Minimized Group Delay FIR Low Pass Filter Design Using Modified Differential Search Algorithm

Sonelal Prajapati[1], Sanjeev Rai[1], Manish Tiwari[1], and Atul Kumar Dwivedi[2]

[1]*Motilal Nehru National Institute of Technology Allahabad, Prayagraj, India,*
[2]*Bundelkhand Institute of Engineering and Technology, Jhansi, India*

https://doi.org/10.26636/jtit.2023.3.1313

Abstract — Designing a finite impulse response (FIR) filter with minimal group delay has proven to be a difficult task. Many research studies have focused on reducing pass band and stop band ripples in FIR filter design, often overlooking the optimization of group delay. While some works have considered group delay reduction, their approaches were not optimal. Consequently, the achievement of an optimal design for a filter with a low group delay value still remains a challenge. In this work, a modified differential search optimization algorithm has been used for the purpose of designing a minimal group delay FIR filter. The results obtained have been compared with the classical techniques and they turned out to be promising.

Keywords — differential search optimization algorithm, FIR filter, optimization, small group delay

1. Introduction

Recently, digital finite impulse response (FIR) filters have caught the interest of researchers in a range of fields, including seismic and satellite signal processing, pattern recognition, biomedical signal processing and more. This is primarily because of their desirable characteristics, such as linear phase, inherent stability, suitability for multi-rate applications, and ease of implementation. FIR filters are known for their stable nature, low sensitivity, and ability to maintain a linear phase [1].

However, one common drawback of FIR filters is that they require higher filter orders to meet specific requirements, thus resulting in increased memory usage and processing delays. To ensure effective and reliable operation of FIR filters, a well-defined design procedure is necessary. Various approaches have been proposed to generate coefficient sets that aim to meet the desired specifications. Design engineers face such challenges as passband ripples (PBR), stopband ripples (SBR), and find it difficult to achieve sharper cutoffs with minimal filter orders. Apart from frequency domain specifications, a properly designed filter should also minimize group delays, as their high values lead to increased power consumption.

The motivation behind this study is to design filters that not only meet the desired frequency specifications, but also provide minimal delay once implemented in their hardware form. Group delay is impacted by the electrical length of the digital signal processing (DSP) system and can cause distortion of the output signal. Linear phase FIR filters offer constant group delay in the passband, if designed symmetrically. However, a major disadvantage is that they provide an overall group delay of $N/2$, where N represents the filter order. This large group delay reduces the speed of processing the signal by the filters, thus necessitating the mitigation of group delay in order to speed up signal processing.

A group delay reduction approach based on multirate techniques was proposed for active noise control systems in [2]. This approach utilizes polyphase decomposition and requires the design of multiple filters and multirate signal processing, consequently increasing design complexity and hardware requirements.

Other methods have explored small group delay for FIR filters using minimax constrained optimization, convex optimization based on semi-definite programming, quasi-convex optimization using second-order cone optimization, and minimax group delay reduction with the implementation in all-pass filters. However, the optimization approaches employed in these methods were not globally optimal, and the considered examples of filters were characterized by high orders, thus requiring significant power and chip area for implementation. Designing a minimum group delay filter using a global optimization approach still remains a rather challenging task.

Previously, evolutionary optimization algorithms, such as the genetic algorithm (GA), particle swarm optimization (PSO), simulated annealing (SA), artificial bee colony (ABC), bacteria foraging algorithm (BFO), cat swarm optimization (CSO), differential evolution (DE), and orthogonal harmony search (OHS) have been utilized for solving complex, nonlinear, multidimensional FIR filter design problems. However, none of these approaches specifically considered the design of a low group delay FIR filter.

In this work, the optimization-based function for group delay reduction is formulated. The conventional differential search algorithm (DSA), bijective differential algorithm (BDSA), and the proposed differential search algorithm (mDSA) are employed to solve the problem of optimizing the design of a minimal group delay FIR filter. The effectiveness of the

proposed approach is evaluated by comparing the designed filters with existing methods.
The article is structured as follows. Section 2 presents the mathematical formulation of the problem, specifically focusing on the design of a minimum group delay FIR filter. This formulation enables the application of the proposed algorithm called mDSA. Section 3 contains a description of specific optimization algorithms, such as DSA, BDSA, and mDSA. In Section 4, the results obained upon implementing DSA, BDSA, and mDSA are discussed. Finally, conclusions concerning the techniques proposed in the article are presented in Section 5.

2. Model of a Minimum Group Delay FIR Filter

The output $y(t)$ of a signal processing system whose impulse response is $h(t)$, for an input $x(t)$ is given by:

$$y(t) = (h * x)(t) = \int_{-\infty}^{\infty} x(d)\, h(t-\tau)\, \mathrm{d}\tau \ , \tag{1}$$

where τ is the unit sample delay in time domain. In the frequency analysis, the same can be represented as:

$$Y(s) = [H(s)]\, [X(s)] \ , \tag{2}$$

where

$$X(s) = \mathcal{L}\Big\{x(t)\Big\} \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} x(t)\, \mathrm{e}^{-st}\, \mathrm{d}t \ , \tag{3}$$

and

$$H(s) = \mathcal{L}\Big\{h(t)\Big\} \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} h(t)\, \mathrm{e}^{-st}\, \mathrm{d}t \ , \tag{4}$$

where $h(t)$ is the time-domain impulse response of the LTI system, $X(s)$, $Y(s)$, $H(s)$ are the Laplace transforms of the input $x(t)$, output $y(t)$, and impulse response $h(t)$, respectively. $H(s)$ is called the transfer function of the LTI system and, like the impulse response $h(t)$, fully defines the input-output characteristics of the LTI system.
The main objective of this work is to formulate an objective function for optimizing the design of a low group delay FIR filter. The main requirement of an appropriately designed filter, as formulated in this work, is lower group delay with minimum pass band ripple, stop band ripples with smaller transition width. The objective function has been framed as deviation from the desired filter response to the designed filter response. The deviation of pass band ripple is:

$$f_1(b) = \mid (max[e(\omega)] - \delta_{p_m}) - 1 \mid \quad \text{for} \quad \omega \leqslant \omega_p \ , \tag{5}$$

where δ_{p_m} is the maximum pass band ripple, w_p is the pass-band edge frequency, $e(\omega)$ is the error difference between the desired filter response $B_d(\omega)$ and the actual filter response $B(\omega)$ which is written as:

$$e(\omega) = [B_d(\omega) - B(\omega)] \ . \tag{6}$$

The second part of the objective function is the deviation from the desired filter response to the designed filter response of stop band ripple which can be written as:

$$f_2(b) = \mid (max\ e(\omega) - \delta_{s_m}) \mid \quad \text{for} \quad \omega \geqslant \omega_s \ , \tag{7}$$

where δ_{s_m} is the maximum stop-band ripple, w_s is the stop-band edge frequency. The frequency response $B(\omega)$ of a linear phase designed FIR filter design is:

$$B(\omega) = \sum_{m=0}^{M-1} b(m)\, \mathrm{e}^{-\mathrm{j}(m\omega)} \ , \tag{8}$$

where $b(m) = h(m)$ for $m = 0, 1, \ldots, M-1$, $h(m)$ is the impulse response, and m is an integer. The third part of the objective function is formulated as group delay. It is calculated as a derivative of phase with respect to angular frequency ω. The group delay response of a filter is a measure of the average delay of the filter as a function of frequency. It is the negative first derivative of the phase response of the filter. If the frequency response of a filter is $H(\mathrm{e}^{\mathrm{j}\omega})$, then the group delay is:

$$\tau_g(\omega) = -\frac{\mathrm{d}\,\phi(\omega)}{\mathrm{d}\,\omega} \ , \tag{9}$$

where $\phi(\omega)$ is the phase, or argument, of $H(\mathrm{e}^{\mathrm{j}\omega})$. Group delay is considered to be the third part of the objective function for optimization:

$$f_3(b) = \tau_g(\omega) \ . \tag{10}$$

The overall objective function optimization is:

$$f = f_1 + f_2 + f_3 \ . \tag{11}$$

The optimization problem given by Eq. (11) has been solved using DSA, BDSA and the proposed mDSA optimization algorithm discussed in Section 3.

3. Differential Search Algorithm

DSA is a well-known population-based meta-heuristic optimization algorithm that is based on the migration behavior of superorganisms [3]. The availability of food resources varies throughout the year due to varying climate conditions. As a result, many live species move. Migration behavior permits living organisms to move from an inefficient, food-limited habitat to a productive, food-rich environment. Ants and birds, for example, follow a regular migration pattern. During migration, species develop superorganisms made up of several individuals. Superorganisms start relocating to more productive locations to suit their needs. They can linger at the new place for a while before moving on to more productive areas. In DSA, an artificial-superorganism is a population that consists of arbitrary solutions to the corresponding problem. During migration, an artificial superorganism decides where to stop. Individuals of the superorganisms that discovered it would immediately reside there and would continue their migration to the global minimum value.
In DSA, the artificial organism can be represented as $X_i, i = 1, 2, \ldots, N_{pop}$ and the superogranism made up of these artificial organisms is defined as $superorganism_g = [X_i]$ where $g = 1, 2, \ldots, G$ contains the members according to the size of the problem such that $x_{ij}, j = 1, 2, \ldots, D$ where N_{pop} represents the number of individuals, and D signifies the size of the problem, while G is the maximum generation or iteration number. In such case, the operation of DSA can be described by the following steps:

1. Initialization of artificial-organism. The first step is to initialize each member of the artificial-organism to its random position such that:

$$x_{ij} = lb_j + \text{rand}[0,1] \times (ub_j - lb_j) \;, \tag{12}$$

where $rand[0,1]$ is the random value between 0 and 1 and ub_j, lb_j are the upper bounds and lower bounds of j-th dimension of the respective problem, respectively.

2. Initialization of control parameters. The DSA algorithm has only two control parameters p_1 and p_2 which are used to determine the number of disturbances corresponding to the position of each member of the artificial-organism. Scale is used to determine the magnitude of change in the position of an individual member of the artificial organism. The value of scale is generated by gamma random number generator $rand_g$ so the values of p_1, p_2, and $scale$ can be calculated as:

$$p_1 = 0.3 \times rand_1 \;\text{ and }\; p_2 = 0.3 \times rand_2 \;, \tag{13}$$

$$scale = rand_g \times (2 \times rand_3) \times rand_4 - rand_5 \;, \tag{14}$$

where $rand_1$, $rand_2$, $rand_3$, $rand_4$, and $rand_5$ are the uniform random number generators. In this way, the artificial organism may radically change its direction within the habitat.

3. Compute the stop-over site. During the migration process, the stop-over site is a randomly shuffling intermediate position assumed by the artificial organism based on the Brownian-like random walk model. In order to calculate the stop-over site, the algorithm creates stop-over vector $S_{i,G}$ which can be calculated as:

$$S_{i,G} = superorganism(X_{i,G}) + scale \times (superorganism(X_{i,G}) - donor) \;, \tag{15}$$

where $donor$ refers to the generation of the direction matrix based on the random shuffling of individual members of the artificial organism moving towards the target, such that:

$$donor = X_{random\_shuffling(i)} \;. \tag{16}$$

4. Selection of next generation. The selection operator $G = G + 1$ is used to calculate the next generation between the artificial organism population and the stop-over vector population. This selection is based on the fitness value:

$$S_{i,G+1} = \begin{cases} S_{i,G}, & \text{if } f(S_{i,G}) \leqslant f(X_{i,G}) \\ X_{i,G}, & \text{if } f(S_{i,G}) > f(X_{i,G}) \end{cases} . \tag{17}$$

5. Check the termination criterion. If the stop-over site is more fruitful then the source owned by the artificial organism, then the artificial organism shifts to the newly discovered position. However, the superorganisms continue to migrate until the global optimum (best solution) is reached.

3.1. Bijective Differential Search Algorithm (BDSA)

The bijective differential search algorithm has been explored by Civicioglu [3] based on stochastic permutation of the original population of the algorithm. It is different from the original differential search algorithm in terms of scale factor R and the section method. In BDSA, the scale factor is defined with the help of the stochastic walk of the population replacing the Brownian walk relied upon in the differential search algorithm.

3.2. Modified Differential Search Algorithm

The DSA and BDSA do not guarantee to find global optima in all types of situations, and the rate of exploitation at the local optima is relatively modest. Therefore, in this work, a modified differential algorithm has been proposed which utilizes a quantum-based selection operator. In the quantum-inspired differential search algorithm [4], the superorganism $|\psi>$ is represented by a pair of complex numbers, as a vector $[\alpha\beta]^T$:

$$|\psi>= \alpha|0> + \beta|1> \;, \tag{18}$$

where α and β are probabilities with the relation between them being $\alpha^2 + \beta^2 = 1$, while $|0>$ and $|1>$ are qbit (i.e. representation of a quantum bit). The superorganism of length n is represented by a qbit.

$$q = \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix} \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix} \begin{bmatrix} \ldots \\ \ldots \end{bmatrix} \begin{bmatrix} \alpha_n \\ \beta_n \end{bmatrix} . \tag{19}$$

One example of a superorganism of this type, having two qbits, can be represented as:

$$q = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix} . \tag{20}$$

In quantum differential equations, there are NP individuals and each individual is composed of D qbits. Because the probability of states of every qbit satisfies the relationship of $\alpha^2 + \beta^2 = 1$, qbits can also be represented by $\sin\theta$ and $\cos\theta$. After generation of the superorganism, other control parameters are initialized as real numbers, which are later on converted to qbits. Now, the stop over site is also calculated with the help of the quantum mechanism. In this operation, diversity of the population is maintained by the donor and it is increased because of the quantum-based representation of population. The Brownian motion of the stop over site is more diverse because of the quantum representation. The next generation is selected with the help of qbits represented by $\sin\theta$ and $\cos\theta$. Therefore, Eq. (17) can be modified as:

$$\theta_{i,G+1} = \begin{cases} \theta_{i,G}, & \text{if } f(\theta_{i,G}) \leqslant f(\theta_{i,G}) \\ \theta_{i,G}, & \text{if } f(\theta_{i,G}) > f(\theta_{i,G}) \end{cases} . \tag{21}$$

These quantum operators increase the diversity of the population, thus expanding the algorithm's search space.

4. Results and Implications

In this work, the proposed technique has been tested and evaluated using Matlab. The performance of the proposed technique has been evaluated under three scenarios. In the first case, it has been tested in terms of standard benchmark functions. In the second case, it has been tested in connection with designing a low pass FIR filter. For each algorithm, the population size has been taken as 100. Each algorithm is executed over 500 iterations. The algorithm's parameters

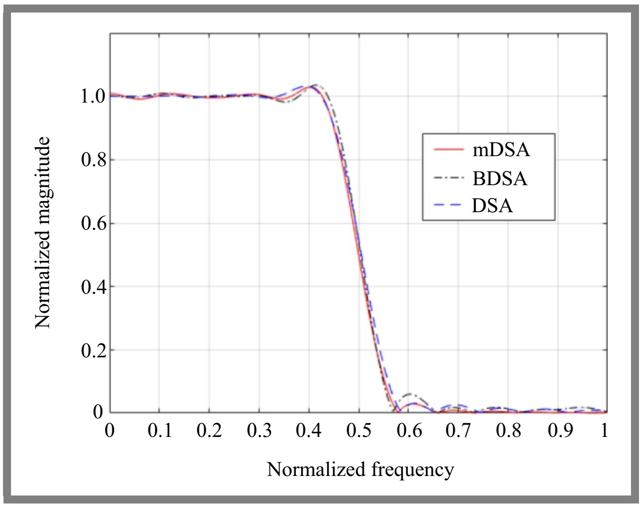

Fig. 1. Normalized magnitude response of the FIR filter.

have been set after number of trials. In order to take into account randomization of the optimization algorithm, 50 trials have been performed for each algorithm, with the minimum and maximum values of the parameters being calculated with a standard deviation. The statistical hypothesis test is performed using the t test to evaluate the significance of the proposed technique.

4.1. Case 1 – Benchmark Function Evaluation

For evaluating the impact of the proposed technique on benchmark functions, five multimodal, non-linear benchmark functions are considered (Tab. 1). The benchmark functions have been tested at 30 dimensions. The DSA, BDSA and the proposed mDSA approaches have been applied to these selected benchmark equations.

The best and normalized values of the benchmark functions are presented in Tab. 2, where optimized results are shown, after normalization. The value in each row 50 is the representation of the minimum value and other values are scaled corresponding to this value. Thus, the proposed mDSA is capable of finding optimum results while performing standard benchmark functions. DSA performs better than BDSA for Schewelfel 2.21 and Whiteley functions. However, for Ackley, Griewank and Rastringin functions, BDSA performs better than DSA. Therefore, it may be concluded that the proposed quantum-inspired differential search algorithm finds a better solution.

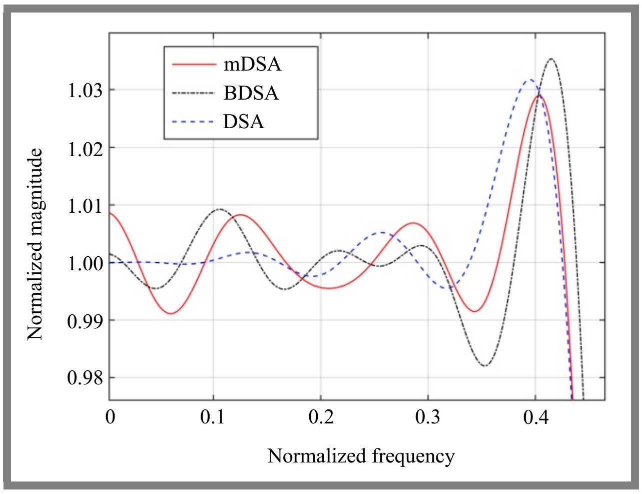

Fig. 2. Normalized PBR response of the FIR filter.

Tab. 1. Benchmark functions.

No.	Function	Search range	Optimized value (min. at)
F_1	Griewank	$-512 \dots 512$	$f(0,\dots,0)=0$
F_2	Ackley	$-32 \dots 32$	$f(0,\dots,0)=0$
F_3	Rastringin	$-5.12 \dots 5.12$	$f(0,\dots,0)=0$
F_4	Schwefel's 2.21	$-100 \dots 100$	$f(0,\dots,0)=0$
F_5	Whitley	$-10.24 \dots 10.24$	$f(1,1,\dots,1)=0$

Tab. 2. The best results of benchmark functions.

Benchmark function	DSA	BDSA	Proposed mDSA
F_1 (Griewank)	129	76	50
F_2 (Ackley)	156	121	50
F_3 (Rastringin)	142	124	50
F_4 (Schwefel's)	130	154	50
F_5 (Whiteley)	80	164	50

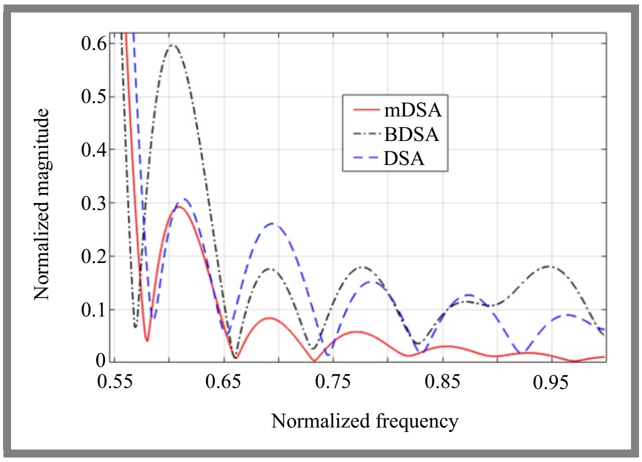

Fig. 3. Normalized SBR response of the FIR filter.

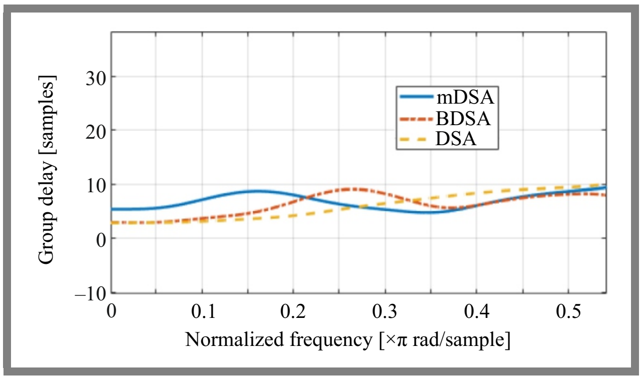

Fig. 4. Normalized group delay response of the FIR filter.

Tab. 3. Specifications for the desired FIR filter.

Parameter	Value
Order of designed filter	20.00
Pass-band edge frequency (normalized)	0.45
Stop-band edge frequency (normalized)	0.55
Normalized transition width	0.10
Normalized maximum pass band ripple	0.01
Normalized maximum stop band ripple	0.01
Maximum group delay	0.00

Tab. 4. The normalized pass band ripple in FIR low pass filter.

Algorithm	Max.	Min.	Avg.	Std. dev.
GA [7]	0.11926	0.01007	0.01932	0.00722
PSO [8]	0.11740	0.08230	0.10071	0.00789
ABC [8]	0.16620	0.09970	0.12800	0.02380
MFO [8]	0.15170	0.10210	0.12580	0.01900
BFO [9]	0.12900	-	-	-
CSO [6]	0.16400	-	-	-
DE [10]	0.04000	-	-	-
OHS [6]	0.14000	-	-	-
IOT [8]	0.11630	0.08890	0.10515	0.00535
DSA	0.03613	0.03177	0.03289	0.00079
BDSA	0.03536	0.03354	0.03412	0.00031
Proposed mDSA	0.02786	0.02634	0.02653	0.00021

Tab. 5. The normalized stop band ripple in FIR low pass filter.

Algorithm	Max.	Min.	Avg.	Std. dev.
GA [7]	0.04521	0.01250	0.02131	0.00681
PSO [8]	0.0186	0.0122	0.014877	0.001104
ABC [8]	0.1189	0.0825	0.09483	0.08711
MFO [8]	0.02773	-	-	-
BFO [9]	0.02773	-	-	-
CSO [6]	0.01980	-	-	-
DE [10]	0.03900	-	-	-
OHS [6]	0.1746	-	-	-
IOT [8]	0.0175	0.0131	0.01417	0.00093
DSA	0.02873	0.02631	0.02756	0.00092
BDSA	0.05891	0.04376	0.05623	0.00090
Proposed mDSA	0.02781	0.02667	0.02730	0.00081

4.2. Case 2 – Low Pass FIR Filter Design

In this work, in order to check the effectiveness of the proposed technique, a low group delay filter has been designed

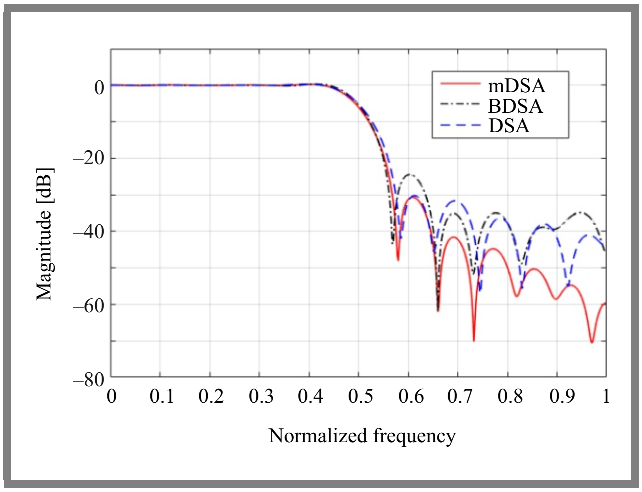

Fig. 5. Magnitude response of the FIR filter.

by applying DSA, BDSA and the proposed optimization techniques. The upper limit of the solution is set as +1 and lower limits of filter coefficients have been set as –1. The design parameters are given in Tab. 3. These filter parameters related to pass band cut-off frequency, stop band cut-off frequency, transition width and stop band ripple are the same as shown in [5], [6]. In this work, pass band ripple of 0.1 is considered, being a better value than that achieved in past research.

Such specifications as pass band ripple, stop band ripple, group delay pass band cut-off frequency, stop band cut-off frequency and transition width of the low pass filter designed with the use of the proposed techniques have been compared with the results obtained using DSA, BDSA and other known techniques, i.e. GA [7], PSO [8], ABC [8], MFO [8], BFO [9], CSO [6], DE [10], OHS [6] and IOT [8].

The comparison concerning pass band ripple is shown in Tab. 4. It can be concluded that the proposed algorithm provides best results compared with other state-of-the-art techniques listed in the table, in terms of maximum, average and minimum PBR values (Figs. 1–5).

The normalized stop band ripple (SBR) obtained using the proposed technique is also compared with DSA, BDSA and other evolutionary optimization techniques listed earlier. It can be summarized from Tab. 5 that the proposed technique provides comparable results in terms of maximum, average and minimum stop band ripple (SBR) values.

The importance of the transition width is presented in Tab. 6. The proposed method offers a transition width hat is very similar to the desired specifications (see Tab. 3). However, the transition width reported in many previous works that are based on state-of-the-art techniques is violating the assumed limits.

The main objective of this research was to design a low group delay filter and the group of the filters designed using the proposed techniques is shown in Tab. 8. The group delay presented in this table is the average value obtained from 50 iterations of the proposed algorithm.

It can be clearly observed from the Tab. 8 that the proposed mDSA approach outperforms all other optimization-based filter design techniques. In the majority of the previously

Tab. 6. Normalized transition width in an FIR low-pass filter.

Algorithm	Max.	Min.	Avg.	Std. dev.
GA	0.186	0.152	0.163	0.002
PSO [11]	0.087	-	-	-
ABC [12]	0.102	-	-	-
BFO [9]	0.092	-	-	-
CSO [6]	0.095	-	-	-
DE [10]	0.180	-	-	-
OHS [6]	0.099	-	-	-
DSA	0.158	0.133	0.134	0.002
BDSA	0.131	0.122	0.120	0.001
Proposed mDSA	0.119	0.098	0.102	0.001

reported FIR filter design approaches, group delay is not considered and symmetric coefficients are used which provide a constant group delay of 10 on a normalized scale. There are some approaches for filter design. These approaches do not provide better ripple specifications at lower orders in the range of 20.

5. Conclusions

In this work, low group delay FIR filters have been designed. The objective function has been formulated by considering the difference between the designed filter and the desired specification. Classical differential algorithms have been used to solve the filter design problem. Furthermore, the bijective differential search algorithm (BDSA) was also used during the design process. Inspired by modern quantum theory, a modified differential search algorithm (mDSA) has been developed. The improved mDSA algorithm enhanced accuracy and convergence parameters by exploring a larger search space. The results obtained have been compared with those achieved by deploying existing FIR filter design methods. The proposed technique has outperformed all other approaches in terms of group delay and other filter specifications.

Tab. 7. Group delay in an FIR low-pass filter.

Algorithm	Order	Group delay			
		Max.	Min.	Avg.	Std. dev.
GA [7]	20	10	10	10	0.000
PSO [8]	20	10	10	10	0.000
ABC [8]	20	10	10	10	0.000
MFO [8]	20	10	10	10	0.000
IOT [8]	20	10	10	10	0.000
CO [1]	32	6.943	4.25	6.532	1.234
MCO [13]	24	7.1	5.25	6.173	1.411
DSA	20	9.362	8.801	9.132	0.0029
BDSA	20	9.044	8.743	8.892	0.0036
Proposed mDSA	20	8.704	8.602	8.704	0.00029

Tab. 8. Optimized filter coefficients.

Filter coefficients	Algorithms		
	DSA	BDSA	Proposed mDSA
C (1.0)	–0.042	–0.072	–0.032
C (2.0)	–0.101	–0.150	–0.095
C (3.0)	–0.140	–0.127	–0.079
C (4.0)	–0.217	–0.063	0.060
C (5.0)	–0.344	–0.165	0.119
C (6.0)	–0.355	–0.374	–0.098
C (7.0)	–0.101	–0.369	–0.416
C (8.0)	0.216	–0.045	–0.459
C (9.0)	0.239	0.257	–0.188
C (10)	–0.021	0.229	0.049
C (11)	–0.174	0.009	0.046
C (12)	–0.052	–0.094	–0.025
C (13)	0.092	–0.028	0.020
C (14)	0.052	0.007	0.092
C (15)	–0.046	–0.031	0.054
C (16)	–0.035	–0.028	–0.022
C (17)	0.026	0.032	–0.024
C (18)	0.021	0.043	0.013
C (19)	–0.017	–0.004	0.008
C (20)	–0.011	–0.023	–0.017
C (21)	0.009	–0.004	–0.013

References

[1] C. Wu, D. Gao, and K.L. Teo, “A Direct Optimization Method for Low Group Delay FIR Filter Design”, *Signal Processing*, vol. 93, no. 7, pp. 1764–1772, 2013 (https://doi.org/10.1016/j.sigpro.2013.01.015).

[2] R.M. Rizk-Allah, A.E. Hassanien, M. Elhoseny, and M. Gunasekaran, “A New Binary Salp Swarm Algorithm: Development and Application for Optimization Tasks”, *Neural Computing and Applications*, vol. 31, no. 5, pp. 1641–1663, 2019 (https://doi.org/10.1007/s00521-018-3613-z).

[3] P. Civicioglu, “Transforming Geocentric Cartesian Coordinates to Geodetic Coordinates by Using Differential Search Algorithm”, *Computers & Geosciences*, vol. 46, pp. 229–247, 2012 (https://doi.org/10.1016/j.cageo.2011.12.011).

[4] W. Deng *et al.*, “An Improved Quantum-inspired Differential Evolution Algorithm for Deep Belief Network”, *IEEE Transactions on Instrumentation and Measurement*, vol. 69, no. 10, pp. 7319–7327, 2020 (https://doi.org/10.1109/TIM.2020.2983233).

[5] A.K. Dwivedi, S. Ghosh, and N.D. Londhe, "Low Power FIR Filter Design Using Modified Multi-objective Artificial Bee Colony Algorithm", *Engineering Applications of Artificial Intelligence*, vol. 55, pp. 58–69, 2016 (https://doi.org/10.1016/j.engappai.2016.06.006).

[6] S.K. Saha *et al.*, "Efficient and Accurate Optimal Linear Phase FIR Filter Design Using Opposition-based Harmony Search Algorithm", *The Scientific World Journal*, art. no. 320489, 2013 (https://doi.org/10.1155/2013/320489).

[7] A.K. Dwivedi, S. Ghosh, and N.D. Londhe, "Review and Analysis of Evolutionary Optimization-based Techniques for FIR Filter Design", *Circuits, Systems, and Signal Processing*, vol. 37, no. 10, pp. 4409–4430, 2018 (https://doi.org/10.1007/s00034-018-0772-1).

[8] T. Mittal, "Design of Optimal FIR Filters Using Integrated Optimization Technique", *Circuits, Systems, and Signal Processing*, vol. 40, no. 6, pp. 2895–2925, 2021 (https://doi.org/10.1007/s00034-020-01602-8).

[9] S.K. Saha, R. Kar, D. Mandal, and S.P. Ghoshal, "Bacteria Foraging Optimization Algorithm for Optimal FIR Filter Design", *International Journal of Bio-Inspired Computation*, vol. 5, no. 1, pp. 52–66, 2013 (https://doi.org/10.1504/IJBIC.2013.053039).

[10] K.S. Reddy and S.K. Sahoo, "An Approach for FIR Filter Coefficient Optimization Using Differential Evolution Algorithm", *AEU - International Journal of Electronics and Communications*, vol. 69, no. 1, pp. 101–108, 2015 (https://doi.org/10.1016/j.aeue.2014.07.019).

[11] K. Boudjelaba, F. Ros, and D. Chikouche, "Potential of Particle Swarm Optimization and Genetic Algorithms for FIR Filter Design", *Circuits, Systems, and Signal Processing*, vol. 33, no. 10, pp. 3195–3222, 2014 (https://doi.org/10.1007/s00034-014-9800-y).

[12] A. K. Dwivedi, S. Ghosh, and N. D. Londhe, "Modified Artificial Bee Colony Optimization-based FIR Filter Design with Experimental Validation Using Field-programmable Gate Array", *IET Signal Processing*, vol. 10, no. 8, pp. 955–964, 2016 (https://doi.org/10.1049/iet-spr.2015.0214).

[13] J. Konopacki, "Design of Sparse FIR Filters with Low Group Delay", *International Journal of Electronics and Telecommunications*, vol. 67, no.1, pp. 121–126, 2021 (https://doi.org/10.24425/ijet.2021.135953).

Sonelal Prajapati, Ph.D.
https://orcid.org/0000-0001-5891-9528
E-mail: sonelal59@gmail.com
Motilal Nehru National Institute of Technology Allahabad, Prayagraj, India
http://www.mnnit.ac.in

Sanjeev Rai, Ph.D.
E-mail: srai@mnnit.ac.in
Motilal Nehru National Institute of Technology Allahabad, Prayagraj, India
http://www.mnnit.ac.in

Manish Tiwari, Ph.D.
https://orcid.org/0000-0002-0361-9373
E-mail: mtiwari@mnnit.ac.in
Motilal Nehru National Institute of Technology Allahabad, Prayagraj, India
http://www.mnnit.ac.in

Atul Kumar Dwivedi, Ph.D.
https://orcid.org/0000-0003-4221-806X
E-mail: akd@bietjhs.ac.in
Bundelkhand Institute of Engineering and Technology, Jhansi, India
http://bietjhs.ac.in