Compressive Sensing-based Differential Channel Feedback Scheme Using Subspace Matching Pursuit Algorithm for B5G Wireless Systems

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Abstract — Millimeter wave (mmWave) massive multi-input multi-output (MIMO) systems are the promising technology for next-generation 5G wireless systems and beyond. Sparse signal recovery and channel feedback are challenging and fundamental problems affecting downlink transmission due to the substantial increase in channel matrix size in mmWave systems. To overcome the overhead of the channel and improve CS recovery effectiveness, this article proposes the joint use of the subspace matching search algorithm and differential operation for channel impulse response (CIR). Here, the current CIR is converted to a differential CIR using operations between the current and previous CIRs. The differential CIR is then compressed and fed back to the base station. Subsequently, this differential CIR is recovered using the subspace matching search algorithm. Such a scheme leverages effective structural sparsity through a combination of subspace and differential operations. The adaptive algorithm adaptively selects relevant subspaces based on coefficients. The simulation results show that the proposed scheme reduces channel overhead by 36% and 24% at compression ratios of 25% and 45%, respectively, over different time slots in mmWave massive MIMO systems.

Keywords — channel impulse response, channel state information, compressive sensing, mmWave

1. Introduction

Massive millimeter MIMO systems are a key technology for 5G communication systems and beyond [1]. Such systems are equipped with many directional antennas to achieve effective beamforming gains [2] required to cope with higher path losses. To overcome these issues, effective beamforming strategies are proposed along with a large number of antenna arrays. There is always a trade-off between performance and system complexity when using such beamforming strategies [3]. Effective channel state information (CSI) estimation is very important to obtain optimal performance of any scheme.

For time division duplexing (TDD) scheme-based mmWave massive MIMO systems, the downlink (DL) channel CSI is estimated immediately by DL using received the transmitted pilot signals due to its channel reciprocity. However, in frequency division duplexing (FDD) systems, due to nonexistence of channel reciprocity [4], the uplink channel (UL) CSI is not always equivalent to the DL CSI. Excessive channel feedback is always required to estimate CSI, making channel feedback very challenging [5]. To address this issue, orthogonal pilot signals are transmitted. However, the use of such orthogonal pilots results in increased resource utilization within the communication system.

Generally, the estimation of mmWave channels always exhibits various sparse scattering characteristics [6]. The mmWave channel matrix always exploits a highly sparse channel matrix with non-zero elements based on the positions of angle of arrival (AoA) and angle of departure (AoD) of the various dominant multipath signals [7]. In massive MIMO systems, effective recovery of the sparse signal is considered as the mmWave-based channel estimation problem.

Compressive sensing (CS) algorithms are an effective approach [8] adopted to reconstruct undetermined linear systems. MmWave channel estimation is generally formulated as a sparse signal recovery problem, which provides a compressive measurement that combines the effects of analog and digital precoders and combiners. CS-based algorithms are introduced to recover these sparse signal recovery problems by effectively quantifying AoA and AoD of various multipath signals for the formation of uniform grids. Some hierarchical codebook-based schemes are designed in such a way as to estimate the millimeter wave path parameters of the channels in MIMO systems.

For CS recovery, the orthogonal matching pursuit (OMP) algorithm is employed to quantize non-uniform angle grids. Based on this evidence, direct estimation of all the entries in the mmWave channel of massive MIMO systems is a challenging task. CS-based channel feedback schemes [9] are proposed to exploit the antenna's spatial correlations by compressing the channel matrix of mmWave channels, thereby reducing the feedback overhead.

The channel impulse response (CIR) is directly compressed and recovered using effective CS algorithms, exploiting them in the time domain to reduce feedback overhead [10]. In mmWave channels, the overhead is very high because of the sparsity nature of the channel impulse response.

2. Related Works

Many CS-based recovery algorithms have been analyzed and proposed in recent studies to reduce channel feedback. The sparse recovery problem [11] is formulated based on the multipath signal by quantizing the angle of arrivals (AoA) and angle of departures (AoDs) into uniform grids. Some effective and adaptive CS-based algorithms are developed to estimate channel parameters. To facilitate this estimation operation, multiresolution codebook-based CS algorithms have been developed [12]. Some methods improve channel recovery performance by identifying non-uniformly quantized angles. The approach presented in [13] also reduces coherence redundancy of the systems. To find the array response vectors and to estimate non-uniform quantized angle grids, the orthogonal matching pursuit (OMP) algorithm is proposed [14].

Other basic search algorithms are also introduced to reduce computational complexity of existing systems [15]. In [16], two different adaptive low-complexity CS algorithms are presented which iteratively estimate channel parameters. A block sparsity-based CS algorithm is proposed in [17], considering multi-user mmWave massive MIMO systems.

In studies [18]–[20], CS algorithms are used for narrowband frequency-flat channels. In such cases, they were analyzed in the time domain instead of the angular or frequency domains. The main challenge in estimating channels in frequency-selective channels is the need for common supports between the mmWave channel sparsity matrices, which inevitably leads to a better trade-off between complexity and performance. In [20], uplink frequency-selective channels are estimated using a two-stage CS algorithm, with precoders and combiners. The major challenge is that, in the time domain, wideband mmWave channels exploit delays in sparse channel vectors. To address the problem of sparse channel recovery, hybrid architectures are proposed, but, unfortunately, they generally require limited training [21].

In paper [22], a two-step time domain estimator based on OMP and the least squares methods is formulated to reduce computational complexity. The sparsity nature of mmWave systems in the angle and delay domains is defined jointly, and the problem is effectively iterated by the CS-based message passing method (MPM) [23] which exploits its structured sparsity to find the nearest neighbor pattern [24], [25]. In addition to exploiting the sparsity nature, other structured channel models, such as low rank [26], uniform grid structure [27], [28], and jointly sparse and low rank structures [29], are also used and estimated by leveraging the sparsity feature. The sparse recovery problem is defined for both time and frequency domains. Different CS algorithms [30] are used for recovery in the time domain the frequency domain, or a combination of both, as proposed in [31]–[33].

Most of the existing works, e.g. [34] and [35], dealing with mmWave channel estimation techniques, focus only on narrowband frequency models. To provide context, techniques focusing on wideband mmWave channels and frequency-selective channels are also analyzed in both frequency and time domains [36]. To analyze downlink frequency-selective channels, CS techniques will be helpful in finding the common support which exhibits the sparsity of mmWave channels in the frequency domain [20].

Other methods use effective precoders and combiners employing both on-grid and off-grid techniques [37]. In general, these wideband mmWave channel estimation techniques widely exploit delay sparsity while using small training overheads. Lastly, two-step channel estimation methods are proposed using least squares estimation along with orthogonal matching search algorithms for CS recovery [38].

In this paper, we propose a modified compressive sensing-based differential channel feedback scheme using the subspace matching pursuit recovery algorithm (SMP-DF CS) to reduce feedback overhead and improve the performance of mmWave massive MIMO systems. The said scheme exploits the temporal correlation of highly time-varying mmWave massive MIMO systems. This temporal correlation property exists in both distributed and centralized systems.

The proposed algorithm effectively reduces the feedback overhead and computational complexity at different compressive ratios and demonstrates improved performance for the differential feedback scheme of mmWave massive MIMO systems.

The main contribution of this article is summarized as follows:

- A joint framework is proposed for a modified SMP-based CS recovery scheme with differential operation for the effective estimation of the channel impulse response in the mmWave channel. This concept effectively leverages the sparsity nature of mmWave channels in the angular domain and reduces channel overhead.
- The CS recovery scheme introduced uses a modified subspace matching pursuit algorithm to improve CIR recovery by effectively searching subspaces in each iteration in order to form the support vectors.
- The proposed adaptive algorithm selects relevant subspaces based on coefficients, rather than choosing each basis for iterations, thus resulting in faster convergence and a better achievable sum rate.

The paper is organized as follows. Section 3 explains the proposed CS-based differential channel feedback scheme for subspace matching using a model of the system model and relevant preliminaries. Section 4 gives a detailed explanation of the performance of NMSE in different SNR regimes. Conclusions are presented in Section 5.

3. Proposed SMP-DF CS Scheme

3.1. System Model

The model of a mmWave massive MIMO system is equipped with N_t and N_R transmitting and receiving antennas, respectively. The received downlink signal transmission r_p, based on the training symbol, is expressed as:

$$r_p = H_p f_p s_p + n_p \; , \tag{1}$$

where H_p is the channel matrix of the downlink mmWave system, f_p is the vector representing beamforming, s_p denotes the received symbol vector, and n_p represents the complex white noise vector with Gaussian distribution. The combined vectors formed to detect the transmitted symbols are given by:

$$y_p = W^H H_p f_p s_p + W^H n_p \; , \tag{2}$$

where vector $w = [W_1, W_2, W_3, \ldots, W_{Nq}]$.

The downlink channel matrix equivalent to its transmitted pilots is given as $H_p = H$ and is based on block fading. The received signal vector is formulated as:

$$Y = [y_1, y_2, y_3, \ldots, y_{N_p}] \; . \tag{3}$$

After receiving the pilot signals, Eq. (3) becomes:

$$Y = W^H HFS + W^H N \; , \tag{4}$$

where s represents the diagonal matrix.

The mmWave signal channel is modelled as:

$$H = \sqrt{\frac{N_T N_R}{L}} \sum_{l=1}^{L} \alpha_l \, . \, a_r(\theta_l) \, . \, a_t^H(\phi_l) \; , \tag{5}$$

where L represents the number of scatters, θ_l and φ_l denote the array response vectors that are associated with azimuth and elevation angles, respectively.

The array response vectors $a_t(\theta_l)$ of the transmitter are:

$$\sqrt{\frac{1}{N_t}} \left[1, e^{j\frac{2\pi}{\lambda} d \sin(\theta_l)}, \ldots, e^{j\frac{2\pi}{\lambda}(N_t-1) d \sin(\theta_l)}\right] . \tag{6}$$

The array response vectors $a_r(\varphi_l)$ of the receiver are formulated as:

$$\sqrt{\frac{1}{N_r}} \left[1, e^{j\frac{2\pi}{\lambda} d \sin(\phi_l)}, \ldots, e^{j\frac{2\pi}{\lambda}(N_r-1) d \sin(\phi_l)}\right] , \tag{7}$$

where d and λ represent the distance between the adjacent antennas in the UPA and the wavelength of the signal, respectively.

The model of the system is finally defined in a compact form as:

$$H = A_r \, . \, H_\alpha \, . \, A_t^H \; , \tag{8}$$

where the terms $A_t = [a_t(\theta_1), a_t(\theta_2), \ldots, a_t(\theta_l)]$ and $A_r = [a_r(\phi_1), a_r(\phi_2), \ldots, a_r(\phi_l)]$ represents the steering matrix of transmitter and receivers of this systems, respectively.

The channel matrix is formulated as follows:

$$H_\alpha = \sqrt{\frac{N_t \, . \, N_r}{L}} \mathrm{diag}\left[\alpha_1, \alpha_2, \ldots, \alpha_L\right] , \tag{9}$$

where H_α is the diagonal matrix based on the steering vectors of the transmitted and received signals.

3.2. Time Varying mmWave Massive MIMO Channel

The temporal time-varying channel impulse response of the mmWave massive MIMO system of the t-th time slot of the n-th transmitting antenna at the base station (BS) is considered. The effective channel model for the received signal at a single-antenna user is given as:

$$h(t)_n = \left[h_n^{(t)}(0), h_n^{(t)}(1), \ldots, h_n^{(t)}(L-1)\right] , \tag{10}$$

where N is the total number of transmitting antennas and L is the maximum channel delay spread.

The channel impulse response (CIR) is always very sparse because of its nature in mmWave communication, as it typically consists of only a few dominant propagation paths. These paths play a significant role in improving channel response.

The sparsity nature of the CIR series $\left[h(t)_n^T\right]_{t=1}^T$ consists of T consecutive time slots that exhibit high temporal correlation values even in massive MIMO channels with fast time-varying MIMO channels. The change in temporal correlations always exists through the support vectors which refer to the position of non-zero elements and their amplitudes.

The time-varying and sparse nature of the CIR equation is formulated by the support vector $p(t)_n$ and the amplitude vectors $a(t)_n$. Then the Eq. (10) becomes:

$$h(t)_n = a(t)_n \circ p(t)_n \; , \tag{11}$$

where $p(t)_n$ is the support vector and $a(t)_n$ is the amplitude vector at time slot t of the n-th transmitting antenna. The "$\circ$" symbol represents the Hadamard product which denotes element-wise multiplication.

In order to model the rapid variations of this mmWave channel, support vectors $p(t)_n$ over time slot t in l elements can be represented as a first-order Markov process. This Markov process is characterized on two different transition probabilities, denoted as P_{01} and P_{10}. These terms are distributed $m(1)_n$ at the initial time slot $t = 1$. For any steady-state in the Markov process, the transition probabilities for all values of t and n are given as:

$$\Pr\left[p(t)_n(l) = 1\right] = m \; . \tag{12}$$

The other transmission probabilities P_{10} are represented as:

$$P_{10} = \frac{\mu \, P_{01}}{1-\mu} \; . \tag{13}$$

CIR amplitude over the time slot t is modeled using a first-order autoregressive model, given as:

$$a(t)_n = \rho \, . \, a(t-1)_n + \sqrt{1-\rho^2 \, w(t)} \; , \tag{14}$$

where r represents the correlation coefficient, ρ is the zero-order Bessel function, f_d represents the maximum Doppler frequency, τ is the duration between the time intervals, and $w(t)$ stands for independent noise vectors, with all elements always assumed to be independent and identically distributed (iid) with a normal distribution.

3.3. SMP-DF CS Scheme

The proposed algorithm (depicted in Fig. 1) enables the adaptation of beamforming techniques and provides effective reduction of channel feedback, as well as improves robustness to channel estimation errors. This scheme directly compresses

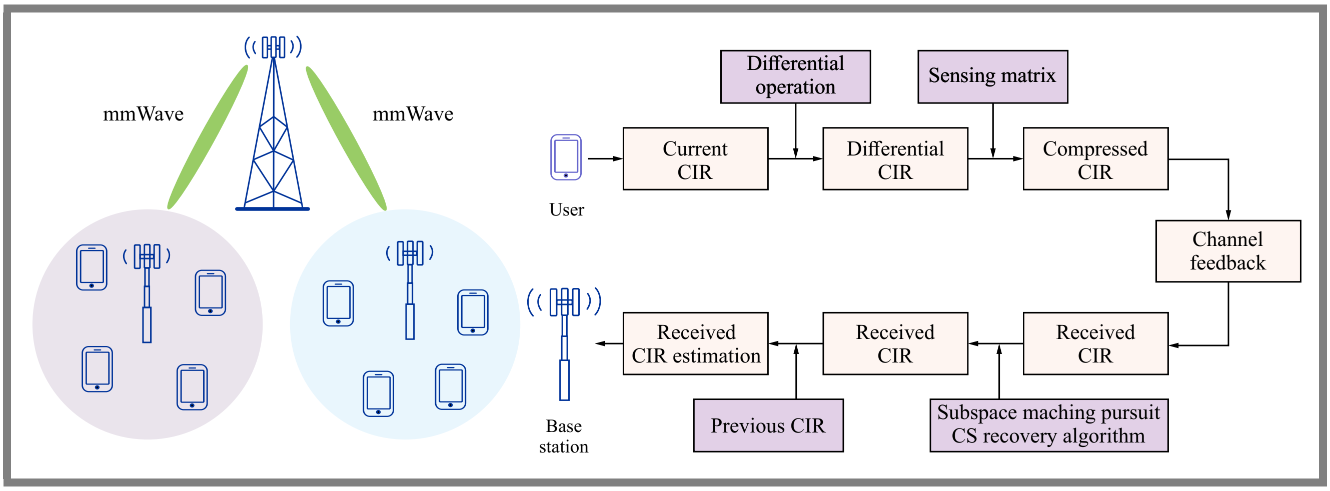

Fig. 1. Subspace matching pursuit CS recovery-based differential channel feedback.

sparse CIR using the sensing matrix based on the CS algorithm.

Before compression, the sparse CIR is converted into a differential CIR by using differential operations between the current and previous CIRs. This differential CIR has better sparsity than the original CIR. It computes the difference between the estimated $h_n(t-1)$ at $(t-1)$ slot and the previous CIR $h(t)$ at t slot. The differential CIR is expressed as [9]:

$$\triangle h_n^{(t)} = h_n^{(t)} - h_n^{(t-1)} . \quad (15)$$

Algorithm 1 Subspace matching pursuit CS algorithm

Input: measurement vector z, measurement matrix Φ, sparsity level s, maximum number of iterations max_iter

Output: recovered signal a, number of used iterations $used\_iter$

Start

$d \leftarrow$ dimension of Φ
$a \leftarrow \mathbf{0}_d$ ▷ Initialize recovered signal
$\rho \leftarrow z$ ▷ Initialize residual
$T \leftarrow \emptyset$ ▷ Initialize support
for $it = 1$ to max_iter **do**
  Compute inner products: $inner\_products \leftarrow |\Phi^T \rho|$
  Update support: $T \leftarrow$ indices of top $2s$ elements of $inner\_products$
  Estimate signal: $b \leftarrow \mathbf{0}_d$, $b_T \leftarrow (\Phi(:,T))^+ z$
  Prune signal estimate: keep top s coefficients; $T \leftarrow$ indices of top s elements of $|b|$
  Update recovered signal: $a \leftarrow \mathbf{0}_d$, $a_T \leftarrow b_T$
  Update residual: $r \leftarrow z - \Phi a$
  if $\|\rho\| < 1\mathrm{E}-3\,\|z\|$ **then**
    break
  end if
end for
$used\_iter \leftarrow it$

End

After substituting the current and previous CIRs according to Eq. (14), the equation becomes:

$$\triangle h_n^{(t)} = p_n^{(t)} \circ \left[\sqrt{(1-\rho^2 w(t)} - (1-\rho) a_n^{(t-1)}\right] + \left[(p_n^{(t)} - p_n^{(t-1)} \circ a_n^{(t-1)}\right] . \quad (16)$$

The first term of Eq. (16), which consists of the CIR amplitude, is almost negligible. Similarly, the second term, representing the non-zero elements, is also very small. To avoid errors in feedback propagation, mobile users initialize the CIR based on a high and effective compression ratio to enable precise recovery at the base station.

After compression of the differential CIR, the channel is fed back to the BS. On the BS side, the proposed scheme is based on three important steps. The received CIR is efficiently recovered using the subspace matching pursuit (SMP)-based CS algorithm.

Based on the compressive sensing (CS) theory, the highly sparse differential CIR is compressed using the sensing matrix. Then, the measurement vector is given as:

$$y = \phi \,.\, \triangle\, h_n^{(t)} , \quad (17)$$

where ϕ represents the sensing matrix and $\triangle h_n^{(t)}$ represents the differential CIR.

On the receiving side of the BS, measurement vector y is fed back through the channel:

$$y = \phi \,.\, \triangle\, h_n^{(t)} + n , \quad (18)$$

where n represents the noise of the channel. Here, the entities are also considered i.i.d. and follow a normal distribution.

On the BS side, the noise vector is added to the measurement vector. The SMP-based CS recovery algorithm is adopted to recover differential CIRs. The SMP-based CS recovery algorithm iteratively refines the differential CIR by updating its support and amplitudes based on the received measurement vector. This Algorithm 1 initializes such parameters as d, which represents the dimension of the sensing matrix, a which is a zero vector, ρ – denoting the measurement vector, and T, which represents the support vector of the signal.

Tab. 1. Simulation parameters for the proposed scheme.

Parameter	Value
Measurement matrix dimensions L	200
Transmission probability at initial time slot ρ	0.05
Initial probability of support vector m	0.1
Maximum Doppler frequency f_d	10 Hz
Time slot duration τ	1 ms
Standard deviation of independent noise vector σ	1
Number of antennas N	32

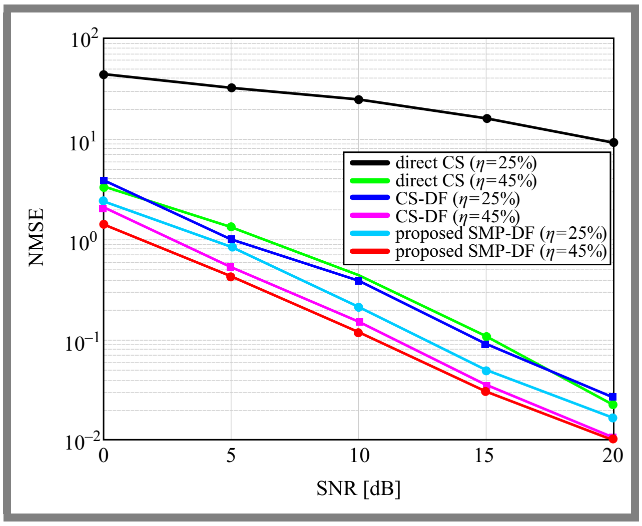

Fig. 2. Comparison of NMSE versus SNR for the proposed scheme and existing CS schemes (η =25% and 45%).

In each iteration stage of this algorithm, the inner products between the residual and columns are computed first to find the dimension of the sensing matrix. Thereafter, the support vector is selected based on the magnitudes of the first two elements. The signal is estimated by optimizing the least squares problem. The signal is estimated based on the coefficient with the highest magnitude. The residue is then updated by calculating the difference between the updated signal and its measurements.

4. Simulation Results

This section presents the numerical results to illustrate the proposed channel estimation algorithm and other existing methods. Here, we have considered the mmWave system architecture where both the transmitter and receiver are equipped with uniform planar arrays. Half of the wavelength of the signal $\frac{\lambda}{2}$ will be considered as the distance between the adjusted antenna elements. Similarly, the values of AoA and AoD of each path are selected using a uniform distribution form $[0, 2\pi]$. Similarly, the gain of each mmWave path follows a normal distribution with a mean value of 1 and a variance of 1, as $N(0, 1)$. It is assumed that the system operates under a carrier frequency of 28 GHz with a bandwidth of 100 MHz. The simulations consider the massive millimeter wave MIMO system model with the parameters shown in Tab. 1.

The proposed SMP-DF CS algorithm is compared with the existing conventional direct and CoSaMP-based differential CS (CS-DF) algorithms at two different compression ratios. In the differential CIR, it is assumed that the initial amplitude and its support vectors may vary from the value 0 to N and that such vectors are always independent. To maintain the appropriate effective channel compression ratio ($\eta = \frac{\mu}{L}$) over the feedback slots, η is maintained at 25% and 45%, respectively. The SMP recovery CS-based differential feedback scheme is compared with a direct CS scheme and a CS recovery-based differential feedback method. For evaluation, the normalized mean square error (NMSE) is calculated using the following equation.

$$\text{NMSE} = E\left[\frac{||\hat{H} - H||^2}{||H||^2}\right], \tag{19}$$

where $\hat{H}$ is the estimate of the true channel H.

A comparison of NMSE for the specific schemes is shown in Fig. 2 for different signal-to-noise ratio (SNR) regimes.

To achieve effective compression ratios, 45% and 15% are considered for initial and subsequent time slots for the first case (η =25%). For the second case (η =45%), the ratios are 65% and 35%, respectively. This result shows that the proposed scheme outperforms other existing schemes by 36% and 24% at the compression ratios of η at 25% and 45%, respectively. This is achieved by leveraging the structural sparsity of signals as a subspace combination. The adaptive pursuit strategies to select the relevant subspace and coefficient iteratively, integrating the differential operation, enhance overall robustness and improve the recovered accuracy.

The achievable sum rate of the proposed system is compared with all other existing direct CS and differential feedback-based CS schemes at different compression ratios of 25% and 45%, respectively. The achievable sum rate is to maximize the data rate supported by the system for all end users given the channel. Figure 3 shows that as SNR increases, with the achievable sum rate also improving for the proposed scheme. The SMP-based differential CS scheme performs the best across the entire SNR range compared to other existing CS

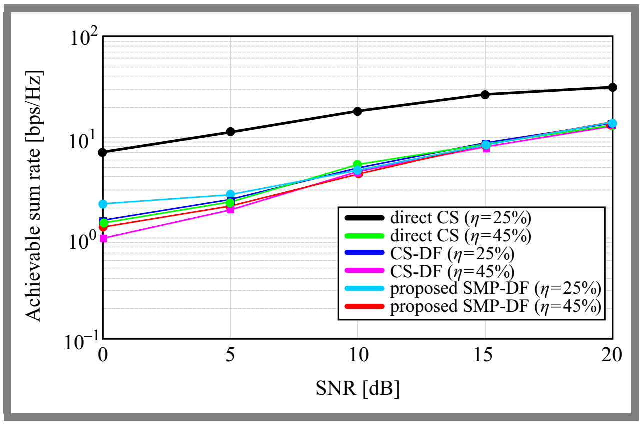

Fig. 3. Achievable sum rate versus SNR of the proposed scheme.

schemes. Furthermore, the gap between the direct CS method and the proposed SMP-based differential feedback method becomes comparatively narrower as SNR increases and the compression ratio η increases from 25% to 45%, respectively.

5. Conclusion and Future Work

This article investigates the sparse channel recovery problem and the issue of channel feedback overhead. To enhance the performance of the CS-based differential channel feedback scheme, the article proposes a subspace matching pursuit recovery algorithm used in conjunction with differential operations. A simulation has been performed to analyze and compare NMSE across different SNR regimes.

Results of the simulation show that the proposed scheme outperforms the other existing direct CS and differential CS schemes, reducing the channel overhead by 36% and 24% at different compression ratios over the time slots of mmWave massive MIMO systems. In future work, the proposed scheme may be extended to intelligent reflecting surface (IRS)-aided mmWave massive MIMO systems to meet the requirements of future-generation wireless systems.

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