Exact Analysis of MIMO Channel Estimation Based on Superimposed Training

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Abstract — In this paper, channel estimation capabilities of a multiple-input multiple-output (MIMO) system using superimposed training sequences are investigated. A new expression for estimation-error variance is derived. It is shown that the training sequences must be balanced and must have specific correlation properties. The latter are required only in a specific zone. Sequences that satisfy these criteria exist and are referred to as zero-correlation zone (ZCZ) solutions. Consequently, by using balanced ZCZ sequences, harmful direct current (DC) offset can be removed. Owing to their zero-cross correlation, interference from other transmitting antennas may be eliminated. Furthermore, a closed-form expression of the estimation-error variance can be obtained due to their impulse-like autocorrelation. To increase the number of antennas in the MIMO system, a new construction of ZCZ sequence set is proposed, in which all sequences are balanced.

Keywords — channel estimation, DC offset, MIMO channel, superimposed training, ZCZ sequences

1. Introduction

In recent years, channel estimation based on superimposed training (ST) has attracted much interest among researchers dealing with wireless communication systems, such as unmanned aerial vehicle-assisted cellular communication systems [1], massive multiple-input multiple-output networks [2], [3], orthogonal frequency division multiplexing systems [4], cyclic block filtered multi-tone [5], and multiple access relay networks [6]. In an ST scheme, a periodic training sequence is arithmetically added to the information sequence prior to transmission.

Contrary to the conventional training approach, no dedicated resource is reserved for training. Thus, transmission efficiency may be improved. On the other hand, some power is allocated to the training sequence, thereby decreasing the signal-to-noise ratio (SNR) [7]. Due to the periodicity of the training sequence, cyclostationary characteristics are induced in the received signal and can be exploited to estimate the channel via first-order statistics [8].

Paper [9] proposes an approach to determine the properties of an ST sequence in order to obtain an optimal estimation performance for a single-input single-output (SISO) channel. To achieve this goal, it was shown that the training sequence must have an impulse-like autocorrelation and must also be balanced, i.e. the sum of all elements in the training sequence must equal zero. However, it was proven that the optimum perfect sequences are not balanced. Without the balance property, the harmful direct current (DC) offset cannot be directly removed and performance degradation may occur. In addition, in a MIMO system, channel estimation is more complicated than in SISO due to the large number of channel paths between different transmitting and receiving antennas.

Therefore, the research from [9] cannot be extended to MIMO systems due to the difficulty in finding a set of perfect sequences with good cross correlation properties. To tackle this issue, a new method was proposed in [10] to estimate MIMO channels using an ST scheme based on balanced zero correlation zone (ZCZ) sequences. These sequences have an impulse-like autocorrelation and zero cross-correlation inside their zero zones.

However, this method suffers from three major issues. Firstly, the analysis performed in this method leads to a very complex expression of the channel estimation error and only approximate results were given. This is because the sequences' matrix is not square. Therefore, only a pseudo-inverse of the matrix can be obtained. Secondly, the adequacy of ZCZ sequences for MIMO channel estimation was not demonstrated. Finally, only a small subset of the ZCZ sequences set used in [10] is balanced. This severely limits their potential application in MIMO channel estimation.

The contributions of this paper are as follows:

We analytically derive the characteristics of the optimal set of training sequences for MIMO channel estimation. These characteristics include the following: balance, impulse-like autocorrelation and zero cross-correlation functions (only in a specific zone). The type of sequences that satisfy the two last conditions (correlation properties) are referred to as ZCZ sequences.

It is shown that, if balanced ZCZ sequences are used for training, an exact closed-form expression for the variance of the error in MIMO channel estimation can be obtained.

A new construction of a ZCZ sequence set is proposed, where all sequences are balanced. This is because ZCZ sequences were originally proposed for code division multiple access systems, where the balance property was not taken into consideration [11], [12]. Consequently, most of the ZCZ sequence sets proposed in the literature either have a limited number of balanced sequences or do not have any balanced sequences at all.

The paper is organized as follows. Section 2 is devoted to modelling the MIMO system under study, to channel estimation using superimposed training sequences and to DC-offset removal. In Section 3, a closed-form solution for the variance of estimation error is derived and a comparison is presented. Section 4 deals with the number of balanced sequences used for training. Concluding remarks are given in Section 5.

2. MIMO System Model Using Superimposed Training

2.1. System Model

Let us consider a MIMO communication system with N_t transmitting antennas and N_r receiving antennas, as depicted in Fig. 1.

Let $b_t(k)$ be a data symbol sequence from the t-th transmitter antenna, where $t = 1, 2, \dots, N_t$. Periodic superimposed sequences $c_t(k)$ with period P are added to $b_t(k)$ to produce the transmitted signal $s_t(k)$ such as:

$$s_t(k) = b_t(k) + c_t(k) \ , \tag{1}$$

where k is the discrete time variable.

Let $h_{rt}(k)$ be the discrete-time finite impulse response associated with the channel between the t-th transmit antenna and the r-th receive antenna, where $r = 1, 2, \dots, N_r$. Signals from each transmit antenna travel through the channel and the output is combined with additive white Gaussian noise (AWGN), $n_r(k)$ and an unknown DC offset d_r component.

The latter should consider due to the imperfection of the direct conversion receiver, especially when the channel estimation

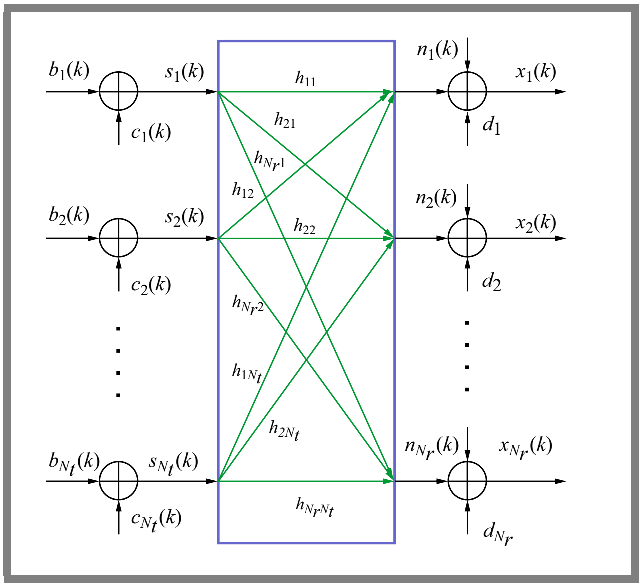

Fig. 1. MIMO system base band model.

is carried out using first order statistics only [9], [10]. The signal at the r-th receive antenna is:

$$x_r(k) = n_r(k) + d_r + \sum_{t=1}^{N_t} \sum_{m=0}^{M-1} h_{rt}(m)\, s_t(k-m) \ , \tag{2}$$

where M is the channel order.

Note that in practice different $h_{rt}(k)$ could have different values of M, so the M in Eq. (2) stands for the largest value. The output signal $x_r(k)$ has the following period P cyclic feature:

$$\begin{aligned} y_r(j) &= E\big[x_r(iP+j)\big], \ \ j = 0, 1, \dots, P-1 \\ &= d_r + \sum_{t=1}^{N_t} \sum_{m=0}^{M-1} h_{rt}(m)\, c_t(j-m)_{\text{mod } P} \ , \end{aligned} \tag{3}$$

where $b_t(k)$ and $n_r(k)$ are assumed to have zero mean and mod P is defined as an arithmetic "modulo-P". Note that, for each value of t, the above expression represents a set of P linear equations for the channel impulse response coefficients $\big\{h_{rt}(k)_{k=0}^{M-1}\big\}$.

Equation (3) can also be written as:

$$y_r(j) = d_r + \sum_{t=1}^{N_t} \sum_{m=0}^{P-1} h_{rt}(m)\, c_t(j-m)_{\text{mod} P} \ , \tag{4}$$

where channel coefficients:
$h_{rt}(P-1), h_{rt}(P-2), \dots, h_{rt}(M)$ are equal to zero.

To simplify future analysis, we choose to write Eq. (4) in the matrix form, as:

$$\boldsymbol{y}_r = \boldsymbol{d}_r + \sum_{t=1}^{N_t} \boldsymbol{C}_t \boldsymbol{h}_{rt} \ , \tag{5}$$

where:

$$\boldsymbol{C}_t = \begin{bmatrix} c_t(0) & c_t(1) & & c_t(P-1) \\ c_t(P-1) & c_t(0) & & c_t(P-2) \\ \vdots & \vdots & \dots & \vdots \\ c_t(1) & c_t(2) & & c_t(0) \end{bmatrix}_{P \times P} \tag{6}$$

and vectors:
$\boldsymbol{y}_r = [y_r(P-1), y_r(P-2), \dots, y_r(0)]_P^T$,
$\boldsymbol{h}_{rt} = [h_{rt}(P-1), h_{rt}(P-2), \dots,$
$h_{rt}(M-1), \dots, h_{rt}(0)]_P^T$,
$\boldsymbol{d}_r = [d_r, d_r, \dots, d_r]_P^T$.

Note that matrix $\boldsymbol{C}_t$ is circulant and
$h_{rt}(P-1), \dots, h_{rt}(M+1), h_{rt}(M)$ are equal to zero.

2.2. MIMO Channel Estimation

To estimate the elements of $\boldsymbol{y}_r$ from a finite amount N of received samples, we use the following estimator:

$$\hat{y}_r(j) = \frac{1}{N_p} \sum_{i=0}^{N_p-1} x_r(iP+j) \ , \tag{7}$$

where $N_P = \frac{N}{P}$.
Note that for finite N, the estimated received vector $\hat{\boldsymbol{y}}_r \approx \boldsymbol{y}_r$. When $N \to \infty$, the above expression becomes an equality (10). Let us now derive the estimated channel vector:

$$\begin{aligned}\hat{\boldsymbol{h}}_{rl} &= \boldsymbol{C}_l^{-1}\hat{\boldsymbol{y}}_r = \boldsymbol{C}_l^{-1}\boldsymbol{d}_r + \sum_{t=1}^{N_t}\boldsymbol{C}_l^{-1}\boldsymbol{C}_t\boldsymbol{h}_{rt} \\ &= \boldsymbol{C}_l^{-1}\boldsymbol{d}_r + \boldsymbol{h}_{rl} + \sum_{t=1,t\neq l}^{N_t}\boldsymbol{C}_l^{-1}\boldsymbol{C}_t\boldsymbol{h}_{rt}\ .\end{aligned} \tag{8}$$

In order for the estimated channel vector $\hat{\boldsymbol{h}}_{rl}$ to be equal to its real value $\boldsymbol{h}_{rl}$, the two remaining terms must be eliminated. Let us start with the DC offset term. As matrix $\boldsymbol{C}_t$ circulant, $\boldsymbol{C}_t^{-1}$ will be circulant as well, $\boldsymbol{C}_t^{-1}\boldsymbol{d}_r = \mathrm{sum}_{c_t}\boldsymbol{d}_r$ with s_{c_t} is the sum of the elements of any row in $\boldsymbol{C}_t^{-1}$. To remove the DC offset term, the sum_{c_t} must be equal to zero. In other words, each of the sequences used in the transmitting side c_t must be balanced. If the sequences used at the transmitters are all balanced, the DC offset can be eliminated and Eq. (5) becomes:

$$\boldsymbol{y}_r = \sum_{t=1}^{N_t}\boldsymbol{C}_t\boldsymbol{h}_{rt}\ . \tag{9}$$

3. Performance Analysis

3.1. Variance of the Estimation Error

Here, we evaluate the channel estimation performance using the estimation error variance. The estimation error of the received vector is:

$$\boldsymbol{e}_{y_r} = \hat{\boldsymbol{y}}_r - \boldsymbol{y}_r = \sum_{t=1}^{N_t}\boldsymbol{C}_t\left(\hat{\boldsymbol{h}}_{rt} - \boldsymbol{h}_{rt}\right) = \sum_{t=1}^{N_t}\boldsymbol{C}_t\boldsymbol{e}_{\boldsymbol{h}_{rt}}\ . \tag{10}$$

Let us now write Eq. (7) in a matrix form, as:

$$\begin{aligned}\hat{\boldsymbol{y}}_r &= \frac{1}{N_p}\sum_{i=0}^{N_p-1}\boldsymbol{x}_r(iP) \\ &= \frac{1}{N_p}\sum_{i=0}^{N_p-1}\sum_{t=1}^{N_t}\boldsymbol{H}_{rt}\big[\boldsymbol{b}_t(iP)+\boldsymbol{c}_t(iP)\big] + \boldsymbol{n}_r(iP)\ ,\end{aligned} \tag{11}$$

where

$$\begin{aligned}\boldsymbol{b}_t(iP) &= [b_t(iP+P-1), b_t(iP+P-2), \ldots, b_t(iP), \\ &\quad \ldots, b_t(iP-(P-1))]_{2P-1}^T \\ \boldsymbol{c}_t(iP) &= [c_t(iP+P-1), c_t(iP+P-2), \ldots, c_t(iP), \\ &\quad \ldots, c_t(iP-(P-1))]_{2P-1}^T \\ \boldsymbol{n}_r(iP) &= [n_r(iP+P-1), n_r(iP+P-2), \\ &\quad \ldots, n_r(iP+1), n_r(iP)]_P^T\end{aligned}$$

and $\boldsymbol{H}_{rt}$ is a filtering matrix for each transmitter-receiver pair given by:

$$\boldsymbol{H}_{rt} = \begin{bmatrix} h_{rt}(0) & h_{rt}(1) & \ldots & \ldots & h_{rt}(P-1) & 0 & \ldots & 0 \\ 0 & h_{rt}(0) & h_{rt}(1) & \ldots & \ldots & h_{rt}(P-1) & \ldots & 0 \\ 0 & 0 & h_{rt}(0) & h_{rt}(1) & \ldots & \ldots & h_{rt}(P-1) & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & h_{rt}(0) & h_{rt}(1) & \ldots & h_{rt}(P-1) \end{bmatrix}_{P\times(2P-1)} \tag{12}$$

By replacing Eq. (11) in Eq. (10) we get:

$$\begin{aligned}\boldsymbol{e}_{y_r} &= \frac{1}{N_p}\sum_{i=0}^{N_P-1}\sum_{t=1}^{N_t}\boldsymbol{H}_{rt}\big[\boldsymbol{b}_t(iP)+\boldsymbol{c}_t(iP)\big] \\ &\quad + \boldsymbol{n}_r(iP) - \sum_{t=1}^{N_t}\boldsymbol{C}_t\boldsymbol{h}_{rt} \\ &= \frac{1}{N_p}\sum_{i=0}^{N_P-1}\sum_{t=1}^{N_t}\boldsymbol{H}_{rt}\boldsymbol{b}_t(iP) + \boldsymbol{n}_r(iP)\ ,\end{aligned} \tag{13}$$

where it can be shown that

$$\sum_{t=1}^{N_t}\boldsymbol{H}_{rt}\boldsymbol{c}_t(iP) = \sum_{t=1}^{N_t}\boldsymbol{C}_t\boldsymbol{h}_{rt}\ \forall i\ .$$

From Eqs. (10) and (13), we get:

$$\sum_{t=1}^{N_t}\boldsymbol{C}_t\boldsymbol{e}_{\boldsymbol{h}_{rt}} = \frac{1}{N_P}\sum_{i=0}^{N_P-1}\sum_{t=1}^{N_t}\boldsymbol{H}_{rt}\boldsymbol{b}_t(iP) + \boldsymbol{n}_r(iP)\ . \tag{14}$$

Now, the goal is to derive the correlation properties of the training sequences to obtain a closed form solution for the variance of the channel estimation error $\sigma^2_{e_{h_{rt}}}$. In this study, we opt to compute the variance of the error in the received vector e_{y_r} and then proceed to evaluate $\sigma^2_{e_{h_{rt}}}$. To simplify notations in the following analysis, let the terms of Eq. (14) be denoted as:

$$\begin{aligned}\boldsymbol{A} &= \frac{1}{N_p}\sum_{i=0}^{N_P-1}\sum_{t=1}^{N_t}\boldsymbol{H}_{rt}\boldsymbol{b}_t(iP) + \boldsymbol{n}_r(iP)\ , \\ \boldsymbol{B} &= \sum_{t=1}^{N_t}\boldsymbol{C}_{rt}\boldsymbol{e}_{\boldsymbol{h}_{rt}}\ .\end{aligned} \tag{15}$$

The variance of term $\mathbf{A}$ is:

$$\sigma_{\mathbf{A}}^2 = \frac{P}{N_P}\left(\sum_{t=1}^{N_t}\sigma_{b_t}^2 + \sigma_{n_r}^2\right) . \tag{16}$$

The analysis is described in Appendix A. The variance of term $\mathbf{B}$ is (see Appendix B for a detailed analysis):

$$\sigma_{\mathbf{B}}^2 = \left[\sum_{t=1}^{N_t} tr\big(R_t(0)e_M\big)\right] = \sum_{t=1}^{N_t} P\sigma_{c_t}^2\sigma_{e_{rt}}^2 , \tag{17}$$

where:

$\sigma_{e_{rt}}^2 = E\left[\sum_{i=0}^{M-1}\left|e_{rt}(i)\right|^2\right]$ and

$\sigma_{c_t}^2 = \frac{1}{P}\sum_{k=0}^{P-1}\left|c_t(k)\right|^2 = \frac{1}{P}R_t(0)$.

The operator $tr(\cdot)$ means “take the trace of the matrix”. In the proposed approach, we consider independent MIMO channels. Hence, all the channels have the same error variance $\sigma_{e_{rt}}^2 = \sigma_e^2$.

$$\sigma_{\mathbf{B}}^2 = P\,\sigma_e^2\sum_{t=1}^{N_t}\sigma_{c_t}^2 . \tag{18}$$

From Eq. (14) $\sigma_{\mathbf{B}}^2 = \sigma_{\mathbf{A}}^2$. Then σ_e^2 is:

$$\sigma_e^2 = \frac{1}{N_P}\frac{\sum_{t=1}^{N_t}\sigma_{b_t}^2 + \sigma_{n_r}^2}{\sum_{t=1}^{N_t}\sigma_{c_t}^2} . \tag{19}$$

Since the total power P is split among the transmit antennas, we can write

$$\sigma_e^2 = \frac{1}{N_P}\frac{\sigma_b^2 + \sigma_{n_r}^2}{\sigma_c^2} , \tag{20}$$

where σ_b^2 and σ_c^2 are the average powers allocated to the information data and the training sequence, respectively.

These average powers are divided between all transmitting antennas. Recall that Eq. (20) was obtained by assuming that the correlation properties of the training sequences must have an impulse-like auto-correlation function and zero cross-correlation in a zone larger than or equal to the channel order M (see Appendix B).

Fortunately, sets of sequences with the aforementioned properties exist and are called ZCZ sequences. To eliminate the DC offset, a balanced ZCZ sequence may be selected from the set before being used as training sequences for MIMO channel estimation.

3.2. Performance Comparison

It is interesting to notice that the channel estimation is less dependent on the number of transmitting antennas. Indeed, this study demonstrates the capability of ZCZ sequences in reducing interference in MIMO channel estimation. The application of balanced ZCZ sequences for MIMO channel estimation was first proposed in [10]. However, as stated in Section 1, this method suffers from three major issues:

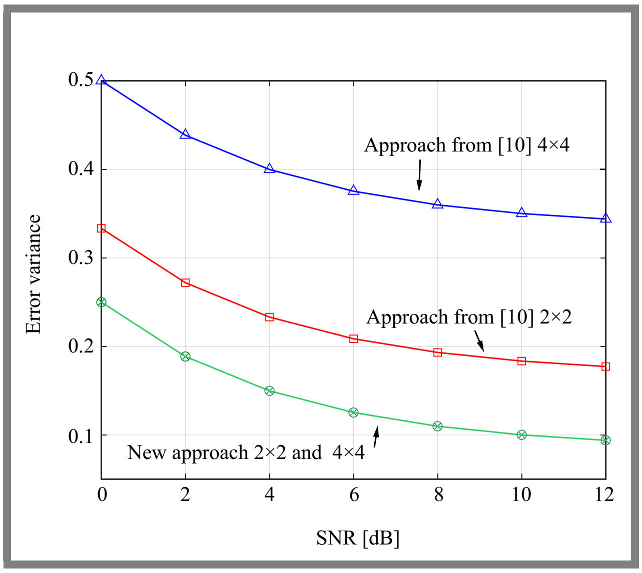

Fig. 2. Channel estimation error vs. SNR in 2×2 and 4×4 MIMO systems.

- only an approximation of the channel estimation error variance was obtained,
- the analysis did not show how the correlation properties of ZCZ sequences lead to the variance expression,
- only a small subset of the ZCZ sequences set used in [10] is balanced.

In this work, an exact expression of the error variance was obtained. In addition, the analysis explicitly shows the role of ZCZ sequences’ correlation properties in deriving the error variance.

Let us now rewrite the obtained error variance as a function of the signal-to-noise ratio (γ) and the power loss factor (α), as:

$$\sigma_e^2 = \frac{1}{N_P}\frac{\sigma_b^2 + \sigma_{n_r}^2}{\sigma_c^2} = \frac{1}{N_P}\frac{\gamma\,\alpha + 1}{\gamma(1-\alpha)} , \tag{21}$$

where γ is defined as $\dfrac{\sigma_b^2 + \sigma_c^2}{\sigma_n^2}$ and the power loss factor as $\alpha = \dfrac{\sigma_b^2}{\sigma_b^2 + \sigma_c^2}$. For the expression of the variance of the channel estimation error obtained in [10]:

$$\sigma_e^2 \approx \frac{M}{N}\frac{N_t\sigma_b^2 + \sigma_n^2}{\sigma_c^2} \approx \frac{M}{N}\frac{\gamma N_t\alpha + 1}{\gamma(1-\alpha)} . \tag{22}$$

To enable a fair comparison of the two expressions, it is assumed that $M = P$, $\alpha = 0.5$, $N = 192$, $P = 16$, and $N_P = N/P = 12$. Figure 2 illustrates a comparison between the error variance obtained in this work and the one from [10], where 2×2 and 4×4 MIMO systems are considered.

We now focus our attention on the third issue related to the number of ZCZ sequences available for training. The ZCZ set used in [10] for training comes from a method proposed in [14], with the latter being focused on the correlation properties of ZCZ sequences, the balance property was not taken into account.

The existing ZCZ sequence sets have only a subset containing balanced sequences. It is the case of the design methods proposed in [15]–[16]. Another constraint is that the balanced sequences in the set are not identifiable. Thus, the task of selecting balanced ZCZ sequences becomes more laborious as the set size increases. Consequently, their potential application in MIMO channel estimation is limited. This issue is addressed in the following section, where a new method of constructing ZCZ sequence sets is proposed in which all ZCZ sequences are balanced.

4. Design of Balanced ZCZ Sequences

A set of ZCZ sequences is denoted by $ZCZ(N, K, Z_o)$, where N is the length of a given sequence, K is the number of the sequences in the set, and Z_o is the zero-zone length in the correlation functions. The proposed method begins with a starter matrix $\mathbf{F}$ and the interleaving technique. There are two sequences in the starter matrix, given as:

$$\mathbf{F} = \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \end{bmatrix} = \begin{bmatrix} - - - + + + - + \\ - + + - + + - - \end{bmatrix}, \tag{23}$$

where + and – denote 1 and –1, respectively. We use the interleaving operation of f_1 and f_2 to obtain:

$$\boldsymbol{Z}^{(1)} = \begin{bmatrix} \boldsymbol{Z}_1^{(1)} \\ \boldsymbol{Z}_2^{(1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_1 \bigotimes \boldsymbol{f}_2 \\ \boldsymbol{f}_1 \bigotimes (-\boldsymbol{f}_2) \end{bmatrix}, \tag{24}$$

where operation $\boldsymbol{f}_1 \bigotimes \boldsymbol{f}_2$ denotes the interleaving of $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$. Set $Z^{(1)}$ is a set of ZCZ sequences with $ZCZ(16, 2, 2)$. In general, an extended set $\boldsymbol{Z}^{(n)}$ with $ZCZ(2^{3+n}, 2, 2^n)$ can be recursively build by interleaving the set of ZCZ sequences $\boldsymbol{Z}^{(n-1)}$ with itself as:

$$\boldsymbol{Z}^{(n)} = \begin{bmatrix} \boldsymbol{Z}_1^{(n)} \\ \boldsymbol{Z}_2^{(n)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{z}_1^{(n-1)} \bigotimes \boldsymbol{z}_2^{(n-1)} \\ \boldsymbol{z}_1^{(n-1)} \bigotimes \left(-\boldsymbol{z}_2^{(n-1)}\right) \end{bmatrix}, \tag{25}$$

where $n \geqslant 2$.
Note that the ZCZ set $\boldsymbol{Z}^{(n)}$ contains two sequences only. To increase the number of sequences, the following operation is applied:

$$\boldsymbol{Z}^{(n,1)} = \begin{bmatrix} \boldsymbol{Z}^{(n)} \boldsymbol{Z}^{(n)} \\ \boldsymbol{Z}^{(n)} \left(-\boldsymbol{Z}^{(n)}\right) \end{bmatrix}, \tag{26}$$

where $\boldsymbol{Z}^{(n)}\boldsymbol{Z}^{(n)}$ denotes the matrix whose i-th row is obtained by concatenating elements of the i-th row of $\boldsymbol{Z}^{(n)}$ and elements of the i-th row of $\boldsymbol{Z}^{(n)}$.

The length of the latter element is 2^{n+2}. The set $\boldsymbol{Z}^{(n,1)}$ is now a set of ZCZ sequences with $ZCZ(2^{3+n+1}, 2^{1+1}, 2^n)$. In general, an extended set $\boldsymbol{Z}^{(n,m)}$ with $ZCZ(2^{3+n+m}, 2^{1+m}, 2^n)$ can be recursively developed by applying Eq. (26) m times to set $\boldsymbol{Z}^{(n)}$:

$$\boldsymbol{Z}^{(n,m)} = \begin{bmatrix} \boldsymbol{Z}^{(n,m-1)} \boldsymbol{Z}^{(n,m-1)} \\ \boldsymbol{Z}^{(n,m-1)} \left(-\boldsymbol{Z}^{(n,m-1)}\right) \end{bmatrix}, \tag{27}$$

where $m \geqslant 2$.
Example: A set of ZCZ sequences with $ZCZ(32, 4, 2)$ can be constructed as follows. By using the interleaving operation of Eq. (24) on the starter matrix of Eq. (23), we obtain:

$$\boldsymbol{Z}^{(1)} = \begin{bmatrix} - - - + - + + - + + + + - - + - \\ - + - - - - + + + - + - - + + + \end{bmatrix}.$$

A set of ZCZ sequences $\boldsymbol{Z}^{(1,1)}$ with $ZCZ(32, 4, 2)$ is obtained by applying Eq. (26). The four sequences of $\boldsymbol{Z}^{(1,1)}$ are:

$$\boldsymbol{Z}_1^{(1,1)} = [- - - + - + + - + + + + - - + - - \\ - - + - + + - + + + + - - + -],$$

$$\boldsymbol{Z}_2^{(1,1)} = [- + - - - - + + + - + - - + + + - \\ + - - - - + + + - + - - + + +],$$

$$\boldsymbol{Z}_3^{(1,1)} = [- - - + - + + - + + + + - - + - + \\ + + - + - - + - - - - + + - +],$$

$$\boldsymbol{Z}_4^{(1,1)} = [- + - - - - + + + - + - - + + + + \\ - + + + + - - - + - + + - - -].$$

It can be easily verified that the set is balanced. In the initial pair of sequences $(\boldsymbol{f}_1, \boldsymbol{f}_2)$, both of them are balanced. Interleaving them will not change their balance, thus all sequences in $\boldsymbol{Z}^{(n)}$ are balanced. The same can be said about the result of concatenating two balanced sequences $\boldsymbol{Z}^{(n,m)}$.

4.1. Correlation Properties

PACF and PCCF properties of the ZCZ codes set $\boldsymbol{Z}^{(1,1)}$ with $ZCZ(32, 4, 2)$ are shown in Fig. 3. To show the zero-zone $Z_o = 2$ in PACF and PCCF, the plot is limited to a delay of $|\tau| \leqslant 20$.

PACF and PCCF properties of a set of ZCZ $\boldsymbol{Z}^{(2,1)}$ with $ZCZ(64, 4, 4)$ are shown in Fig. 4.

5. Conclusion

In this paper, the performance of a MIMO channel estimation procedure based on superimposed training was analyzed. It was shown that to remove the DC offset, training sequences must all be balanced. In addition, to derive an exact expression of the error-variance of channel estimation, training sequences must also have special correlation properties: impulse-like autocorrelation and zero cross-correlation in a specific zone. It turned out that sequences referred to as ZCZ sequences

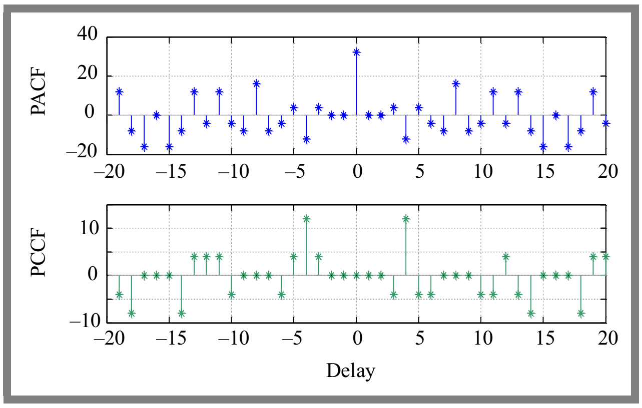

Fig. 3. PACF and PCCF properties of ZCZ codes set $\boldsymbol{Z}^{(1,1)}$.

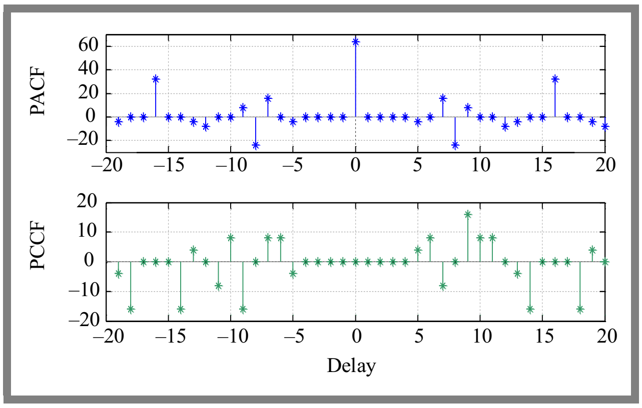

Fig. 4. PACF and PCCF properties of ZCZ codes set $\boldsymbol{Z}^{(2,1)}$.

are ideal for this application. Indeed, the interference in MIMO channel estimation can be eliminated if balanced ZCZ sequences are used for training.

As future work, we believe that the application of the proposed approach for channel estimation performance in MIMO-OFDM may result in an interesting performance analysis focusing primarily on accuracy and simplicity.

Appendix A

Variance of term $\mathbf{A}$ is defined as:

$$\sigma_{\boldsymbol{A}}^2 = E\left[\left|\boldsymbol{A}\right|^2\right] = tr\left\{E\left[\boldsymbol{A}.\boldsymbol{A}^H\right]\right\}. \tag{28}$$

By using property $(AB)^H = B^H A^H$, we get:

$$\begin{aligned}\sigma_{\boldsymbol{A}}^2 = \frac{1}{N_P^2} tr\Bigg\{ \sum_{i=0}^{N_{P-1}} \sum_{j=0}^{N_{P-1}} \sum_{t=1}^{N_t} \sum_{l=1}^{N_t} &\boldsymbol{H}_{rt} E\left[\boldsymbol{b}_t(iP)\boldsymbol{b}_l^H(jP)\right] \\ &\boldsymbol{H}_{rl}^H + \boldsymbol{H}_{rt} E\left[\boldsymbol{b}_t(iP)\boldsymbol{n}_r^H(jP)\right] \\ &+ E\left[\boldsymbol{n}_r(iP)\boldsymbol{b}_l^H(jP)\right]\boldsymbol{H}_{rl}^H \\ &+ E\left[\boldsymbol{n}_r(iP)\boldsymbol{n}_r^H(jP)\right]\Bigg\}. \end{aligned} \tag{29}$$

Assuming that $\boldsymbol{b}(k)$ and $\boldsymbol{n}(k)$ are uncorrelated:

$$E\left[\boldsymbol{b}_t(iP)\boldsymbol{n}_r^H(jP)\right] = 0_{(2P-1)\times P} \tag{30}$$

and

$$E\left[\boldsymbol{n}_r(iP)\boldsymbol{b}_l^H(jP)\right] = 0_{P\times(2P-1)}\,. \tag{31}$$

Equation (29) becomes:

$$\begin{aligned}\sigma_{\boldsymbol{A}}^2 = \frac{1}{N_P^2} tr\Bigg(\sum_{i=0}^{N_P-1} \sum_{j=0}^{N_P-1} \sum_{t=1}^{N_t} \sum_{l=1}^{N_t} &\boldsymbol{H}_{rt} E\left[\boldsymbol{b}_t(iP)\boldsymbol{b}_l^H(jP)\right] \\ &\boldsymbol{H}_{rl}^H + E\left[\boldsymbol{n}_r(iP)\boldsymbol{n}_r^H(jP)\right]\Bigg). \end{aligned} \tag{32}$$

Assuming $\boldsymbol{b}(k)$ are i.i.d., for $l \neq t$ we have:

$$E\left[\boldsymbol{b}_t(iP)\boldsymbol{b}_l^H(jP)\right] = 0_{P\times(2P-1)}\,. \tag{33}$$

Thus, Eq. (32) becomes:

$$\begin{aligned}\sigma_{\boldsymbol{A}}^2 = \frac{1}{N_P^2} tr\Bigg\{ \sum_{i=0}^{N_P-1} \sum_{j=0}^{N_P-1} \sum_{t=1}^{N_t} \sum_{l=1}^{N_t} &\boldsymbol{H}_{rt} E\left[\boldsymbol{b}_t(iP)\boldsymbol{b}_t^H(jP)\right] \\ &\boldsymbol{H}_{rt}^H + E\left[\boldsymbol{n}_r(iP)\boldsymbol{n}_r^H(jP)\right]\Bigg\}. \end{aligned} \tag{34}$$

It can be shown that:

$$E\left[\boldsymbol{b}_t(iP)\boldsymbol{b}_t^H(jP)\right] = \begin{cases} \sigma_{b_t}^2 \boldsymbol{I}_{2P-1}, & i = j \\ \sigma_{b_t}^2 \boldsymbol{J}_{2P-1}, & j = i+1 \\ \sigma_{b_t}^2 \boldsymbol{J}_{2P-1}^T, & j = i-1 \\ \boldsymbol{O}_{2P-1} & \text{otherwise} \end{cases} \tag{35}$$

where matrix $\boldsymbol{J}$ is given by
$\boldsymbol{J} = \text{semicirc}\,[0, 0, \ldots, 1, 0, \ldots, 0]$.

The semicirc operation produces a semi-circulant matrix, where element 1 in the first line appears in position $(P+1)$ [9]. The expectation of the second term in Eq. (34) is:

$$E\left[\boldsymbol{n}_r(iP)\boldsymbol{n}_r^H(jP)\right] = \begin{cases} \sigma_{n_r}^2 \boldsymbol{I}_P, & i = j \\ \boldsymbol{O}_P, & i \neq j \end{cases} \tag{36}$$

Equation (34) becomes:

$$\begin{aligned}\sigma_{\boldsymbol{A}}^2 =& tr\Bigg\{ \sum_{t=1}^{N_t} \frac{1}{N_P} \sigma_{b_t}^2 \boldsymbol{H}_{rt}\boldsymbol{H}_{rt}^H \\ &+ \frac{N_P - 1}{N_P^2} \sigma_{b_t}^2 \boldsymbol{H}_{rt}\boldsymbol{J}_{2P-1}^{+}\boldsymbol{H}_{rt}^H + \frac{1}{N_P}\sigma_{n_r}^2 \boldsymbol{I}_P \Bigg\}, \end{aligned} \tag{37}$$

where $\boldsymbol{J}_{2P-1}^{+} = \boldsymbol{J}_{2P-1} + \boldsymbol{J}_{2P-1}^T$.

Knowing that $tr\{A\} + tr\{B\} = tr\{A + B\}$, the above equation takes the following form:

$$\sigma_A^2 = \sum_{t=1}^{N_t} \frac{\sigma_{b_t}^2}{N_P} tr\left\{\boldsymbol{H}_{rt}\boldsymbol{H}_{rt}^H\right\} + \frac{(N_P-1)\sigma_{b_t}^2}{N_P^2} tr\left\{\boldsymbol{H}_{rt}\boldsymbol{J}_{2P-1}^{+}\boldsymbol{H}_{rt}^H\right\} + \frac{\sigma_{n_r}^2}{N_P} tr\left\{\boldsymbol{I}_P\right\} . \tag{38}$$

In Eq. (38), term $tr\left\{\boldsymbol{H}_{rt}\boldsymbol{H}_{rt}^H\right\} = P\sum_{k=0}^{M-1}\left|h_{rt}(k)\right|^2 = P$, since each channel energy has been normalized to one, $tr\left\{\boldsymbol{H}_{rt}\boldsymbol{J}_{2P-1}^{+}\boldsymbol{H}_{rt}^H\right\} = 0$ follows directly from matrices $\boldsymbol{H}_{rt}$ and $\boldsymbol{J}_{2P-1}^{+}$, and $tr\{\boldsymbol{I}_P\} = P$.

Therefore, Eq. (38) may be written as:

$$\sigma_A^2 = \frac{P}{N_P}\left(\sum_{t=1}^{N_t}\sigma_{b_t}^2 + \sigma_{n_r}^2\right) . \tag{39}$$

Now, we have a simple expression of the variance of term $\mathbf{A}$.

Appendix B

Similarly, the variance of term $\boldsymbol{B}$ is defined as:

$$\begin{aligned}\sigma_B^2 &= E\left[|\boldsymbol{B}|^2\right] = tr\left\{E\left[\boldsymbol{B}.\boldsymbol{B}^H\right]\right\} \\ &= tr\left\{E\left[\sum_{t=1}^{N_t}\sum_{l=1}^{N_t}\boldsymbol{C}_t\boldsymbol{e}_{rt}\boldsymbol{e}_{rl}^H\boldsymbol{C}_l^H\right]\right\}\end{aligned} \tag{40}$$

By using $tr\{E[A]\} = E\big[tr\{A\}\big]$ and $tr\{AB\} = tr\{BA\}$, we get:

$$\sigma_B^2 = E\left[\sum_{t=1}^{N_t}\sum_{l=1}^{N_t} tr\left\{\boldsymbol{C}_l^H\boldsymbol{C}_t\boldsymbol{e}_{rt}\boldsymbol{e}_{rl}^H\right\}\right] . \tag{41}$$

Since $\boldsymbol{C}_t$ is a circulant matrix, $\boldsymbol{C}_t^H$ is also circulant. Thus, $\boldsymbol{C}_t^H\boldsymbol{C}_t = \boldsymbol{C}_t\boldsymbol{C}_t^H$ and σ_B^2 is:

$$\sigma_B^2 = E\left[\sum_{t=1}^{N_t}\sum_{l=1}^{N_t} tr\left\{\boldsymbol{C}_t\boldsymbol{C}_l^H\boldsymbol{e}_{rt}\boldsymbol{e}_{rl}^H\right\}\right] . \tag{42}$$

Let us now further develop the above expression. Firstly, the term $\boldsymbol{e}_{rt}$ is:

$$\boldsymbol{e}_{rt} = \big[e_{rt}(P-1), e_{rt}(P-2), \ldots, e_{rt}(M), \ldots, e_{rt}(1), e_{rt}(0)\big]_P^T . \tag{43}$$

Since we already know that taps:

$$h_{rt}(M), h_{rt}(M+1), \ldots, h_{rt}(P-1)$$

in the channel impulse response are all zero, their estimated values $\hat{h}_{rt}(M), \hat{h}_{rt}(M+1), \ldots, \hat{h}_{rt}(P-1)$ are considered to be equal to zero. Thus, elements $e_{rt}(P-1), e_{rt}(P-2), \ldots, e_{rt}(M)$ in Eq. (43) are replaced by zeros. It can be shown that matrix $\boldsymbol{e}_{rt}\,\boldsymbol{e}_{rl}^H$ can be written as:

$$\boldsymbol{e}_{rt}\boldsymbol{e}_{rl}^H = \begin{bmatrix} \mathbf{0}_{P-M} & \mathbf{0}_{(P-M)\times M} \\ \mathbf{0}_{M\times(P-M)} & \boldsymbol{e}_M \end{bmatrix}_{P\times P} , \tag{44}$$

where $\boldsymbol{e}_M$ is given by Eq. (45), and secondly, we have term $\boldsymbol{C}_t\boldsymbol{C}_l^H$ given by Eq. (46).

$$\boldsymbol{e}_M = \begin{bmatrix} e_{rt}(M-1)e_{rl}^*(M-1) & & e_{rt}(M-1)e_{rl}^*(1) & e_{rt}(M-1)e_{rl}^*(0) \\ \vdots & & \vdots & \vdots \\ e_{rt}(1)e_{rl}^*(M-1) & \cdots & e_{rt}(1)e_{rl}^*(1) & e_{rt}(1)e_{rl}^*(0) \\ e_{rt}(0)e_{rl}^*(M-1) & & e_{rt}(0)e_{rl}^*(1) & e_{rt}(0)e_{rl}^*(0) \end{bmatrix}_{M\times M} \tag{45}$$

$$\boldsymbol{C}_t\boldsymbol{C}_l^H = \left[\begin{array}{ccc|cccc} & & & R_{tl}(M) & & R_{tl}(P-2) & R_{tl}(P-1) \\ & & & R_{tl}(M-1) & & R_{tl}(P-3) & R_{tl}(P-2) \\ & \boldsymbol{R} & & \vdots & \cdots & \vdots & \vdots \\ & & & \vdots & & R_{tl}(P-M-1) & R_{tl}(P-M) \\ & & & \vdots & & \vdots & \vdots \\ & & & \cdots & & R_{tl}(M-1) & R_{tl}(M) \\ \hline R_{tl}(M) & R_{tl}(M-1) & \vdots & & & & \\ \vdots & \vdots & \cdots & & & & \\ R_{tl}(P-M) & R_{tl}(P-M-1) & \vdots & & & & \\ \vdots & \vdots \quad \cdots & \vdots & & & & \\ R_{tl}(P-2) & R_{tl}(P-3) & R_{tl}(M-1) & & & \boldsymbol{R} & \\ R_{tl}(P-1) & R_{tl}(P-2) & R_{tl}(M) & & & & \end{array}\right]_{P\times P} \tag{46}$$

$$\boldsymbol{R}=\begin{bmatrix} R_{tl}(0) & R_{tl}(1) & \dots & R_{tl}(M-1) \\ R_{tl}(1) & R_{tl}(0) & \dots & R_{tl}(M-2) \\ \vdots & \vdots & \vdots & \vdots \\ R_{tl}(M-1) & R_{tl}(M-2) & \dots & R_{tl}(0) \end{bmatrix}_{M\times M}, \tag{47}$$

and $R_{tl}(\tau)$ is the periodic cross-correlation function (PCCF) defined as:

$$R_{tl}(\tau) = \sum_{k=0}^{P-1} c_t(k)\, c_l(k-\tau)\ . \tag{48}$$

When $l = t$, PCCF becomes the periodic auto-correlation function (PACF). If all the training sequences used for channel estimation in the MIMO system have the following correlation properties:

$$R_{tl}(\tau) = \begin{cases} R_t(0), & \tau = 0,\ \ t = l \\ 0,\ \ 1 \leqslant \tau \leqslant M-1, & t = l\ , \\ 0,\ \ 0 \leqslant \tau \leqslant M-1, & t \neq l \end{cases} \tag{49}$$

then $\mathbf{R}$ will be:

$$\boldsymbol{R}=\begin{bmatrix} R_t(0) & 0 & \dots & 0 \\ 0 & R_t(0) & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & R_t(0) \end{bmatrix}_{M\times M} \quad \text{for } l = t \tag{50}$$

and

$$\boldsymbol{R} = \mathbf{0}_{M\times M}, \quad \text{for } l \neq t \tag{51}$$

the product $\boldsymbol{C}_t\boldsymbol{C}_l^H\boldsymbol{e}_{rt}\boldsymbol{e}_{rl}^H$ can now be written as:

$$\boldsymbol{C}_t^H\boldsymbol{C}_t\boldsymbol{e}_{rt}\boldsymbol{e}_{rt}^H=\begin{bmatrix} \mathbf{0}_{P-M} & \boldsymbol{D}_{(P-M)\times M} \\ \mathbf{0}_{M\times(P-M)} & R_t(0)\boldsymbol{e}_M \end{bmatrix}_{P\times P} \quad \text{for } l = t \tag{52}$$

$$\boldsymbol{C}_l^H\boldsymbol{C}_t\boldsymbol{e}_{rt}\boldsymbol{e}_{rl}^H=\begin{bmatrix} \mathbf{0}_{P-M} & \boldsymbol{D'}_{(P-M)\times M} \\ \mathbf{0}_{M\times(P-M)} & 0\times\boldsymbol{e}_M \end{bmatrix}_{P\times P} \quad \text{for } l \neq t. \tag{53}$$

where $\boldsymbol{D}_{(\boldsymbol{P}-\boldsymbol{M})\times\boldsymbol{M}}$ and $\boldsymbol{D}'_{(\boldsymbol{P}-\boldsymbol{M})\times\boldsymbol{M}}$ are matrices whose elements are different from zero. Note that for $l \neq t$, the trace of the matrix above will be equal to zero. Then, $\sigma^2_{\boldsymbol{B}}$ becomes:

$$\begin{aligned} \sigma_B^2 &= \left[\sum_{t=1}^{N_t} tr\left\{R_t(0)\boldsymbol{e}_M\right\}\right] \\ &= \sum_{t=1}^{N_t} P\sigma_{c_t}^2\sigma_{e_{rt}}^2\ . \end{aligned} \tag{54}$$

We have now a simple expression of the variance of term $\mathbf{B}$.

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