Design of a Fractional Order Low-pass Filter Using a Differential Voltage Current Conveyor

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Abstract — In this paper, an active implementation of a differential voltage current conveyor (DVCC) based on a low-pass filter operating in the fractional order domain is presented. The transfer function for a fractional order system is dependent on the rational approximation of s^{α}. Different methods used for calculating the rational approximation, including Carlson, Elkhazalil, and curve fitting, are evaluated here. Finally, to validate the theoretical results, a fractional order Butterworth filter is simulated in the Pspice environment using the 0.5 micrometer CMOS technology with an R-C network-based fractional order capacitor. Additionally, using the Monte Carlo analysis, the impact of current and voltage faults on DVCC response is investigated. It has been inferred that realization with a wider bandwidth is possible.

Keywords — current conveyor, differential voltage, differentiator, fractional order, integrator, RC network, simulation

1. Introduction

In the field of signal processing and communication engineering, active filters are among the most common types of filters used. There are five types of such filters, referred to as low-pass, high-pass, band-pass, band reject and all-pass filters. The name “fractional order filters” refers to the application of the fractional order system in those filters. Integration and differentiation with values that are not integers is the focus of a mathematical discipline dealing with “fractional components” [1]. Different types of fractional order differentiation and integration approaches have been developed, but when dealing with fractional order differentiation and integration systems, the Riemann-Liouville definition is among the most useful solutions. A fractance device exhibits fractional order impedance properties. In today’s world, the application of fractance, also known as fractional order elements, is a topic of numerous research works, e.g. [2]–[5]. The method is applied across a wide range of engineering and research domains. The qualities of a resistor, an inductor, and a capacitor can all be demonstrated with a single element using mathematical modeling. Several examples of these types of approximations include Mastuda’s approximation, Chareff’s approximation, the continued fraction expansion approach, and others. These approaches may be relied upon for computing rational approximation.

In this article, a fractional order low-pass filter that uses a smaller number of components is proposed. Different mathematical models, such as continued fraction expansion, regular Newton process, and Taylor series expansion with a higher order using R-L, R-C networks, all rely on fractance devices [6]. A genetic algorithm-based approach for designing fractional order elements is discussed in [7]. The use of simulation software in fractional order signal processing is presented in [8]. A field programmable analog array has been used for the realization of a fractance device in [9]. An exploratory study of different fractance device synthesis techniques is made available in [10]–[12]. A fractance device is also used in the design of fractional order controllers studied in [13]. Several equivalent models of the constant phase element are discussed in [14].

Many of the commercially available active devices find application in the design of fractional order elements. Operational transconductance amplifiers (OTA) are used for fractional order differentiation [15]. The application of fractance in biomedical engineering is studied in [16], where the authors have followed a novel topology for the realization of OTA-based circuits [16]. Paper [17] covers the design of a fractional order system with a Butterworth low-pass filter. It should be noted that the responses of fractional order filters can be changed solely by changing the order of fractional derivatives. Article [17] introduces a fractional order Butterworth low-pass filter based on DVCC that can be used for various applications, i.e. in microwave frequencies [18].

The realization of fractional filters is one of the most common and successful filter design techniques, and some important developments in this area have been observed in recent years. It has been noted that the practical implementation of fractance devices for microwave signals is constrained by the fact that existing approximation circuits for fractional devices may contain numerous lumped elements for a single fractional element [18]. In this area, work [19] presents model of fractional differentiators’ with reduced integer order using two steps.

The paper is structured in the following manner. DVCC is covered in detail in Section 2. Section 3 presents an in-depth analysis of the fractional order low-pass filter and of different approximation techniques. In Section 4, the designing of a fractional order Butterworth filter that is based on DVCC is presented. Circuit simulations and their results are presented in Section 5. The conclusions are presented in the Section 6.

2. Differential Voltage Current Conveyor

Since the beginning of the twenty-first century, many electronic applications have been constructed using current mode topology, due to the numerous advantages it offers, including high performance, high slew rate, higher dynamic range, better bandwidth, and power savings.

Differential voltage current conveyors (DVCC) have four terminals on their analogue blocks, and the voltages and currents that they carry are described with the use of the following matrix [17]:

$$\begin{bmatrix} I_{Y1} \\ I_{Y2} \\ V_X \\ I_Z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_{Y1} \\ V_{Y2} \\ I_X \\ V_Z \end{bmatrix} . \quad (1)$$

A block diagram representation of a DVCC is as shown in Fig. 1. A CMOS implementation of a DVCC is shown in Fig. 2 [17].

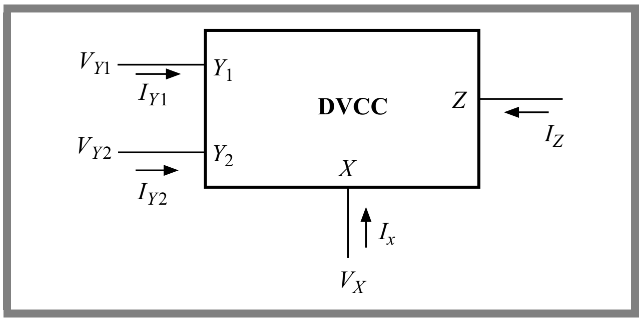

Fig. 1. DVCC block diagram.

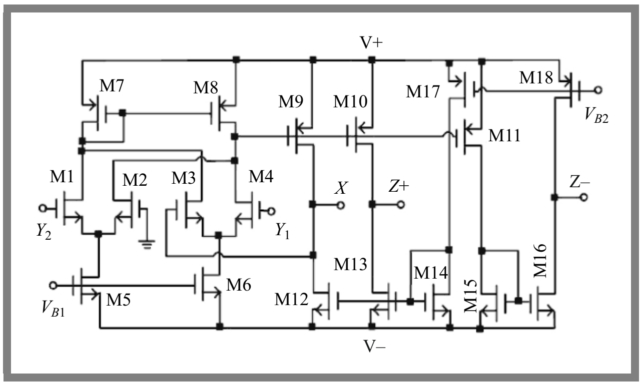

Fig. 2. DVCC block implementation using CMOS technology.

3. Fractional Order Low-pass Filter

The transfer function of a fractional order filter is obtained as [17]:

$$T(s) = \frac{k_3}{s^{\alpha+\beta} + k_1 s^{\alpha} + k_2} , \quad (2)$$

where $k_1 = \frac{R}{L}$ and $k_2 = k_3 = \frac{1}{LC}$.

A fractional order Butterworth low-pass frequency response filter is given by:

$$f_c = \frac{1}{2\pi} k_1^{\frac{1}{\alpha+\beta}} . \quad (3)$$

3.1. Carlson Method

Carlson and Halijak [2] proposed an approximation method for approximating fractance using the regular Newton process. The general expression of the approximation is given by:

$$G_{k+1}(s) = G_k(s) \frac{(n-1)G_k^n(s) + (n+1)H(s)}{(n+1)G_k^n(s) + (n-1)H(s)} , \quad (4)$$

where n is the order of approximation and k is the iteration number.

The interval of frequencies in which the approximation is valid is always centered at unity. We assume $G_0(s) = 1$ in the rational approximation for:

$$s^{-0.5} = \frac{s^4 + 36s^3 + 126s^2 + 84s + 9}{9s^4 + 84s^3 + 126s^2 + 36s + 1} . \quad (5)$$

When using this method, approximations for the fractance device of even integer order are the only ones that can be obtained. User's requirements determine the order in which the approximation is performed. Nevertheless, in the event that the order quantity increases, the realization will become more challenging and will call for additional hardware.

3.2. Reyad El-Khazali Approximation

This approximation is proposed by Reyad El-Khazali *et al.* [20]. A fractance operator can be approximated using a cascade connection of biquadratic transfer functions with the form given by:

$$\left(\frac{s}{\omega_g}\right)^{\alpha} = \prod_{i=1}^{n} H_i\left(\frac{s}{\omega_i}\right) = \prod_{i=1}^{n} \frac{N_i\left(\frac{\frac{s}{\omega_i}}{\omega_g}\right)}{D_i\left(\frac{\frac{s}{\omega_i}}{\omega_g}\right)} , \quad (6)$$

where ω_i, $i = 1, 2, \ldots, n$ is the center frequency of each biquadratic module and ω_g is their geometric mean. By selecting the first center frequency ω_1, the remaining center frequencies can be calculated using a recursive formula:

$$\omega_i = \omega_x^{2(i-1)} \omega_1, \quad i = 2, 3, 4, \ldots, n . \quad (7)$$

The value of ω_x is calculated by obtaining the maximum real solution of the following equation:

$$a_0 a_2 \eta \gamma^4 + a_1(a_2 - a_0)\gamma^3 + \left(a_1^2 - a_2^2 - a_0^2\right)\eta\gamma^2 + a_1(a_2 - a_0)\gamma + a_0 a_2 \eta = 0 , \quad (8)$$

where $\eta = \tan\frac{\alpha\pi}{4}$. The expression for the quadratic is:

$$\left(\frac{s}{\omega_g}\right)^{\alpha} \cong \frac{a_0\left(\frac{s}{\omega_i}\right)^2 + a_1\left(\frac{s}{\omega_i}\right) + a_2}{a_2\left(\frac{s}{\omega_i}\right)^2 + a_1\left(\frac{s}{\omega_i}\right) + a_0} , \; i = 1, 2, 3, \ldots . \quad (9)$$

The values of coefficients $a_0,\ a_1,\ a_2$ are:

$$a_0 = \alpha^\alpha + 2\alpha + 1$$
$$a_2 = \alpha^\alpha - 2\alpha + 1 \tag{10}$$
$$a_1 = (a_2 - a_0)\tan\frac{(2+\alpha)\,\pi}{4}\,.$$

After the coefficients have been determined, it is possible to obtain a rational approximation for the value of s^α. In order to determine the expression for $s^{-\alpha}$, it is necessary to switch the polynomials that are found in the numerator and the denominator. For $n = 2,\ \alpha = 0.5$, the transfer function is given by:

$$s^{-0.5} = \frac{0.5s^4 + 35.89s^3 + 231s^2 + 137.4s + 7.328}{7.328s^4 + 137.4s^3 + 231s^2 + 137.4s + 0.5}\,. \tag{11}$$

3.3. Frequency Response-based Curve Fitting Approximation

Kishore Bingi *et al.* [21] proposed this approximation as a solution to the problem of fitting in a frequency domain. The ease with which the technique can be put into practice is one of the most important aspects of this method. The findings of the simulation demonstrated that the outcomes produced by this approximation are superior to those produced by Oustaloup and Matsuda approximations. The design procedure is as follows:

1) Compute the frequency response data of s^α in the frequency range $[\omega_l, \omega_h]$ that is requested. This can be accomplished by using the built-in Matlab function frd();
2) Calculate the exact frequency response data of s^α by employing the Sanathanan-Koerner (SK) least squares iterative approach;
3) Choose a value for the order parameter of the rational approximation N;
4) To determine the values of numerator and denominator coefficients, use the Matlab fitfrd() built-in function. The model that is obtained will be in the form of a state space;
5) Use Matlab's ss2tf to convert the state space model into a transfer function.

By selecting $N = 4$, the rational approximation is obtained as shown below:

$$s^{-0.5} = \frac{s^4 + 1137s^3 + 42750s^2 + 86810s + 10130}{51.13s^4 + 10390s^3 + 84630s^2 + 44570s + 638.8}\,. \tag{12}$$

A comparison of the amplitude and phase responses of all approximation techniques is shown in Figs. 3 and 4.

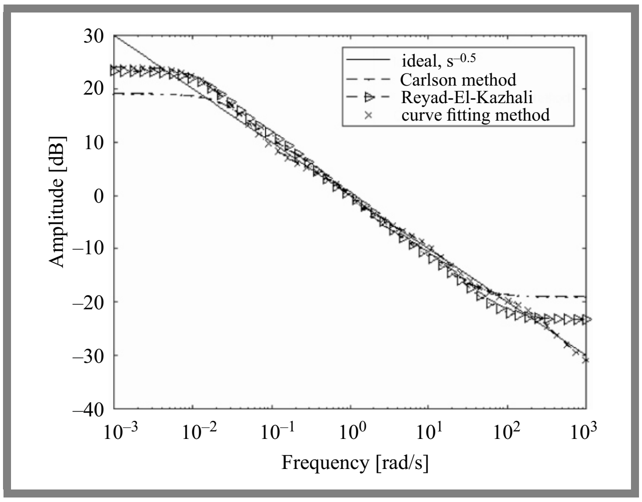

Fig. 3. Amplitude response comparison.

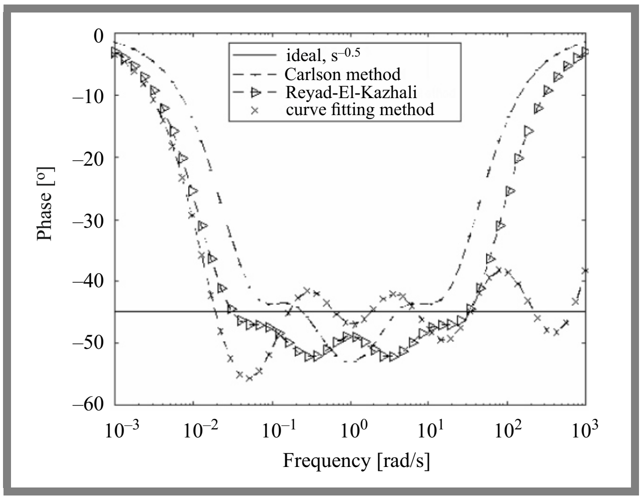

Fig. 4. Phase response comparison.

4. DVCC-based Fractional Order Butterworth Filter Design

A DVCC-based fractional order low-pass filter using two fractional order elements is shown in Figs. 5–6. In this paper, the fractional order values are chosen as $\alpha = 0.5$ and $\beta = 1$.

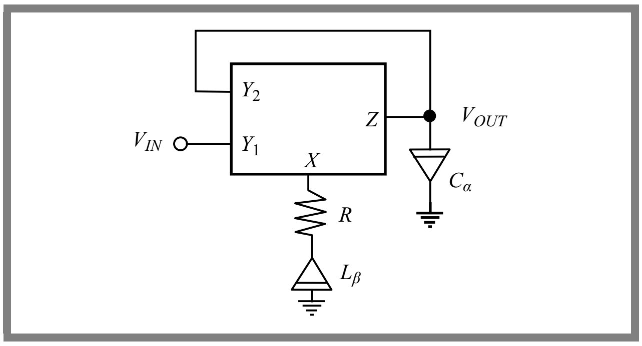

Fig. 5. Fractional order low-pass filter diagram.

For $\alpha = 0.5$, the transfer function given in Eqs. (4), (11), (12) has been realized using the network synthesis procedure. The Pspice diagram used for the realization of the low-pass filter using the proposed RC circuit is shown in Fig. 7. The circuit can be realized by five resistors and four capacitors in place of the fractional capacitor of the fractional order

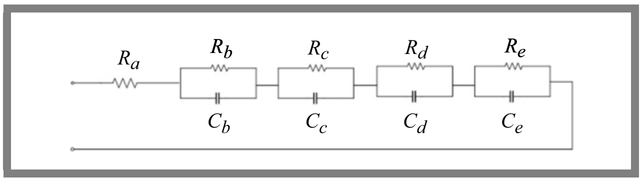

Fig. 6. Fractional order low-pass filter diagram.

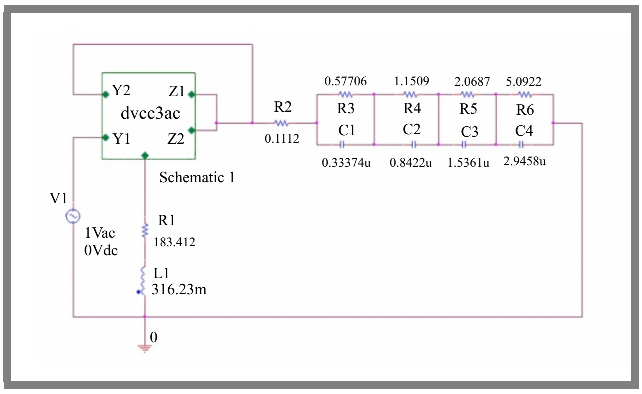

Fig. 7. Carlson-based filter realization using Pspice.

Butterworth lowpass filter design. The element values synthesized using different approximation techniques are presented in Tab. 1.

Tab. 1. Component values for researched filters.

Element	Carlson	El-Khazali	Curve fitting	[17]
R_a	0.111 Ω	0.0682 Ω	0.0196 Ω	14.2 Ω
R_b	0.577 Ω	7.751 Ω	13.137 Ω	31.7 Ω
R_c	1.150 Ω	1.809 Ω	3.0515 Ω	47.8 Ω
R_d	2.068 Ω	0.474 Ω	0.8651 Ω	117 Ω
R_e	5.092 Ω	0.085 Ω	0.698 Ω	929 Ω
C_b	0.3374 F	0.4585 F	0.0674 F	0.66 μF
C_c	0.8422 F	1.0794 F	0.3856 F	2.3 μF
C_d	1.5361 F	3.0432 F	1.573 F	4.3 μF
C_e	2.9458 F	5.4854 F	4.7504 F	5.5 μF

5. Results and Discussion

The fractional order Butterworth low-pass filter is simulated using Pspice software. The Butterworth condition is satisfied by $k_2 = 31622.77$ and $k_1 = 580$ and the circuit parameters are R = 183.412 Ω, $L = 316.23$ mH, $C = 100$ μ**F**.

Tab. 2. Comparison of the bandwidth for different methods of filter design.

Method	Bandwidth
Curve fitting method	2.15 kHz
El-Khazali method	2.19 kHz
Carlson method	2.03 kHz
[17]	204.6 Hz

The CMOS schematic of DVCC is simulated by a 0.5 μm CMOS model. The width-to-length ratios of DVCC are listed in the table, and the supply voltages are $V_{DD} = -V_{SS} =$ 1.5 V, with biasing voltages equaling –0.52 V and 0.33 V, respectively. Denormalization values of 10 kΩ and 5 nF are taken to make the synthesized values realistic. Table 2 shows the bandwidth of existing filters and the proposed solution, while Figs. 8–10 illustrate simulations of frequency characteristics of proposed filter.

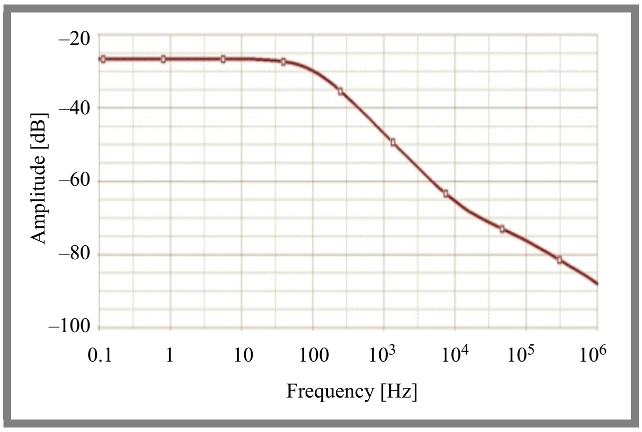

Fig. 8. Amplitude response of the proposed realization using PSpice.

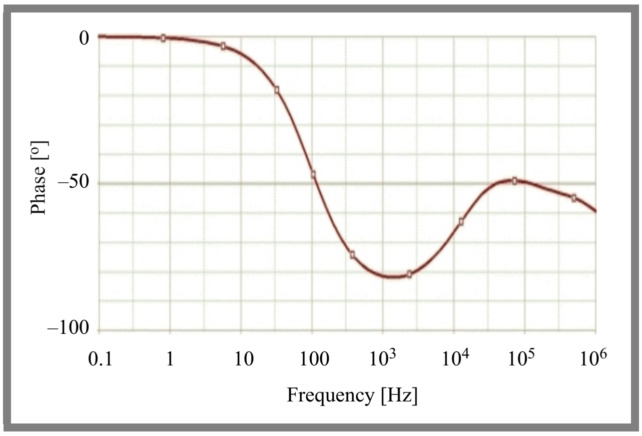

Fig. 9. Phase response of the proposed realization using PSpice.

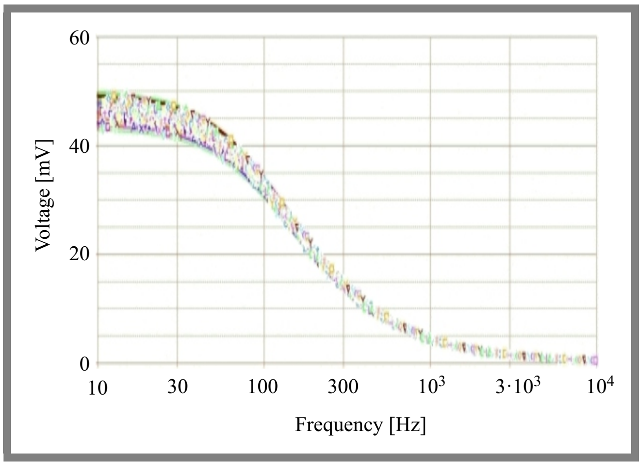

Fig. 10. Monte Carlo simulations.

6. Conclusions

A fractional order low-pass filter using Carlson-based approximation technique is realized in this paper. Initially, different rational approximation techniques are studied. The fifth-order approximation is considered to maintain reasonable hardware complexity. A block for DVCC is created in PSpice. Using network synthesis procedures, a passive equivalent of the fractional order capacitor is designed. It has been observed that the Butterworth filter can be realized with a higher bandwidth value compared with the previous procedures, and bandwidth obtained with the use of the El-Kazali method is superior compared with other methods. The performance of curve fitting and El-Khazali methods is almost the same. The method can be extended to the design of other types of filters.

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