Electromagnetic Scattering of Inhomogeneous Plane Wave by Ensemble of Cylinders

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Abstract—**The interaction between an ensemble of cylinders and an inhomogeneous plane wave is introduced and is determined, in the present paper, through a rigorous theoretical approach. Scattered electromagnetic field generated by an indefinite number of infinite circular cylinders is analyzed by the application of the generalized vector cylinder harmonics (VCH) expansion. The exact mathematical model relied upon to represent this scenario considers the so-called complex-angle formalism reaching a superposition of vectorial cylindrical-harmonics and Foldy-Lax Multiple scattering equations (FLMSE) to account for the multiscattering process between the cylinders. The method was validated by comparing the numerical results obtained with the use of the finite element method and a homemade Matlab code.**

Keywords—electromagnetic scattering, inhomogeneous wave dispersion, lossy media, multi-cylinders scattering, vectorial cylindrical-harmonics.

1. Introduction

In the last decades, researchers were focusing on solving Maxwell's equations in order to determine the field scattered by a single or by multiple objects with a specific geometry. Geometrically, electromagnetic scattering has already been analyzed in a thorough manner. In fact, spherical, spheroidal, cylindrical, conical, and ellipsoidal objects may be found in literature, in several works devoted to canonical scattering [1]–[6]. In addition to the cases mentioned above, many other complex scenarios have been examined, such as spheres, cylinders, and axially symmetric objects [7]–[13]. Ensembles of different configurations of scatterers are also considered [14]–[17]. Moreover, different physical and chemical characteristics of materials constituting the scatterer have been analyzed [18]–[21]. Over the years, the problem of describing the interaction between the electromagnetic field and a set of cylinders has led to the development of numerous exact theoretical models that are increasingly simplified [8]–[10].

In this paper, an accurate method for showing an elliptically polarized inhomogeneous plane wave as an expansion of vector cylindrical-harmonics (VCH) is presented. Moreover, in order to analyze the multi-scattering process, the so-called T-matrix approach [22], [23] is used, applying Foldy-Lax multiple scattering equations (FLMSEs) [24], [25] to ensure continuity of tangential components of the electromagnetic fields on the surface of each scatterer. The general representation of an electromagnetic wave as an inhomogeneous wave has continuously attracted a lot of researchers' interest. An electromagnetic wave that propagates in a lossy medium is represented by a complex wave vector with two components: a phase vector and an attenuation vector. A completely lossless medium is an ideal scenario that does not exist in nature. In fact, a wave typically propagates in a lossy medium.

In 1987, the first study related to an inhomogeneous elliptically polarized plane wave was presented by Ivlev, describing the propagation in its elemental structure and investigating the energy fallout as well [26], [27]. The first application proposed subsequently by Ivlev was on an indefinite cylinder. The Adler-Chu-Fano formulation with phase and attenuation vectors [28] was adopted in these studies, leading to meaningful and elegant results with a high degree of complexity.

The present study shows how the representation of the incident field as VCHs superposition could be reached with much greater simplicity through the use of the complex-angle [29] formulation. Also, this approach will be generalized for a scenario involving scattering caused by a cylinder immersed in a lossy medium. Our goal is to use these simplified models and to introduce a concept of losses in the propagation medium in a minimally invasive way. The results will be suitable for updating models, codes, and software presented in the literature to describe real situations with a minimal impact on the code itself. The paper provides numerical comparisons of different phenomena involving cylindrical vector waves. In addition, the study focuses on the analysis of an ensemble of infinite lossy cylinders immersed in a lossy medium, and on its results. A multi-paradigm numerical computing environment (Matlab) was used to implement the various formulations, while the proper model was simulated with the

use of Comsol Multiphysics – commercial software based on FEM.

The article is divided into 4 parts. Section 2 introduces two principal formalisms required to properly represent an inhomogeneous wave and illustrates the formulas needed to switch between both of them. Subsequently, theoretical aspects are described in order to obtain a representation of an inhomogeneous elliptically polarized electric field as a superposition of VCHs. Section 3 shows numerical validations concerning results obtained in Matlab and the ones generated with the use of Comsol Multiphysics. In the same section, new results are also presented for the scattering of an elliptically polarized plane wave at oblique incidence from n-cylinders in a lossy medium. Finally, in Section 4, conclusions are given, along with potential future developments.

2. Theoretical Approach

From literature, two formalisms are known to be used for representing an inhomogeneous wave propagating in a lossy medium. The first one, characterized by better features as well, is the formalism known as Adler-Chu-Fano formulation. Its propagation vector has a complex nature with $\boldsymbol{k}_i = \boldsymbol{\beta}_i + i\boldsymbol{\alpha}_i$ represented by phase and attenuation vectors $\boldsymbol{\beta}_i, \boldsymbol{\alpha}_i \in \Re$, respectively. The other one has a complex propagation vector as well, represented by the superposition of real and imaginary parts $\boldsymbol{k}_i = k_{\mathbf{R}} + ik_{\mathbf{I}}$. This vector forms a complex angle with the axis of the Cartesian reference system $\bar{\vartheta}_i = \vartheta_{\mathbf{R}} + i\vartheta_{\mathbf{I}}$ [29], see Fig. 1. The symbol $\bar{\vartheta}$ was used to underline the complex nature of the angle.

This study shows that a field expressed as a superposition of basic cylindrical waves through the use of the complex-angle formalism may be represented with relative simplicity. The following wave, in which vectors α_i and β_i are creating angles ζ_i and η_i in relation to the z axis, are also placed on the same plane passing through the z axis, and are forming a real angle φ with the x axis (see Fig. 1). In

this particular case, the following relations exist between the two formalisms [29]:

$$\cos \vartheta_{\mathbf{R}} = \frac{k_{\mathbf{R}}\beta \cos \xi + k_{\mathbf{I}}\alpha \cos \eta}{\sqrt{k_{\mathbf{R}}^2\beta^2 - k_{\mathbf{I}}^2\alpha^2 + 2(k_{\mathbf{R}}k_{\mathbf{I}})^2}} \, , \qquad (1)$$

$$\sin \vartheta_{\mathbf{R}} = \frac{k_{\mathbf{R}}\beta \sin \xi + k_{\mathbf{I}}\alpha \sin \eta}{\sqrt{k_{\mathbf{R}}^2\beta^2 - k_{\mathbf{I}}^2\alpha^2 + 2(k_{\mathbf{R}}k_{\mathbf{I}})^2}} \, , \qquad (2)$$

$$\vartheta_{\mathbf{I}} = \frac{1}{2}\text{atanh}\left(\frac{2\beta\alpha}{k^2}\right) \, , \qquad (3)$$

where η and ξ are the angles that the vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, respectively, form with the z axis. Equations (1) and (2) play a fundamental role in assigning a value to $\vartheta_{\mathbf{R}}$ avoiding its indetermination. Plane $\varphi = 0$ was considered due to its simplicity, although the following considerations can be easily expanded to each plane with $\varphi \neq 0$.

The solution of the scalar Helmholtz equation provides the following scalar formula [30]–[36]:

$$\psi_m = Ae^{im\varphi}Z_m(k_\rho\rho)e^{ik_z z - i\omega t} \, , \qquad (4)$$

where ρ, ϕ, z are three variables independent of the cylindrical coordinate system, A is a complex constant, while k_ρ and k_z are the projections of the propagation vector on the plane $z = 0$ and the on the z axis, respectively. The last two components are defined as shown below:

$$k_\rho^2 + k_z^2 = k^2 \, , \qquad (5)$$

with $k_x = k_\rho \cos \varphi$ and $k_y = k_\rho \sin \varphi$ being projections of the transversal vector k_ρ on the plane $z = 0$. The function $Z_m(k_\rho\rho)$ describes the first, second, third, and fourth Bessel functions as $J_m(k_\rho\rho)$, $Y_m(k_\rho\rho)$, $H_m^{(1)}(k_\rho\rho)$, and $H_m^{(2)}(k_\rho\rho)$, respectively. Hence, the harmonic vector is characterized as [31]–[36]:

$$\mathbf{M} = \nabla \times (\hat{\mathbf{z}}_0 \psi) \, , \qquad \mathbf{N} = \frac{1}{k}\nabla \times \mathbf{M} \, . \qquad (6)$$
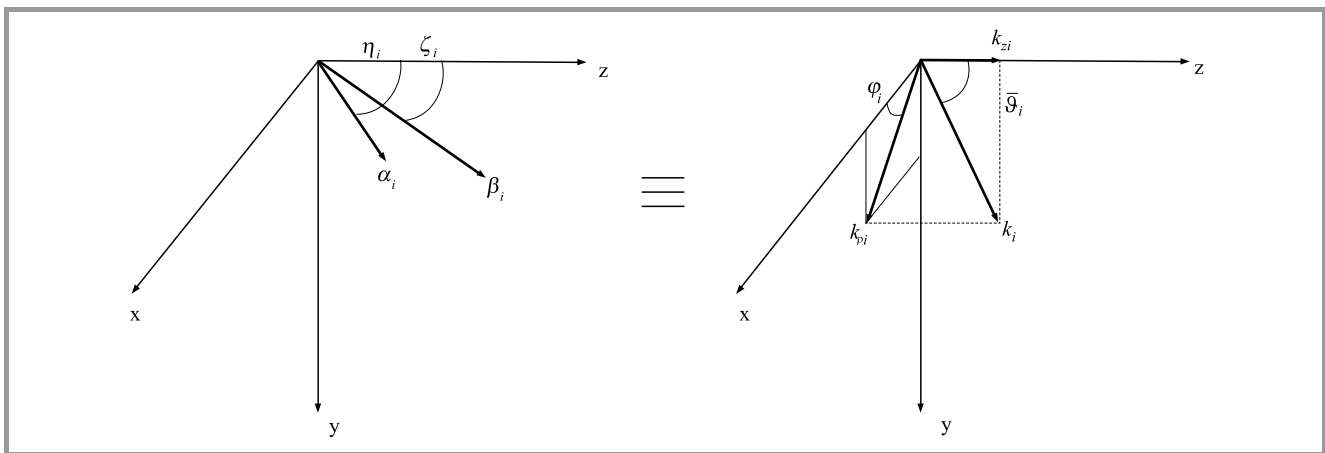

Fig. 1. The figure on the left represents the complex wave vector of an inhomogeneous plane wave with phase and attenuation vectors. The figure on the right represents the same vectors with the complex-angle formulation.

It is always possible to define electric and the magnetic fields as a superposition of these vectorial functions:

$$\mathbf{E} = \sum_{m=-\infty}^{+\infty} (a_m \mathbf{M}_m + b_m \mathbf{N}_m) \ , \tag{7}$$

$$\mathbf{H} = \frac{k}{i\omega\mu} \sum_{m=-\infty}^{+\infty} (a_m \mathbf{N}_m + b_n \mathbf{M}_m) \ . \tag{8}$$

Let us consider a simple inhomogeneous plane wave, using the formalism presented for the first time by Frezza *et al.* [36], [37]. Any obliquely polarized elliptical field, with respect to the surface of a cylinder, can be represented as a linear combination of two components, one vertical ($\mathbf{v}_0$) and one horizontal ($\mathbf{h}_0$), each multiplied by its polarization coefficient (E_{vi} and E_{hi}, respectively):

$$\mathbf{E}(\mathbf{r}) = \left[E_{vi}\mathbf{v}_0(\bar{\vartheta}_i, \varphi_i) + E_{hi}\mathbf{h}_0(\bar{\vartheta}_i, \varphi_i) \right] e^{i\mathbf{k}\cdot\mathbf{r}}$$
$$= \sum_{m=-\infty}^{+\infty} \left[a_m \mathbf{M}_m(k^*\mathbf{r}) + b_m \mathbf{N}_m(k^*\mathbf{r}) \right] \ , \tag{9}$$

imposing the following definitions [36]:

$$a_m = \frac{E_{hi}}{k_\rho}(-i)^{m-1} e^{-im\varphi_i} \ , \tag{10}$$

$$b_m = -\frac{E_{vi}}{k_\rho}(-i)^{m} e^{-im\varphi_i} \ , \tag{11}$$

$$\mathbf{k}_i = k^* \left(\sin\bar{\vartheta}_i \cos\varphi_i \mathbf{x}_0 + \sin\bar{\vartheta}_i \varphi_i \mathbf{y}_0 + \cos\bar{\vartheta}_i \mathbf{z}_0 \right) \ , \tag{12}$$

$$\mathbf{M}_m = \mathbf{m}_m e^{im\varphi} e^{ik_z z - i\omega t} \ , \tag{13}$$

$$\mathbf{N}_m = \mathbf{n}_m e^{im\varphi} e^{ik_z z - i\omega ,t} \tag{14}$$

with:

$$\mathbf{m}_m = im\frac{Z_m(k_\rho \rho)}{\rho}\boldsymbol{\rho}_0 - k_\rho \frac{\partial \rho Z_m(k_\rho \rho)}{\partial \rho}\boldsymbol{\varphi}_0 \ , \tag{15}$$

$$\mathbf{n}_m = i\frac{k_z k_\rho}{k}\frac{\partial \rho Z_m(k_\rho \rho)}{\partial \rho}\boldsymbol{\rho}_0 - \frac{m k_z}{k}\frac{Z_m(k_\rho \rho)}{\rho}\boldsymbol{\varphi}_0 + \tag{16}$$
$$+ \frac{k_\rho^2}{k}Z_m(k_\rho \rho)\mathbf{z}_0 \ ,$$

having indicated, by k^*, the complex conjugate of the wavenumber k and by $\boldsymbol{\rho}_0$, $\boldsymbol{\varphi}_0$ the unit vectors of the cylindrical coordinate system. Considering several parallel cylinders in free space, let us analyze their scattering with the defined incident field, as shown in Fig. 2.
An arbitrarily assigned number L of dielectric cylinders, with relative permittivities ε_j, with $j = 1, \ldots, N$, infinite length and radii r_j in a free-space filled by a lossy medium, in general dissipative, with relative permittivity ε_e, relative permeability μ_e, and electric conductivity σ_e is considered. The incident field, as usual, is an elliptically polarized inhomogeneous plane wave. In order to apply Foldy-Lax multiple scattering equations, the external field on the surface of the q-th cylinder, also referred to as the exciting field, needs to be taken into consideration. The exciting
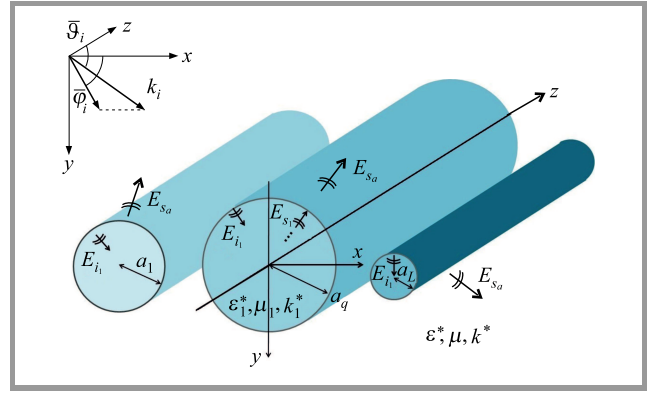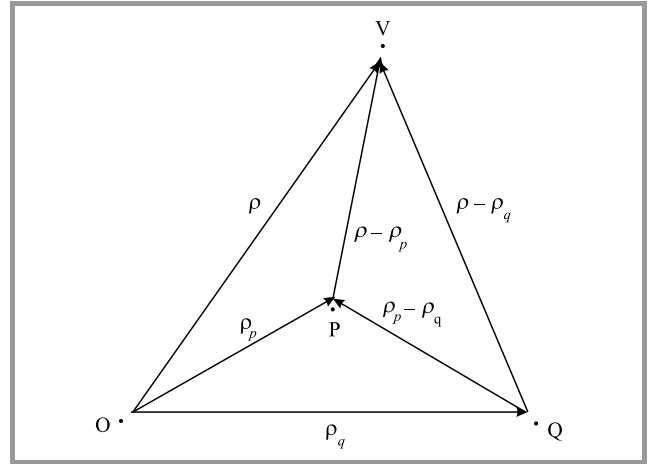
Fig. 2. Depiction of the problem.

Fig. 3. Given three reference systems (O, P, Q), the following figure shows the reciprocal directions of vectors $\boldsymbol{\rho}$ with respect to an external observation point V.

field ($\mathbf{E}_{ex}^q$) is the superposition of the incident field ($\mathbf{E}_i$) and all fields scattered by the cylinders ($\mathbf{E}_s^p$), see Fig. 3:

$$\mathbf{E}_{ex}^q = \mathbf{E}_i + \sum_{\substack{p=1 \\ p\neq q}}^{L} \mathbf{E}_s^p \ . \tag{17}$$

The incident field may be expressed as a function of vector cylindrical harmonics centered on the q-th cylinder [36]:

$$\mathbf{E}_i(k\boldsymbol{\rho}_q) = [E_{vi}\boldsymbol{v} + E_{hi}\boldsymbol{h}] e^{i\mathbf{k}\cdot\boldsymbol{\rho}_q} e^{i\mathbf{k}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}_q)} = \tag{18}$$
$$= \sum_{m=-\infty}^{+\infty} \left[\tilde{a}_m \mathbf{M}_m^{(1)}(k, \boldsymbol{\rho}-\boldsymbol{\rho}_q) + \tilde{b}_m \mathbf{N}_m^{(1)}(k, \boldsymbol{\rho}-\boldsymbol{\rho}_q) \right] \ ,$$

with:

$$\tilde{a}_m = a_m e^{i\mathbf{k}\cdot\boldsymbol{\rho}_q} \ , \qquad \tilde{b}_m = b_m e^{i\mathbf{k}\cdot\boldsymbol{\rho}_q} \tag{19}$$

with k indicating the wavenumber of the host medium (see Fig. 2) and with $\boldsymbol{\rho}_q$ indicating the radial direction centered in the q-th cylinder (see Fig. 3). The exiting field of the q-th cylinder is:

$$\mathbf{E}_{ex}^q(k\boldsymbol{\rho}_q) =$$

$$\sum_{m=-\infty}^{+\infty} \left[w_m^q \mathbf{M}_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) + v_m^q \mathbf{N}_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) \right] , \quad (20)$$

while the scattered electric field from $p \neq q$-th cylinder is:

$$\mathbf{E}_s^p(k\boldsymbol{\rho}_p) = \sum_{m'=-\infty}^{+\infty} \left[T_{m'}^M w_{m'}^p \mathbf{M}_{m'}^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_p) \right. \quad (21)$$

$$\left. + T_{m'}^N v_{m'}^p \mathbf{N}_{m'}^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_p) \right] ,$$

with $T_{m'}^M$ and $T_{m'}^N$ indicating the scattering coefficients in the dielectric cylinder case, i.e. the T-matrix coefficients [24], [25]:

$$T_m^M = -\frac{J_m'(k_{i\rho}a)}{H_m'^{(1)}(k_{i\rho}a)} , \quad (22)$$

$$T_m^N = -\frac{J_m(k_{i\rho}a)}{H_m^{(1)}(k_{i\rho}a)} , \quad (23)$$

while coefficients $w_{m'}^p$ and $v_{m'}^p$ represent the unknowns of our problem. By applying the addition theorem to the VCHs function, we obtain:

$$M_{m'}^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) = \sum_{m=-\infty}^{+\infty} A_{mm'} M_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) , \quad (24)$$

$$N_{m'}^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) = \sum_{m=-\infty}^{+\infty} A_{mm'} N_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) , \quad (25)$$

$$M_{m'}^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) = \sum_{m=-\infty}^{+\infty} B_{mm'} M_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) , \quad (26)$$

$$N_{m'}^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) = \sum_{m=-\infty}^{+\infty} B_{mm'} N_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) , \quad (27)$$

with:

$$A_{mm'} = H_{m-m'}^{(1)}(k|\boldsymbol{\rho}_p - \boldsymbol{\rho}_q|)e^{-i(m-m')\varphi_{pq}} , \quad (28)$$

$$B_{mm'} = J_{m-m'}^{(1)}(k|\boldsymbol{\rho}_p - \boldsymbol{\rho}_q|)e^{-i(m-m')\varphi_{pq}} . \quad (29)$$

By replacing all fields inside the FLMSEs and using the orthogonal properties of the VCHs, the following linear system is obtained:

$$w_m^q = \tilde{a}_m + \sum_{m'=-\infty}^{+\infty} \sum_{\substack{p=1 \\ p\neq q}} A_{mm'} T_{m'}^M w_{m'}^p , \quad (30)$$

$$v_m^q = \tilde{b}_m + \sum_{m'=-\infty}^{+\infty} \sum_{\substack{p=1 \\ p\neq q}} A_{mm'} T_{m'}^N v_{m'}^p . \quad (31)$$

At this point, the linear system may be solved and coefficients w_m^q and v_m^q may be determined. With the field scattered by the q-th cylinder, writable as a superposition of VCHs, as:

$$\mathbf{E}_s^q = \sum_{m=-\infty}^{+\infty} \left[e_m^q \mathbf{M}_m^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) + f_m^q \mathbf{N}_m^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) \right] , \quad (32)$$

the coefficients of the q-th cylinder may be written as follows:

$$e_m^q = T_m^M w_m^q , \quad (33)$$

$$f_m^q = T_m^N v_m^q , \quad (34)$$

and the total scattered field may be obtained:

$$\mathbf{E}_s = \sum_{q=1}^{L} \mathbf{E}_s^q . \quad (35)$$

3. Validation and Numerical Results

Results of the comparison between the formulation and the canonical case of electromagnetic scattering were validated. In this research paper, three infinite lossy dielectric cylinders have been considered with a radius of $a_1 = a_2 = a_3 = 0.125$ m. The three cylinders are centered at the following z plane coordinates: $C_1 = [-0.5, 0]$, $C_2 = [0, 0]$, and $C_3 = [0.5, 0]$ m. A 1 V/m plane wave with a frequency of 300 MHz is impinging on these cylinders.
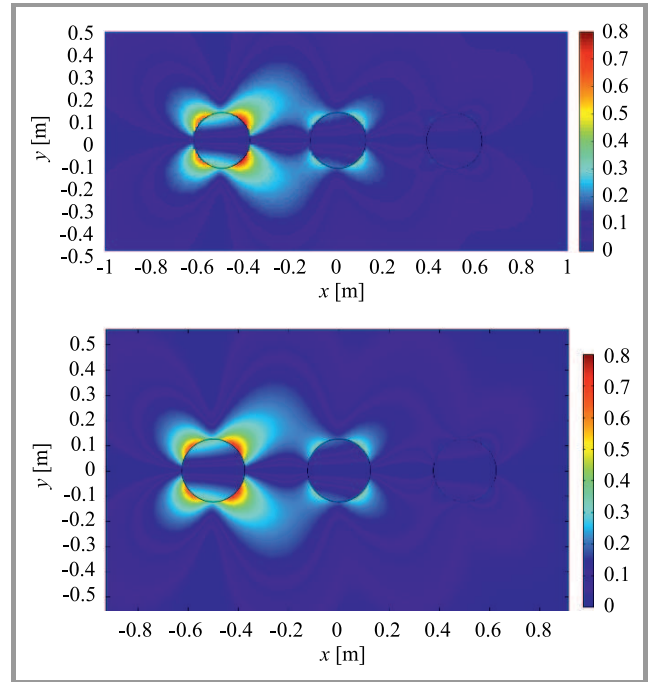

Fig. 4. Comparison of spatial distribution (on the x-y plane) of the absolute value of the x component of the scattered electric field $|E_{s_x}|$. In particular, Matlab (top) and Comsol (bottom) results are shown. (For color pictures see the electronic version of the paper).

In Figs. 4 and 5, results obtained with Matlab and Comsol in the case of $k_e = 1 - i$ 1/m for the environment and $k_c = 2 - 0.5i$ 1/m for all cylinders (arbitrary parameters) for an incident wave directed at $\vartheta_i = \pi/2$ and $\varphi_i = 0$ rad are displayed. In particular, the absolute values of the x and y components of the scattered electric field (the z component is null) are shown. As noted, the results are perfectly compatible.
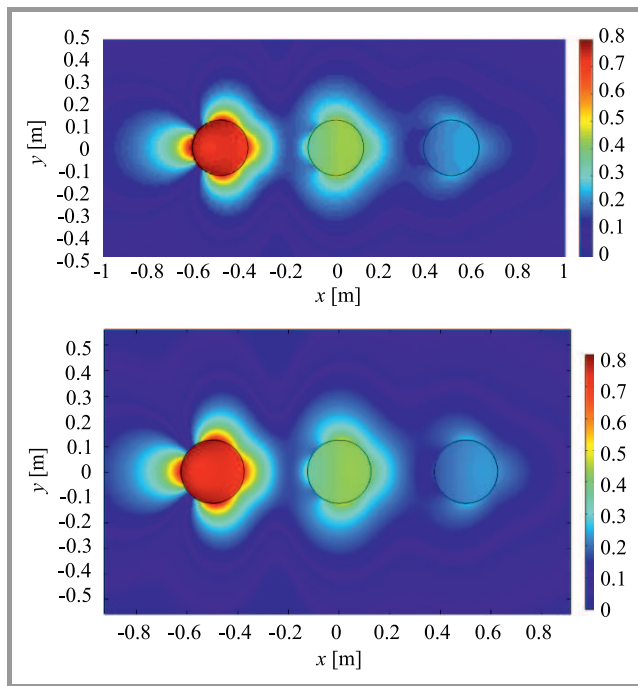
Fig. 5. Comparison of spatial distribution (on the *x-y* plane) of the absolute value of the *y* component of the scattered electric field $|E_{s_y}|$. In particular, Matlab (top) and Comsol (bottom) results are shown.

Subsequently (see Fig. 6), the most general case of an inhomogeneous plane wave with ϑ complex value was shown. In particular, we have considered the following values for the complex angle: $\vartheta = \pi/6(1 + 0.5i)$ rad and $\varphi = \pi/4$ rad (arbitrary parameters). The remaining parameters have remained the same as in the previous case. To obtain these
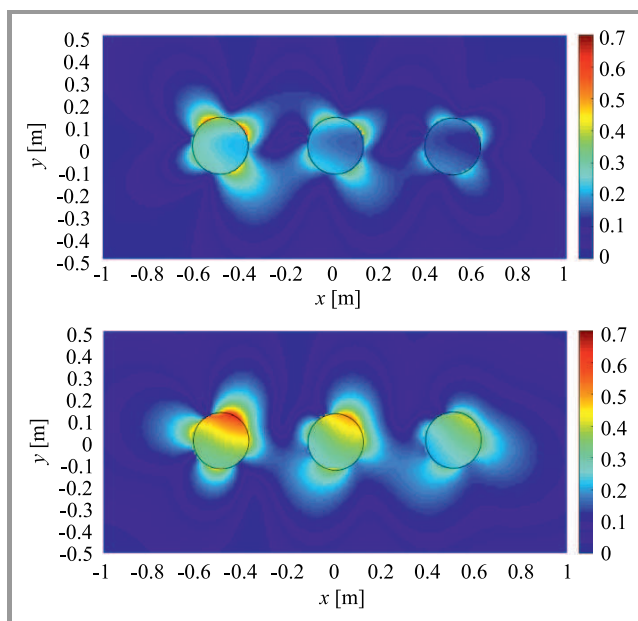

Fig. 6. Spatial distribution of the absolute value (on the *x-y* plane) of the *x* (top) and *y* (bottom) components of the scattered electric field $|E_{s_x}|$, $|E_{s_y}|$ obtained with Matlab.

results, $N = 5$ was used as the number of terms for the VCHs series [38].

In Fig. 6, the contribution of the complex angle is highlighted by the direction of the scattered wave which creates an attenuation effect spreading in the same direction.

4. Conclusions

An accurate method allowing to express an inhomogeneous, elliptically polarized plane wave in terms of vectoral cylinder harmonics, needed to solve the problem of multi-scattering generated by an ensemble of cylinders, is presented. Determination of the expansion coefficients and application of the so-called Foldy-Lay equations required to use the complex-angle formalism contribute to the determination of the solution to the electromagnetic problem. A light and elegant formalism has been reached thanks to such an approach. The procedure was validated with some numerical results as well as with comparisons performed via simulations in the COMSOL environment. In particular, the results were compared with regard to scattering created by three lossy dielectric cylinders of a circular cross-section and infinite length. Perfect compatibility was reached in all scenarios. The cylindrical harmonics defined with the complex angle share the same properties as simple cylindrical harmonics, thanks to the minimal invasiveness of the formalism.

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