappa_2 + 1)}{\Omega_2} \sum_{i_3=0}^{\infty} \left(\frac{\kappa_2(\kappa_2 + 1)}{\Omega_2} \right)^{i_3} \times \frac{1}{(i_3!)^2} x_5^{2i_3+1} e^{-\frac{\kappa_2+1}{\Omega_2} x_5^2}, \quad x_5 \geq 0, \quad (28)$$

and let random variable x_6 have Nakagami- m distribution [29]:

$$p_{x_6}(x_6) = \frac{2}{\Gamma(m_3)} \left(\frac{m_3}{\Omega_3} \right)^{m_3} x_6^{2m_3-1} e^{-\frac{m_3}{\Omega_3} x_6^2}, \quad x_4 \geq 0, x_6 \geq 0, \quad (29)$$

where Ω_i , $i = 1, 2, 3$, are powers of RVs x_i , $i = 4, 5, 6$, κ_1 and κ_2 are Rician factors for variables x_4 and x_5 , and m_3 is Nakagami- m fading severity parameter of RV x_6 .

3.2. Product of Three Random Variables

Here, the random variable x is defined as a product of x_i , $i = 4, 5, 6$:

$$x = \prod_{i=4}^6 x_i. \quad (30)$$

The first time derivative of x is:

$$\dot{x} = \dot{x}_4 x_5 x_6 + x_4 \dot{x}_5 x_6 + x_4 x_5 \dot{x}_6, \quad (31)$$

the average value of x is:

$$\bar{x} = \bar{x}_4 x_5 x_6 + x_4 \bar{x}_5 x_6 + x_4 x_5 \bar{x}_6 = 0, \quad (32)$$

because [29]

$$\bar{x}_4 = \bar{x}_5 = \bar{x}_6 = 0. \quad (33)$$

The variance of \dot{x} is given by:

$$\sigma_{\dot{x}}^2 = x_5^2 x_6^2 \sigma_{\dot{x}_4}^2 + x_4^2 x_6^2 \sigma_{\dot{x}_5}^2 + x_4^2 x_5^2 \sigma_{\dot{x}_6}^2, \quad (34)$$

with:

$$\sigma_{\dot{x}_4}^2 = \pi f_m^2 \frac{\Omega_1}{\kappa_1 + 1}, \quad (35)$$

$$\sigma_{\dot{x}_5}^2 = \pi f_m^2 \frac{\Omega_2}{\kappa_2 + 1}, \quad (36)$$

$$\sigma_{\dot{x}_6}^2 = \pi f_m^2 \frac{\Omega_3}{m_3}. \quad (37)$$

After transformation of Eqs. (35)–(37) into Eq. (34), the variance is:

$$\begin{aligned} \sigma_{\dot{x}}^2 &= \pi^2 f_m^2 \left(x_5^2 x_6^2 \frac{\Omega_1}{\kappa_1 + 1} + \frac{x^2}{x_5^2} \frac{\Omega_2}{\kappa_2 + 1} + \frac{x^2}{x_6^2} \frac{\Omega_3}{m_3} \right) \\ &= \pi^2 f_m^2 x_5^2 x_6^2 \frac{\Omega_1}{\kappa_1 + 1} \\ &\quad \times \left(1 + \frac{x^2}{x_5^4 x_6^2} \frac{\Omega_2}{\kappa_2 + 1} \frac{\kappa_1 + 1}{\Omega_1} + \frac{x^2}{x_5^2 x_6^4} \frac{\Omega_3}{m_3} \frac{\kappa_1 + 1}{\Omega_1} \right). \end{aligned} \quad (38)$$

Joint PDF of x , \dot{x} , x_5 and x_6 is:

$$p_{x\dot{x}x_5x_6}(x\dot{x}x_5x_6) = p_{\dot{x}}(\dot{x}/xx_5x_6) \times p_x(x/x_5x_6) p_{x_5}(x_5) p_{x_6}(x_6), \quad (39)$$

and joint PDF of x and \dot{x} :

$$\begin{aligned} p_{x\dot{x}}(x\dot{x}) &= \int_0^{\infty} dx_5 \int_0^{\infty} dx_6 p_{x\dot{x}x_5x_6}(x\dot{x}x_5x_6) \\ &= \int_0^{\infty} dx_5 \int_0^{\infty} dx_6 p_{\dot{x}/xx_5x_6}(\dot{x}/xx_5x_6) p_x(x/x_5x_6) p_{x_5}(x_5) p_{x_6}(x_6), \end{aligned} \quad (40)$$

with:

$$p_x(x/x_5x_6) = \left| \frac{dx_4}{dx} \right| p_{x_4} \left(\frac{x}{x_5x_6} \right), \quad (41)$$

$$\frac{dx_4}{dx} = \frac{1}{x_5x_6}. \quad (42)$$

The expression for $p_{x\dot{x}}(x\dot{x})$ in Eq. (40), after some replacements is:

$$\begin{aligned} p_{x\dot{x}}(x\dot{x}) &= \int_0^{\infty} dx_5 \int_0^{\infty} dx_6 p_{\dot{x}}(\dot{x}/xx_5x_6) \\ &\quad \times \frac{1}{x_5x_6} p_{x_4} \left(\frac{x}{x_5x_6} \right) p_{x_5}(x_5) p_{x_6}(x_6). \end{aligned} \quad (43)$$

3.3. LCR of Product of the Second Set of Random Variables

Level crossing rate of x is defined by Eq. (16) [39]. LCR of product x from Eq. (30), with $p_{x\dot{x}}(x\dot{x})$ from Eq. (43), is:

$$\begin{aligned} N_x &= \int_0^{\infty} dx_5 \int_0^{\infty} dx_6 \left(\int_0^{\infty} dx \dot{x} p_{\dot{x}}(\dot{x}/xx_5x_6) \right. \\ &\quad \times \left. \frac{1}{x_5x_6} p_{x_4} \left(\frac{x}{x_5x_6} \right) p_{x_5}(x_5) p_{x_6}(x_6) \right). \end{aligned} \quad (44)$$

After introducing Eqs. (27)–(29) and Eq. (38) into Eq. (44), we obtain LCR as:

$$\begin{aligned}
 N_x &= \frac{1}{\sqrt{2\pi}} \pi f_m \frac{\Omega_1^{\frac{1}{2}}}{(\kappa_1 + 1)^{\frac{1}{2}}} x^{2i_1+1} \frac{2(\kappa_1 + 1)}{\Omega_1} \sum_{i_2=0}^{\infty} \left(\frac{\kappa_1(\kappa_1 + 1)}{\Omega_1} \right)^{i_2} \\
 &\times \frac{1}{(i_2!)^2} \frac{2(\kappa_2 + 1)}{\Omega_2} \sum_{i_3=0}^{\infty} \left(\frac{\kappa_2(\kappa_2 + 1)}{\Omega_2} \right)^{i_3} \frac{1}{(i_3!)^2} \frac{2}{\Gamma(m_3)} \\
 &\times \left(\frac{m_3}{\Omega_3} \right)^{m_3} \int_0^{\infty} dx_5 \int_0^{\infty} dx_6 x_5^{-2i_2-1+2i_3+1} x_6^{-2i_2-1+2m_3-1} \\
 &\quad \times e^{-\frac{\kappa_1+1}{\Omega_1} \frac{x^2}{x_5^2} - \frac{\kappa_2+1}{\Omega_2} x_5^2 - \frac{m_3}{\Omega_3} x_6^2} \\
 &\times \left(1 + \frac{x^2}{x_5^4 x_6^2} \frac{\Omega_2}{\kappa_2 + 1} \frac{\kappa_1 + 1}{\Omega_1} + \frac{x^2}{x_5^2 x_6^4} \frac{\Omega_3}{m_3} \frac{\kappa_1 + 1}{\Omega_1} \right)^{\frac{1}{2}}. \quad (45)
 \end{aligned}$$

Now we need to use the Laplace approximation theorem for the solution of double integrals, defined in Eqs. (18)–(20), for x_5 and x_6 , and x_{50} and x_{60} as solutions [39], to solve last integrals in Eq. (45).

For this case the following is valid:

$$\begin{aligned}
 g(x_5, x_6) &= x_5^{-2i_2+2i_3} x_6^{-2i_2+2m_3-2} \\
 &\times \left(1 + \frac{x^2}{x_5^4 x_6^2} \frac{\Omega_2}{\kappa_2 + 1} \frac{\kappa_1 + 1}{\Omega_1} + \frac{x^2}{x_5^2 x_6^4} \frac{\Omega_3}{m_3} \frac{\kappa_1 + 1}{\Omega_1} \right)^{\frac{1}{2}}, \quad (46)
 \end{aligned}$$

$$f(x_5, x_6) = -\frac{\kappa_1 + 1}{\Omega_1} \frac{x^2}{x_5^2 x_6^2} - \frac{\kappa_2 + 1}{\Omega_2} x_5^2 - \frac{m_3}{\Omega_3} x_6^2, \quad (47)$$

$$\frac{\partial f(x_5, x_6)}{\partial x_6} = \frac{2(\kappa_1 + 1)}{\Omega_1} \frac{x^2}{x_5^3 x_6^3} - \frac{2(\kappa_2 + 1)}{\Omega_2} x_5, \quad (48)$$

$$\frac{\partial f(x_5, x_6)}{\partial x_6} = \frac{2(\kappa_1 + 1)}{\Omega_1} \frac{x^2}{x_5^3 x_6^3} - \frac{2m_3}{\Omega_3} x_6. \quad (49)$$

3.4. Numerical Examples and Discussion

Level crossing rate of the product of two Rician random variables and a Nakagami- m random variable is calculated and shown in the next few figures. The influence that Rician factors κ_1 and κ_2 , Nakagami- m fading severity parameter m_3 , Rician multipath fading powers Ω_1 and Ω_2 , and Nakagami- m fading power Ω_3 exert on LCR is discussed.

LCR, normalized by f_m , depending on the signal envelope x , is presented in Fig. 4, for different values of Rician factor κ_1 and Rician fading power Ω_1 . It is obvious from the picture that LCR achieves the maximum for small values of signal envelope x , and starts to decrease for higher values of x . So, it is evident that the impact of the signal envelope on LCR is bigger for small values of the signal envelope. It is also visible from this figure that LCR increases along with the increase in Rician factor κ_1 and in power Ω_1 . It is known that system performance is better for smaller values of LCR.

In Fig. 5, the normalized LCR is shown versus signal envelope for different values of parameters κ_1 and Ω_2 . It is

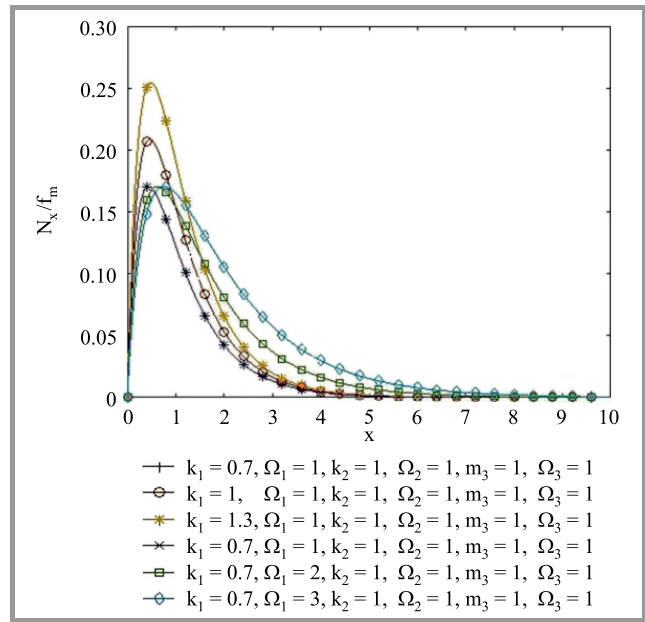


Fig. 4. LCR normalized by f_m versus signal envelope x for various values of parameters κ_1 and Ω_1 .

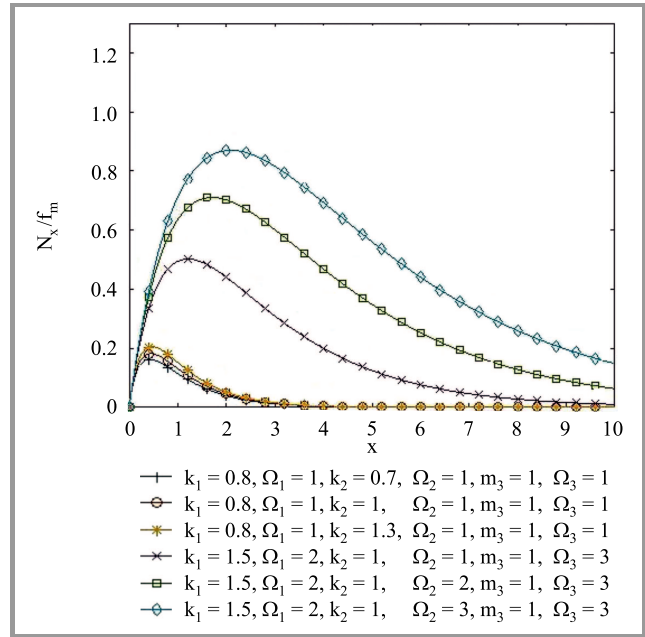


Fig. 5. LCR normalized by f_m versus signal envelope x for several values of parameters κ_2 and Ω_2 .

possible to see from this figure that when κ_1 grows, LCR increases as well, but the increase is insignificant. On the other hand, with the increase in Ω_2 , LCR increases visibly, the curves become wider and the maximums move towards higher values of the signal envelope x .

The last figure, Fig. 6, presents the LCR, normalized by f_m , depending on signal envelope for various values of Nakagami- m fading severity parameter m_3 , and Nakagami- m fading power Ω_3 . It can be noticed that LCR grows with an increase in fading power Ω_3 and with a reduction in the Nakagami- m fading parameter m_3 .

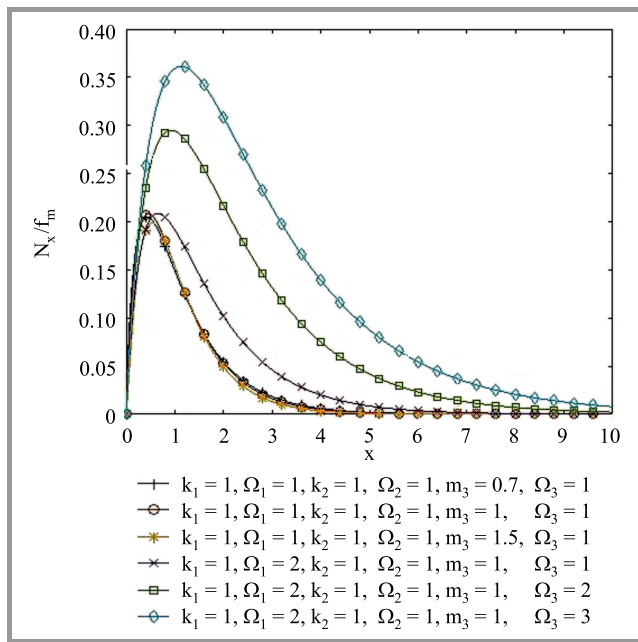


Fig. 6. LCR normalized by f_m versus signal envelope for different values of parameters m_3 and Ω_3 .

4. Conclusion

The product of RVs is applied in multiple relay channels in the presence of composite fading. In this work, we focused on a wireless relay communications channel with three sections, where the product of three RVs describes the amplitude at the output of the cascaded fading channel with three sections. A closed form LCR has been calculated for that channel. The formula obtained has been checked for different values of fading and power parameters. The results are valuable for scientists and system designers dealing with fading models for different wireless channels. It is possible to verify the proposed distribution of the products of other fading amplitudes by measuring parameters in real wireless relay channels in the presence of multipath fading, and also due to the fact that Nakagami- m and Rician distributions are of the general variety. By entering adequate values of fading parameters, other fading distributions in the individual sections may be obtained.

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