

Low-Complexity DOA Estimation Method Based on Joined Coprime Array

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Abstract — In this article, the elimination of ambiguity of a joined coprime array has been examined, with a focus on such of its properties as large aperture size and complete degree of freedom (DOF). The existing methods suffer from a high degree of computation complexity due to the loss constant characteristic and high peak searching. Therefore, in this paper, a DOA estimation method for a joined coprime array, characterized by a low degree of computational complexity, is proposed. The variance of the diagonal eigenvalues of the estimated covariance matrix is designed to enhance the accuracy of the covariance matrix of the joined coprime array. Then, the Capon beamforming methods is employed for peak searching. The simulation shows that the proposed method accomplishes accurate estimation with shorter computation times and fewer operations compared to other DOA estimation methods.

Keywords — DOA estimation, eigenvalues, joined coprime array, low complexity

1. Introduction

An antenna array is a set of aerials arranged in a specific configuration for receiving incoming RF waveforms, such as power, amplitude, and source location [1], [2]. Estimation of the direction of arrival (DOA) of source signals is one of the practical applications of array signal processing that has been recognized as a critical factor in wireless communication, as well as in sonar, radar and other radiocommunication systems relying on antenna arrays [3], [4]. While uniform linear array (ULA) is the traditional array configuration relied upon by such DOA estimation methods as beamforming [5], as well as by Capon [6], MUSIC [7], root-MUSIC [8], and ESPRIT [9] approaches, it is characterized by a limited array aperture due to the small distance between the individual elements, leading to detecting a lower number of sources and poor estimation performance.

Sparse arrays, such as coprime arrays (CAs) proposed in [10], provide a larger virtual aperture with a less pronounced mutual coupling effect and outperform ULAs that rely on a lower number of elements, thus achieving better estimation performance [11]. CA is composed of two ULAs with the distance between its elements being larger than half the wavelength. Research projects concerned with DOA estimation and relying on the CA configuration may be divided into two groups: those focusing on the difference co-array (DCA) methods and

sub-array-based methods. The objective of DCA-based methods is to obtain a higher degree of freedoms (DOFs), which requires a large number of snapshots, thus leading to complex computations [12]–[16].

In sub-array-based methods, the CA is decomposed into two sparse arrays to maintain uniform characteristics of these two subarrays and to achieve a low degree of complexity of computations for DOA estimation. Noting that the distance between the inter-elements of the pair of subarrays is larger than half the wavelength, additional high peaks appear in the MUSIC spectrum. These result from the large distance between the inter-elements in the subarrays [17], [18]. These peaks are called ambiguous angles and are taken into consideration while defining the steering vector [19]. Ambiguities appear when there is a set of various DOAs in the discipline, due to the rank deficiency of the steering matrix [13], [20].

In [17], the two subarrays are treated individually and the MUSIC algorithm is applied thereto. The ambiguity is eliminated by finding the overlapping true angles or the closest peaks from the spectra of these two subarrays. Authors of [18] showed that by using a limited area search to find the arbitrary peaks and exploiting the linear relationship among ambiguous DOA estimates, may contribute to reducing the computational complexity. In paper [21], the actual DOAs are linked to several corresponding angles reaching the conventional ULA. These DOAs are estimated using the ESPRIT algorithm, and the approximated DOAs are restored based on the relation between the actual and corresponding DOAs. The resolved DOA is estimated by linking the outcomes of the subarrays. Sub-array-based methods also some from such shortcomings, such as a limited number of the resolved signals (due to the number of DOF generated by the two subarrays with a few physical elements), the loss of mutual information (since the two subarrays depend only on the self-information, with such an approach leading to the degradation of estimation performance), as well as the a high degree of computational complexity of the process of pairing the results achieved by both subarrays. To overcome these issues, a joined coprime array is proposed [13] to form a non-uniform linear array with a high aperture and the MUSIC algorithm is harnessed to process the data obtained with the use of the joined coprime array considered to be a single non-uniform linear array. The joined array can compute full DOFs with a high estimation precision, since it exploits both self-obtained and

mutual information. The ambiguity angles are eliminated and, therefore, only the peaks of the real DOAs are found in the spectrum estimation. However, this method still suffers from a considerable computational complexity overhead.

In this paper, a DOA estimation method, based on a joined coprime array and characterized by a lower degree of complexity is proposed. The variance of the diagonal eigenvalues of the estimated covariance matrix is constructed to enhance the accuracy of joined coprime array's covariance matrix. Then, the Capon beamforming method is applied to reduce peak searching.

The paper is structured as follows. Section 2 presents a model of the joined coprime array-based system. Section 3 describes the proposed DOA estimation method. In Section 4, the performance of the system is analyzed, while Section 5 offers the conclusions drawn.

Note: In this article we use upper-case bold characters to represent matrices and lower-case for vectors. $[\cdot]^T$, $[\cdot]^*$ and $[\cdot]^H$ stand for the transpose, conjugate and conjugate transpose of a vector or matrix, respectively, while $\text{diag}(\cdot)$ and $\text{vec}(\cdot)$ mean a diagonal matrix and the vectorization operator.

2. System Model

Suppose a coprime array comprises a pair of uniform linear subarrays with the N , M elements having an inter-element spacing of M_d , N_d respectively, where d is half wavelength ($\lambda/2$). N and M are integer numbers and the GCD of $(N, M) = 1$. The two subarrays share one element, i.e the reference element. Thus, the total number of elements in an antenna array is $K = N + M - 1$ and the array aperture size is $(M - 1)N$. Figure 1 shows the conventional configuration of a coprime array.

When both subarrays are joined in a collinear position, the array's aperture size may be increased to $(M - 1)N + (N - 1)M$ [13]. Figure 2 shows the configuration of a combined coprime array, with its elements located at:

$$\begin{aligned} \mathbb{P}_1 &= \{(0, \dots, (M - 1)N_d)\}, \\ \mathbb{P}_2 &= \{(0, \dots, (N - 1)M_d)\} + (M - 1)N_d. \end{aligned} \quad (1)$$

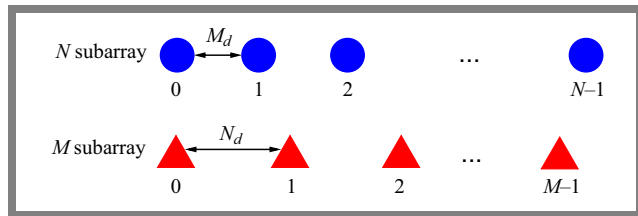


Fig. 1. Conventional coprime array.

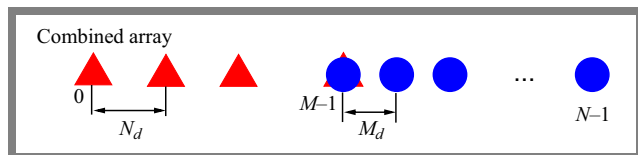


Fig. 2. Combined coprime array.

The joined coprime array is a union of \mathbb{P}_1 and \mathbb{P}_2 . Let us assume there are Q uncorrelated, narrowband and far field sources which are contacting the antenna array from the directions $\theta_1, \theta_2, \dots, \theta_Q$. The data $\mathbf{x}(t)$ observed at the antenna array may be described as:

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \mathbf{s}(t) + \begin{bmatrix} \mathbf{n}_1(t) \\ \mathbf{n}_2(t) \end{bmatrix}, \quad (2)$$

where:

$$\mathbf{A}_1 = [\mathbf{a}_1(\theta_1), \dots, \mathbf{a}_1(\theta_Q)], \quad (3)$$

$$\mathbf{A}_2 = [\mathbf{a}_2(\theta_1), \dots, \mathbf{a}_2(\theta_Q)]. \quad (4)$$

\mathbf{A}_1 and \mathbf{A}_2 are the steering matrix of the two subarrays M and N , respectively, with the steering vector related to signals from direction θ_Q described by:

$$\mathbf{a}_1(\theta_Q) = \left[1, e^{\frac{j2\pi M d \sin(\theta_Q)}{\lambda}}, \dots, e^{\frac{j2\pi M(N-1)d \sin(\theta_Q)}{\lambda}} \right]^T, \quad (5)$$

$$\mathbf{a}_2(\theta_Q) = \left[e^{\frac{j2\pi M(N-1)d \sin(\theta_Q)}{\lambda}}, \dots, e^{\frac{j2\pi(N(M-1)+M(N-1))d \sin(\theta_Q)}{\lambda}} \right]^T. \quad (6)$$

$\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_Q(t)]^T$ is the signal vector, where $t = [1, 2, \dots, T]$, with T representing the number of snapshots. $\mathbf{n}_1(t)$ and $\mathbf{n}_2(t)$ are the additive white Gaussian noise of m and N subarrays, respectively.

Then, the covariance matrices of the two subarrays are estimated with T snapshots, as follows:

$$\hat{\mathbf{R}} = \frac{1}{T} \sum_{t=1}^T \mathbf{x}(t)\mathbf{x}^H(t). \quad (7)$$

The eigen decomposition (ED) of the estimated covariance matrix is performed to decompose it into eigenvalues and its corresponding eigenvectors.

3. Proposed Method

3.1. Low Complexity DOA Estimation

An ambiguity-free reduced computational complexity DOA estimation method based on a coprime array is proposed, exploiting the self and mutual information of the covariance matrix. The diagonal eigenvalues of the covariance matrix are extracted to improve the estimated accuracy of the covariance matrix and then to boost the performance of the DOA estimation process. Additionally, instead of using the MUSIC algorithm which requires high peak searching, the Capon beamforming algorithm is relied upon to lower computational complexity.

Diagonal loading is proposed to modify the eigenvalues of the covariance matrix, since such an approach may control false peaks in the coprime array. First, ED is performed on the covariance matrix in Eq. (7), yielding the following eigenvalues and eigenvectors:

$$\hat{\mathbf{R}} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H, \quad (8)$$

where $\mathbf{\Lambda}$ is the eigenvalue arranged in a descending order:

$\mathbf{\Lambda} = \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_Q \geq \lambda_{Q+1} = \lambda_{Q+1} = \dots = \lambda_K = \sigma^2$
and \mathbf{U} is its corresponding eigenvector. The diagonal loading of the eigenvalues is:

$$\lambda_{DL} = \sqrt{\sum_{i=1}^k \frac{\lambda_i^2}{k}}. \quad (9)$$

Then, the covariance matrix is modified according to:

$$\mathbf{R}_D = \mathbf{R} + \lambda_{DL} \mathbf{I}. \quad (10)$$

The spectrum may be estimated, using Capon, in the following manner:

$$P_{\text{Capon}}(\theta) = \frac{1}{[\mathbf{a}_1^H(\theta)\mathbf{a}_2^H(\theta)] \text{inv}(\mathbf{R}_D) [\mathbf{a}_1(\theta)\mathbf{a}_2(\theta)]}. \quad (11)$$

The true DOA can be estimated by obtaining the peaks of P_{Capon} , with no ambiguous angles appearing with high peaks. The proposed DOA estimation method reduces the number of operations, i.e. multiplications and additions, required. The computational complexity is calculated from solving the covariance matrix, ED, diagonal loading, scan searching and finding peaks to determine the estimated angles.

3.2. Antenna Array Beamwidth

Beamwidth is a common antenna directivity measure, with the result related to the full width of the main lobe. The narrower the beamwidth, the better the resolution. For two signal directions to be resolved accurately, the mutual separation must be lower than the beamwidth. The beamwidth of an antenna may be measured as a function of the array's aperture size.

The electric field layout of a uniform antenna array can be expressed as:

$$|\varepsilon(\theta)| = \left| \frac{\sin\left(K\frac{\pi d}{\lambda}\sin\theta\right)}{\sin\left(\frac{\pi d}{\lambda}\sin\theta\right)} \right|. \quad (12)$$

To determine the beamwidth ($\theta_{3\text{dB}}$), Eq. (12) is normalized to $\frac{1}{\sqrt{2}}$ to find the solution of θ . The solution will turn out to be $0.89\frac{\lambda}{D}$, where D is the total aperture distance and can be approximated as KD [22]. For half wavelength spacing, it may be approximated as $\frac{2}{K}$, thus beamwidth can be denoted as:

$$\Delta\theta_{3\text{dB}} = \frac{1.78}{N} \text{ [rad]} \text{ or } \Delta\theta_{3\text{dB}} = \frac{102}{N} \text{ [deg]}. \quad (13)$$

4. Performance Analysis

The performance of the proposed DOA method is evaluated and compared with other estimation methods in terms of computational complexity, minimum angle separation, and the root mean square error (RMSE).

In relation to computational complexity, i.e. covariance matrix estimation, feature decomposition and peak searching processes, the proposed method requires:

$$O(K^2T + 4K^2 + 3K + KS),$$

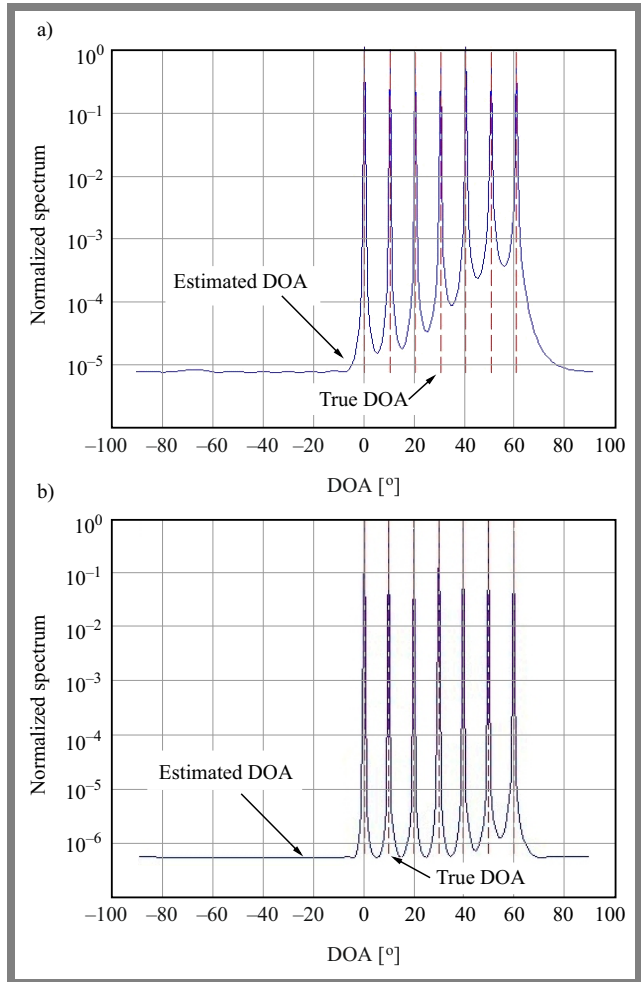


Fig. 3. Spectrum estimation of seven sources: a) reference method [13], b) proposed method.

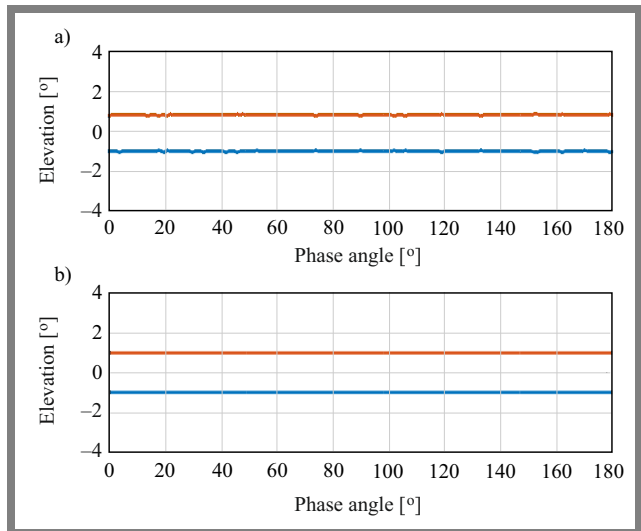


Fig. 4. Phase effect on angle separation: a) when the separation is $(-0.88^\circ \text{ to } 0.88^\circ)$ and b) when the separation is $(-1^\circ, 1^\circ)$.

while method [13] requires:

$$O(K^2T + K^3 + K(K - Q)S),$$

where K is the total number of elements of a coprime array, such as:

$$K = M + N - 1,$$

while T and s are time of spectral search snapshots, respectively.

With three RF sources, SNR is 10 dB, and $T = 200$. With $M = 5$ and $N = 7$, the number of operations required to implement the proposed method equals $11 \cdot 10^5$ for 11 elements, while for [13] and the outcomes are $16 \cdot 10^5$ and $27 \cdot 10^5$, respectively.

Processing time is calculated using the Matlab software and is 0.02966 s for the proposed method, compared to 0.03747 s for [13]. The number of operations and the running time of the proposed method are lower than for other DOA estimation approaches, implying that the proposed method requires less resources.

Several scenarios have been tested to assess the effectiveness of the proposed method.

4.1. Spectrum Resolution

The first simulation scenario shows the spectrum resolution with m and N subarrays being set to 4 and 5, respectively, where the total number of elements is 8. The number of sources that can be resolved is 7, since the number of DOF is 8. SNR is set to 10 dB, while the number of snapshots is set to 200. Figure 3 shows the spectrum estimation of the proposed method and a reference solution [13]. One may notice that the proposed method is capable of detecting 7 source signals with directions ($0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ$) more accurately, since the resulting DOA has higher peaks.

The second analysis determines the DOA of three sources coming from directions ($-42.33^\circ, 3.4^\circ, 28^\circ$) (Tab. 1). One may observe that the proposed method is capable of resolving all the sources with a low error rate.

The third test estimates the DOA of two signals with different minimum separations between them, as shown in Tab. 1. The distance must not be shorter than 1.76° (e.g. from -0.88° to 0.88°), since the beamwidth is $102/N = 1.76^\circ$. Four cases have been explored to find the DOA with the minimum distance and SNR is 5 dB, $T = 100$. The number of trials was set to 15. One may notice that the proposed method is capable of resolving the DOA with a less-than-beamwidth degree of precision, with an error rate of 0.2267 and source separation of 1.5° .

To show the effect of the initial phase on the minimum separation of the sources, Fig. 4a-b presents the initial phase versus angle separation between the sources, with separation equaling ($-0.88^\circ, 0.88^\circ$) and ($-1^\circ, 1^\circ$), respectively. One may notice that the initial phase does not affect the received signal (Tab. 2).

4.2. RMSE Performance

For RMSE evaluation, the proposed method, the solution from the reference approach [13], and the spatial smoothing of the general coprime array are compared. RMSE is defined

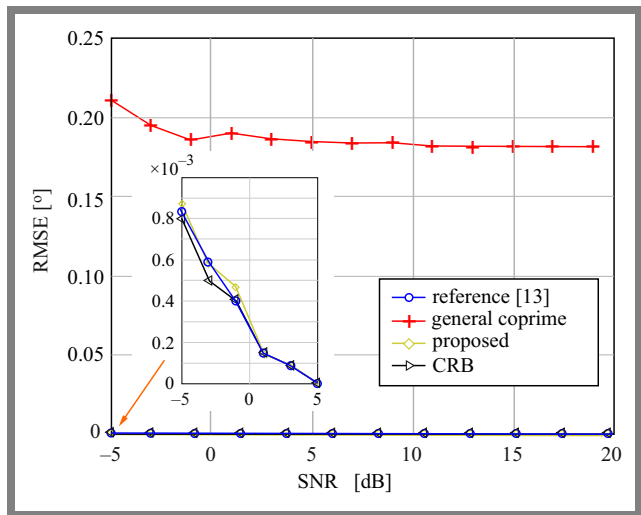


Fig. 5. RMSE vs. SNR.

Tab. 1. DOA estimates obtained.

Method	1	2	3
True DOA	-42.33	3.4	28
[13]	-42.3	3.39	28
Proposed	-42.3	3.399	27.9

by the following formula:

$$RMSE = \sqrt{\frac{1}{QM_c} \sum_{m=1}^{M_c} \sum_{q=1}^Q (\hat{\theta}_{q,m} - \theta_q)^2}, \quad (14)$$

where $\hat{\theta}_{q,m}$, θ_q represent the estimated and true DOA, respectively, and M_c is the number of Monte Carlo trials. In this simulation, M_c is set to 300.

Furthermore, the Cramér-Rao Bound (CRB) is also graphed for reference. CRB provides a lower bound on the variances of unbiased parameter estimates. It is widely used in DOA estimations [23]. CRB is independent of algorithms and is typically associated with array configuration designs. Here, the CRB formula from [24] is used to determine the lower bound that can be attained by the variances of the proposed DOA estimations.

Figure 5 shows RMSE performance versus SNR, where SNR varies within the range of -5 to 20 dB, and two source signals with the directions of ($10^\circ, 20^\circ$) are evaluated. One may notice that the proposed method has the same performance as [13], since the array aperture size is the same in both methods. Therefore, both can resolve the signals accurately. It can also be noticed that the direction of the RMSE curves of the DOA estimation algorithms is consistent with the trend of the CRB curve.

Figure 6 shows RMSE performance with respect to the snapshots (within the range of $10 \dots 500$), SNR is 5 dB, and two sources with directions ($10^\circ, 20^\circ$). The proposed method can perform well is capable of efficiently estimating DOA with a low number of snapshots (less than 50). When the number of snapshots is higher, good results are obtained by all the DOA estimation methods considered.

Tab. 2. Estimated DOA with minimum angle separation.

Cases	True DOA		Estimated DOA	
	1	2	1	2
Case 1	-0.75°	0.75°	-0.9867°	0.9667°
Case 2	-0.80°	0.80°	-0.8067°	0.7867°
Case 3	-0.88°	0.88°	-0.8867°	0.8867°
Case 4	-1.0°	1.00°	-1.0000°	1.0000°

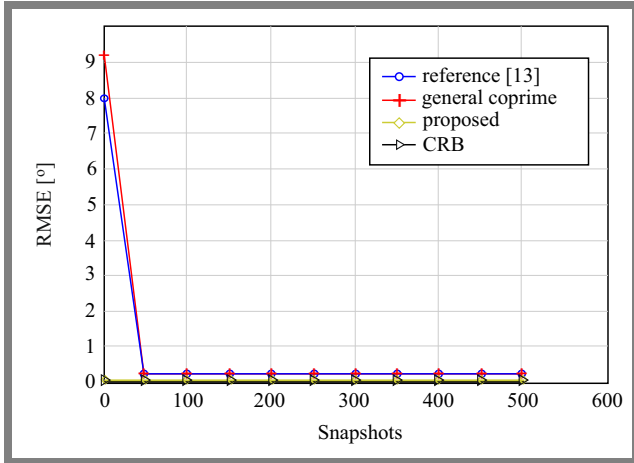


Fig. 6. RMSE vs. snapshots.

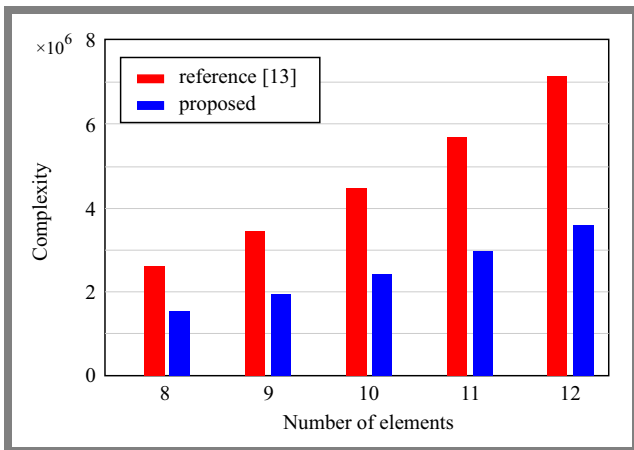


Fig. 7. Complexity versus the number of array elements.

4.3. Computational Complexity

The computational complexity of the proposed DOA estimation method is compared with other DOA estimation approaches, including MUSIC under ULA geometry, spatial MUSIC for a general coprime array, as well as the Zheng method from [13]. The computation time is shown in Tab. 3. For simulation purposes, the number of the elements is set to 10 for all array types, SNR is 10 dB, and the number of snapshots is 200. The number of sources is seven, with their directions equaling 0°, 10°, 20°, 30°, 40°, 50°, and 60°. One may notice that the running time of the proposed method is shorter than in the case of other DOA estimation methods operating under the same conditions.

The proposed method reduces the number of operations required to estimate the direction of the source signal. The

Tab. 3. Computation time comparison.

DOA methods	Time
MUSIC with ULA	0.02534
SS-MUSIC with general coprime	0.52719
Reference [13]	0.02692
Proposed DOA estimation method	0.01708

relation between computational complexity and the number of aerials K is shown in Fig. 7, for $K = 5, 100$ snapshots and a search step of 0.1. One may conclude from Fig. 7 that the proposed method has lower computational complexity than the approach described in [13].

5. Conclusions

In this paper, a low computational complexity DOA estimation method based on a joined coprime array design is proposed. The diagonal elements of the estimated covariance matrix are modified by computing the variance of the eigenvalues. Then, Capon is performed to find the spectrum of the DOA. Computational complexity is reduced by decreasing the amount of eigenvector multiplication and angle scanning operations. The tests show that the proposed method offers higher resolution values and shorter processing times when compared with other DOA estimation methods.

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