Synthesizing Wide-beam Array Patterns Using Phase-only Control and Trapezoidal Amplitudes for Satellite-based Internet Access

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Abstract — Low Earth orbit satellite systems are capable of providing global Internet access due to their high downlink rate and low link budget. In such systems, wide beam array patterns are used to efficiently cover the required areas. In this paper, two efficient methods based on phase-only element excitation control for designing antenna arrays with required broaden beams are introduced. The first method, which is a simple algebraic approach, uses quadratic phase excitation while the amplitudes are chosen to be trapezoid. In the second method, an optimization algorithm is used to optimize the phase excitations of the array elements, while the amplitudes are still kept as a trapezoidal taper. Moreover, the use of trapezoidal-based amplitude excitations in both presented methods provides many desirable features compared to other conventional tapers. This is mainly due to the unique geometrical shape of the trapezoid taper, where the central coefficients have magnitudes of ones and the sided coefficients have decayed magnitudes. Simulations are presented to validate the proposed methods in which the beam width and maximum level of the radiated field were compared with those obtainable from the conventional standard Woodward-Lawson array.

Keywords — antenna array, pattern synthesis, satellite application, wide beams

1. Introduction

Future global Internet access requires low Earth orbit (LEO) satellite communication systems due to their ability to provide higher downlink capacity and a smaller link budget. In this application, the need for antenna arrays which have wide beam patterns are of a great interest. When the element excitation amplitudes and/or phases of an array are properly chosen, the shape of its radiation pattern can be achieved with required width. Thus, beamforming is an essential process in the antenna arrays of satellite communication systems to achieve a higher downlink capacity that is needed to succeed such systems.

Generally, the beam widths of the array patterns are inversely proportional to the apertures of the antenna array. Consequently, the beam widths become narrow for larger array apertures. Larger satellite arrays are essential to provide greater array directivities and gains that help to achieve higher downlink capacity.

However, the satellite coverage areas decrease as the array aperture increases, and at the same time, widened beams are required to cover specific service areas.

The novelty of this paper is to introduce two new methods to efficiently synthesize widened beams for LEO satellite communication systems. In widened beams satellite applications, the flat-top level of the radiated fields is assumed to be uniform to ensure equal received power density within the coverage areas [1]–[3].

Many techniques have been proposed to synthesize wide beams [4]–[10]. In [11], [12], simple analytical techniques were introduced for the synthetization of widened beams. They are based on the quadratic and random selection of the phase-only element excitation control with fixed uniform amplitudes. However, these methods were not successful enough and their results were not promising when there were significant fluctuations in the obtained beams. Thus, the power density of these methods will not be equally received within the service areas. Moreover, a random selection of phase-only element excitations is not an effective approach, and it is mainly dependent on the trial-and-error process.

In [13]–[16], more powerful techniques based on evolutionary algorithms were used to synthesis widened beams.

In all of these aforementioned techniques, the element excitation amplitudes and phases are optimized jointly or separately to produce the required widened beams. Joint amplitude and phase excitation control methods are the most complicated [17], while separate control of amplitude or phase excitations is more simplified [18]. Phase-only control methods have been found to be more preferable than amplitude-only control [19].

In this paper, two new methods based on phase-only element excitation control are presented. The first method is based on a simple analytical approach where quadratic phase and trapezoid amplitude excitations are used to synthesis the widened beams. In the second method, a genetic optimization algorithm is used to optimize the elements of the phase excitations of the array instead of its quadratic values.

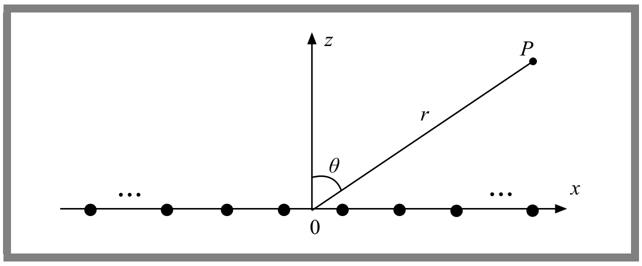

Fig. 1. Linear antenna array with isotropic elements.

Moreover, the use of fixed trapezoid amplitude excitations in the proposed two methods provides many desirable features in the radiation characteristics, such as uniform received power density across the served areas and low sidelobe levels.

2. The Proposed Method

The far-field radiation pattern of the linear antenna array with isotropic elements used can be represented mainly by the array factor. If the elements are placed symmetrically along the x-axis as shown in Fig. 1, then it is broadside array factor on the $x - z$ plane is expressed as follows:

$$AF(\theta, x_n, I_n, \varphi_n) = \sum_{n=1}^{N} I_n \, e^{j \frac{2\pi}{\lambda} x_n \sin\theta + \varphi_n} \,, \tag{1}$$

where λ is wave length, N is the total number of array elements, and θ is the direction of arrival angle from the broadside. From Eq. (1), it is clear that the factor of the array depends on three variable parameters that can be used to control the radiation patterns.

These design parameters are x_n which are the element locations $x_n = [x_1, x_2, \ldots, x_N]^T$, I_n which are the element excitation amplitudes $I_n = [I_1, I_2, \ldots, I_N]^T$, and φ_n which are the element excitation phases $\varphi_n = [\varphi_1, \varphi_2, \ldots, \varphi_N]^T$.

This three-dimensional-variables problem requires an efficient optimization algorithm to optimally determine element locations, amplitude excitations, and phase excitations. Usually, the locations were fixed to avoid iterative changes in the mechanical positions of the elements of the matrix. In this work, the locations were uniformly distributed at multiple integers of $\frac{\lambda}{2}$. Therefore, the elements are separated equally and evenly around the center of the array and Eq. (1) becomes:

$$AF(\theta, x_n, I_n, \varphi_n) = 2 \sum_{n=1}^{\frac{N}{2}} I_n \, e^{j\varphi_n} \cos\left[\frac{2n-1}{2}\pi \sin\theta\right] . \tag{2}$$

The element excitation amplitudes I_n can be chosen according to the newly introduced trapezoidal taper window [20], [21]. The trapezoid taper is unique, and it has two different amplitude excitations. The uniform amplitudes with M elements in the center of the array, and two decayed amplitudes with $N - M$ elements at the array sides. Thus, the I_n can be given by [21]:

$$I_n = \begin{cases} \dfrac{n + \frac{N}{2}}{-\frac{M}{2} + \frac{N}{2}} & -\frac{N}{2} \leqslant n \leqslant -\frac{M}{2} \\ 1 & -\frac{M}{2} \leqslant n \leqslant \frac{M}{2} \\ \dfrac{\frac{N}{2} - n}{\frac{N}{2} - \frac{M}{2}} & \frac{M}{2} \leqslant n \leqslant \frac{N}{2} \end{cases} . \tag{3}$$

From Eq. (2), it is clear that there are only $\frac{N}{2}$ variable excitation phases that must be determined instead of the original three-dimensional variables x_n, I_n, φ_n that were presented in Eq. (1). Furthermore, the $\frac{N}{2}$ variable excitation phases are reduced to only $\frac{N-M}{2}$ when using unit-amplitudes and zero-phasing with M central trapezoidal elements.

In the first proposed method, these $\frac{N}{2}$ variable excitation phases are chosen according to the quadratic distribution, while in the second proposed method, they are taken as the optimization variables. Here in this research work, the peak sidelobe levels (SLL) along with the beam width constraints serve as the optimization objectives.

The objective function can be written as:

$$\begin{aligned} \text{Cost} = & \frac{\max(|AF|)_{\theta \in A}}{\max(|AF|)} \\ & + \max(FNBW - FNBW_D) \\ & + \sum_{i=1}^{I} \frac{\max(|AF(\theta^i_{null}|)}{\max(|AF|)} \,, \end{aligned} \tag{4}$$

where A is the sidelobe area which is located outside of the main beam.

The first term on the right side of the equation is the normalized peak sidelobe level, the second term is the first-null-to-null-beamwidth where $FNBW_D$ is the desired one. The third term is the required null directions toward the interfering signals, where I is the total number of the required null placement.

Then, the optimization problem can be expressed as:

$$\left.\begin{aligned} & \text{find } \varphi_n = [\varphi_1, \varphi_2, \ldots, \varphi_N]^T \\ & \min(cost) \\ \text{subject to } & \frac{-\pi}{2} \leqslant \varphi_n \leqslant \frac{\pi}{2} \;\; \text{for } n = 1, 2, \ldots, \frac{N}{2} \end{aligned}\right\} \tag{5}$$

3. Simulation Results

Consider a linear symmetric antenna array that has $N = 20$ elements with an interelement spacing of $\frac{\lambda}{2}$. In the following simulations, the optimization parameters of the genetic algorithm are chosen referring to [20], [21]. The used trapezoid taper for element excitation amplitudes has $M = 4$ elements with unit-amplitudes and zero-phases at the center, while the remaining phases that need to be determined is $N - M = 16$ elements which they are located at both array sides.

Comparisons are made with other non-optimization methods by using the same example and appropriate parameters setting.

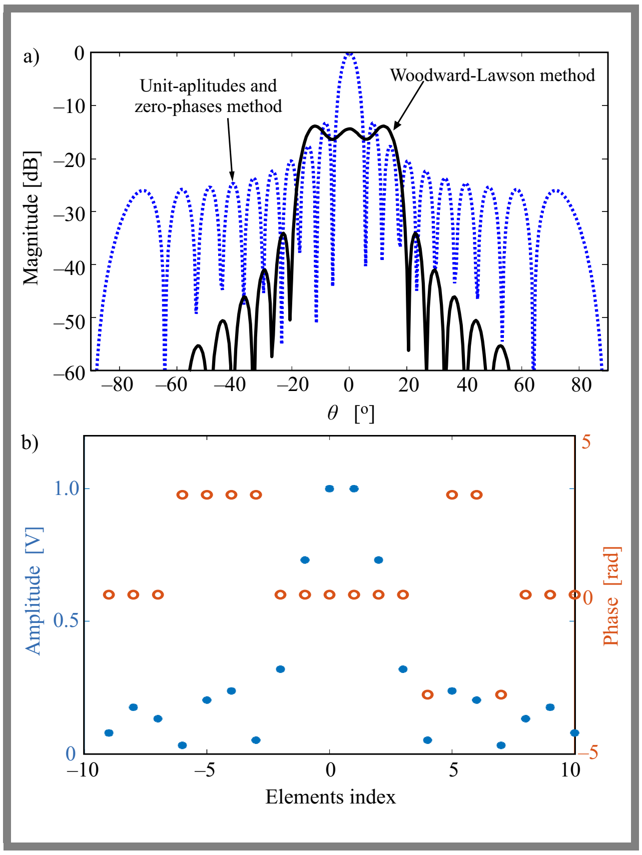

Fig. 2. Beam patterns a) and their corresponding Woodward-Lawson amplitudes and phases b) for $FNBW_D = 40°$.

For the classical unit-amplitude and quadratic-phases method, $I_n = 1$ and

$$\varphi_n = 4\,\varphi_{max}\left(\frac{x_n}{A_L}\right)^2 \text{ for } n = 1, 2, \ldots, \frac{N}{2}.$$

Here, $\varphi_{max} = 3\pi$ which is the maximum allowed phase value at the two end elements and x_n is the location of the n-th element along the array aperture length A_L.

For the trapezoid-amplitude and quadratic-phases method, I_n values are computed according to Eq. (3), $M = 4$, and φ_n values are as mentioned in above. For the standard Woodward-Lawson method, the values of I_n and φ_n are chosen according to Woodward taper [22]. These aforementioned methods were studied and compared under different values of beam widths.

In the first example, the required beam width of the designed linear array is assumed to be equal to $FNBW_D = 40°$. Figures 2–4 show the required amplitudes and phases of the element excitations along with their corresponding beam patterns for the Woodward-Lawson method, unit amplitudes and quadratic-phases method, trapezoid-amplitudes and quadratic phase method, and trapezoid-amplitudes and optimized phase method. From these three figures, it can be seen that the required widened beams have been achieved at the cost of lower directivities in the broadside directions.

The level of the main beam, for the method of unit amplitude and zero phases, was normalized to 0 dB, while the beam patterns of other methods were normalized to the same value. As can be seen in Fig. 4, a minimum drop at $\theta = 0°$ occurs for

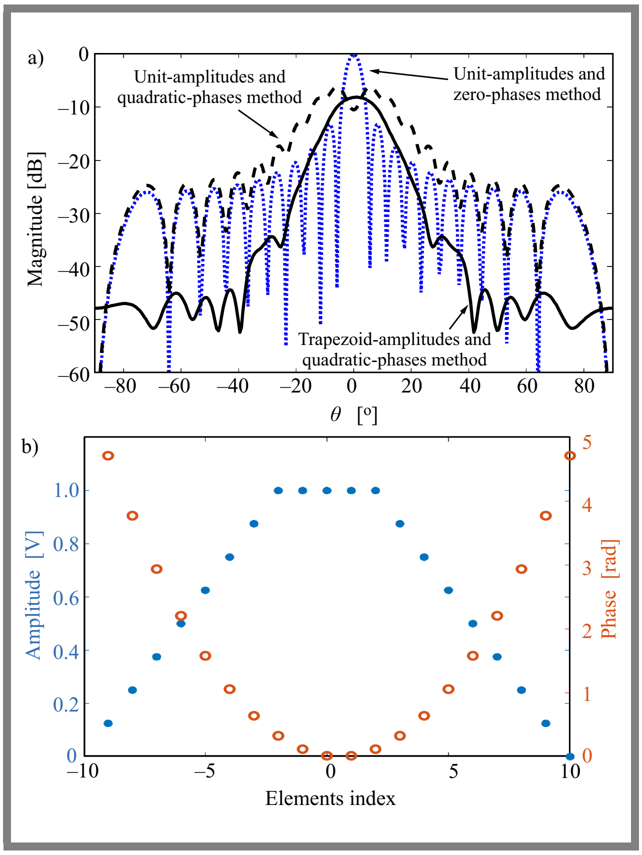

Fig. 3. Beam patterns a) and their corresponding trapezoid amplitudes and quadratic-phases b) for $\varphi_{max} = 3\pi$.

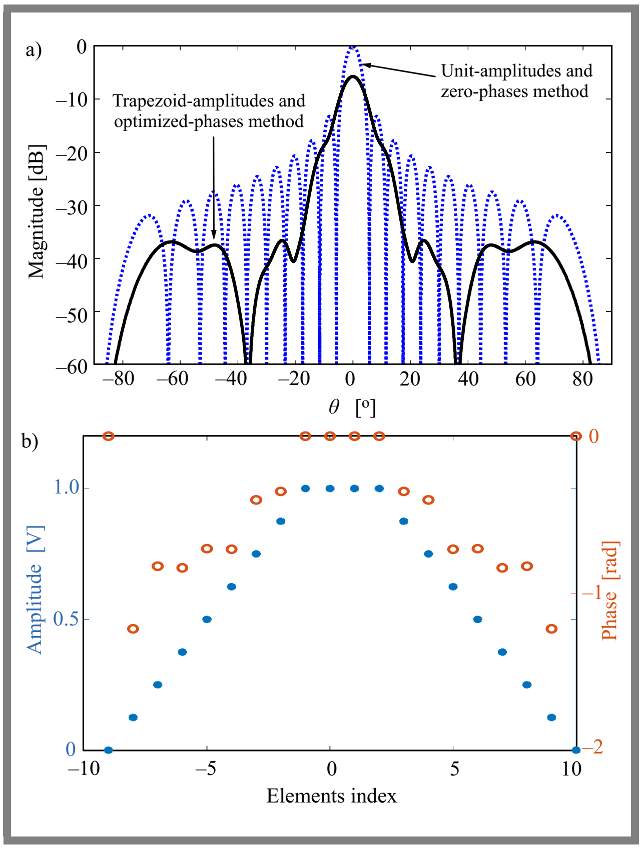

Fig. 4. Beam patterns a) and its corresponding trapezoid amplitudes and optimized phases b) for $FNBW_D = 40°$.

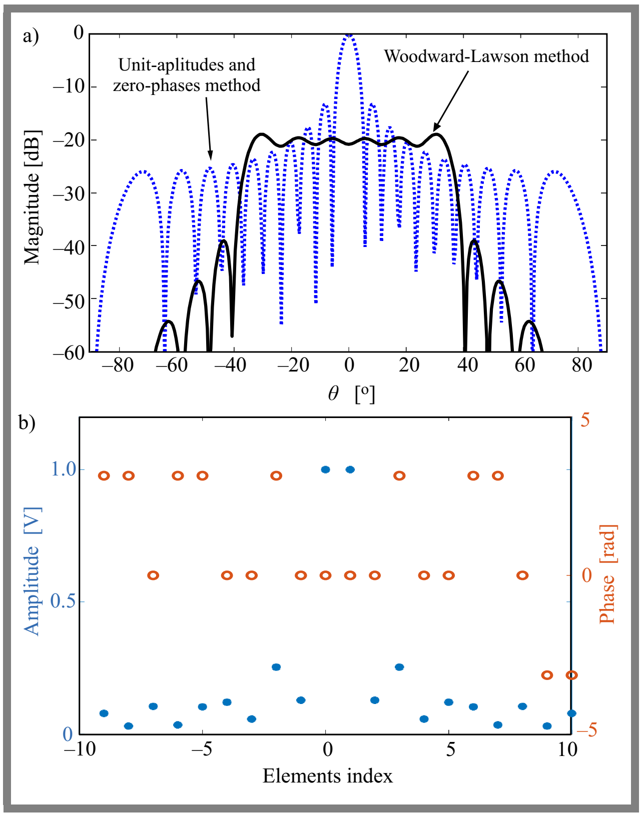

Fig. 5. Beam patterns a) and its corresponding Woodward-Lawson amplitudes and phases b) for $FNBW_D = 80°$.

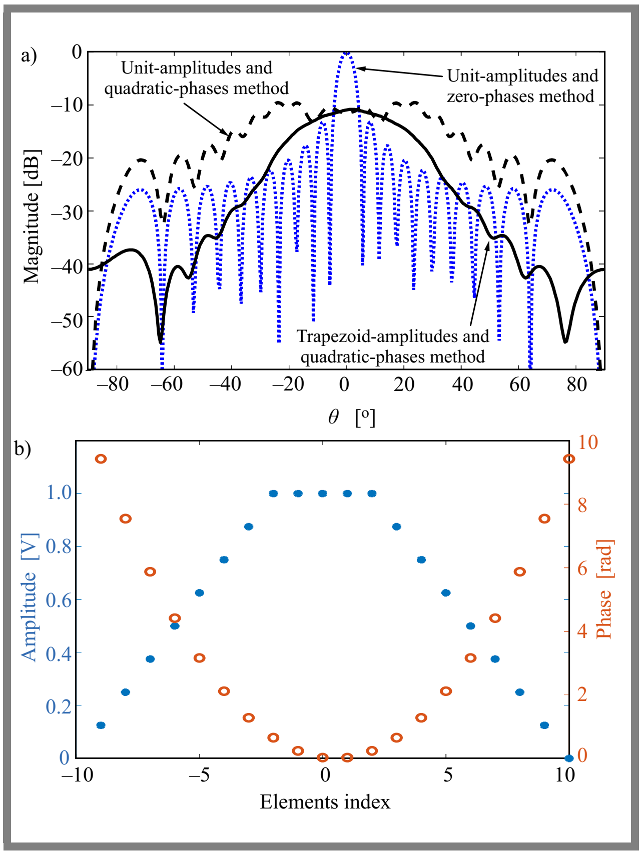

Fig. 6. Beam patterns a) and their corresponding trapezoid amplitudes and quadratic-phases b) for $\varphi_{max} = 6\pi$.

the proposed method of trapezoid amplitudes and optimized-phases. These radiation characteristics were numerically computed and compared in the following example.

In the second example, the performances in terms of array complexity (i.e. needed RF components such as variable attenuators and phase shifters), first-null-to-null beam width (FNBW), directivity, aperture's taper efficiency, and peak sidelobe level (SLL) of these aforementioned methods were compared as shown in Tab. 1.

In the next example, the required first null-to-null beam width is assumed to be equal to $FNBW_D = 80°$ and its results are shown in Figs. 5–7 and Tab. 2.

From Figs. 5–7, it can be seen that the maximum levels of the widened beams further drop as the $FNBW_D$ are increased. The directivities were also significantly reduced with compared to the classic method of unit amplitudes and zero-phases.

However, the proposed methods still provide a lower reduction with compared to that of Woodward-Lawson method. This is evident when comparing the magnitudes of Fig. 7 with that of Fig. 5 at $\theta = 0°$.

Finally, the proposed method of trapezoid amplitudes and optimized phases is extended to include the two-dimensional rectangular planar array instead of its linear counterpart. The results are shown in Fig. 8 for the required null-to-null beam width $FNBW_D = 80°$ and an array size of $N \times N = 20 \times 20$ elements.

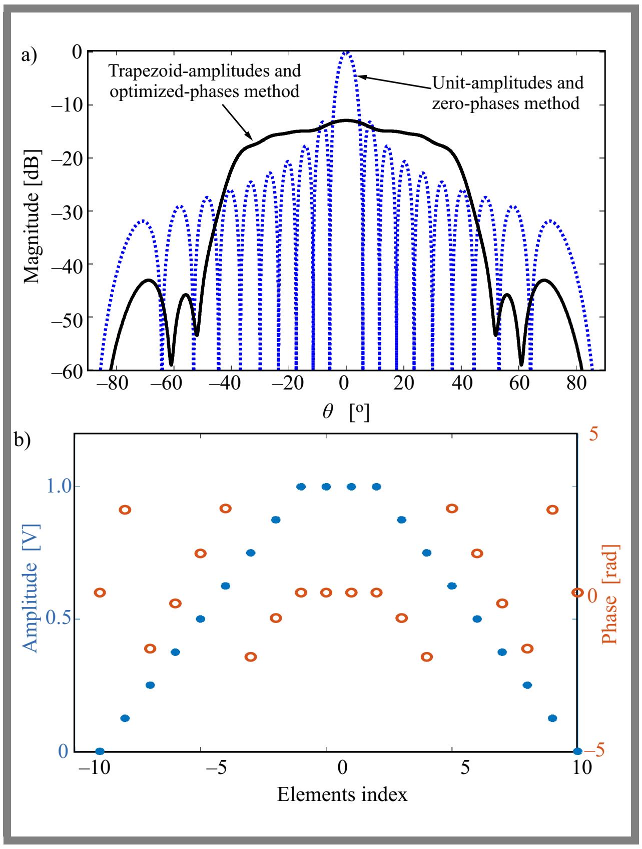

Fig. 7. Beam patterns a) and its corresponding trapezoid amplitudes and optimized phases b) for $FNBW_D = 80°$.

Tab. 1. Performance measures of the methods tested methods for the required $FNBW_D = 40°$.

Method	Feeding network complexity	First null-to-null beam width (FNBW) [°]	Element excitations		Directivi-ty [dB]	Aperture's ta-per efficiency	Peak sidelobe level [dB]
			Amp.	Phase [°]			
Classical unit-amplitudes and zero-phases	Zero transducer and zero phase shifters	The FNBW value is 11.42°. This is narrower than the required one	1 1 1 1 1 1 1 1 1 1	0 0 0 0 0 0 0 0 0 0	26.04	1	–13.2
Woodward-Lawson method [22]	N transducers and N phase shifters	The FNBW value is 40°. This is the same as the required FNBW	0.07 0.17 0.13 0.03 0.20 0.23 0.05 0.31 0.73 1.00	0 0 0 180 180 180 180 0 0 0	13.28	0.23	–34.4
Unit-amplitudes and quadratic-phases method [11]	$N/2$ phase shifters	The FNBW value is 34°. This is narrower than the required one	1 1 1 1 1 1 1 1 1 1	270.00 216.14 168.28 126.39 90.49 60.58 36.64 18.69 6.73 0.74	13.76	0.24	–13.5
Proposed trapezoid-amplitude and quadratic-phases method	$(N-M)/2$ transducers and $N/2$ phase shifters	The obtained FNBW value is 51.4°. This is wider than the required one and mainly depending on the value of φ_{max}	0.12 0.25 0.37 0.50 0.62 0.75 0.87 1.00 1.00 1.00	270.00 216.14 168.28 126.39 90.49 60.58 36.64 18.69 6.73 0.74	16.38	0.33	–34.3
Proposed trapezoid-amplitudes and optimized-phases method	$(N-M)/2$ transducers and $(N-M)/2$ phase shifters	The FNBW value is 40°. This is the same as the required FNBW	0.00 0.12 0.25 0.37 0.50 0.62 0.75 0.87 1.00 1.00	0.00 –70.27 –47.43 –48.02 –41.05 –41.27 –23.26 –20.16 0.00 0.00	18.37	0.64	–36.7

From Fig. 8, it can be seen that the amplitude excitations are exactly as the trapezoid taper, where it has three unit amplitudes on both sides of the array center and then decaying toward the array ends in four array quadrants. While phase excitations are optimized according to the cost function to obtain widen beam that extends from $-40°$ (i.e., corresponds to a value of –0.64) up to $40°$ on both $u-v$ planes. The magnitude of the resultant array pattern is wide enough as required at the cost of little reduction in antenna array directivity.

Tab. 2. Performance measures of the methods tested methods for the required $FNBW_D = 80°$.

Method	Feeding network complexity	First null-to-null beam width (FNBW) [°]	Element excitations		Directivity [dB]	Aperture's taper efficiency	Peak sidelobe level [dB]
			Amp.	Phase [°]			
Classical unit-amplitudes and zero-phases	Zero transducer and zero phase shifters	The FNBW value is 11.42°. This is narrower than the required one	1 1 1 1 1 1 1 1 1 1	0 0 0 0 0 0 0 0 0 0	26.04	1	–13.2
Woodward-Lawson method [22]	N transducers and N phase shifters	The FNBW value is 80°. This is the same as the required FNBW	0.07 0.03 0.10 0.03 0.10 0.12 0.05 0.25 0.12 1.00	180 180 0 180 180 0 0 180 0 0	7.13	0.11	–40.0
Unit-amplitudes and quadratic-phases method [11]	$N/2$ phase shifters	The obtained FNBW value is 57.2°. This is narrower than the required one and mainly depending on the value of φ_{max}	1 1 1 1 1 1 1 1 1 1	540 432 336 252 180 121 73 37 13 1.4	6.97	0.11	–10.0
Proposed trapezoid-amplitude and quadratic-phases method	$(N-M)/2$ transducers and $N/2$ phase shifters	The obtained FNBW value is 75.4°. This is narrower than the required one and mainly depending on the value of φ_{max}	0.125 0.250 0.375 0.500 0.625 0.750 0.875 1.000 1.000 1.000	540 432 336 252 180 121 73 37 13 1.4	10.69	0.17	–30.0
Proposed trapezoid-amplitudes and optimized-phases method	$(N-M)/2$ transducers and $(N-M)/2$ phase shifters	The FNBW value is 80°. This is the same as the required FNBW	0.125 0.250 0.375 0.500 0.625 0.750 0.875 1.000 1.000 1.000	0.00 –23.31 69.75 76.06 –1.35 50.77 112.96 44.02 0.00 0.00	11.63	0.25	–43.0

4. Conclusions

It has been shown that the wide beam patterns with required first null-to-null beam widths and low sidelobe levels can be efficiently generated by controlling the phase-only excitations of the array elements either algebraically by using a simple quadratic phase method or optimally by using a genetic optimization method. In both methods, the amplitudes were constraint as a trapezoidal taper.

Results of using the first proposed method of trapezoid-amplitudes and quadratic-phases showed significant improvements in terms of reducing the sidelobe level, improving the taper's efficiency, and enhancing the array directivity compared to other conventional methods. For the case of

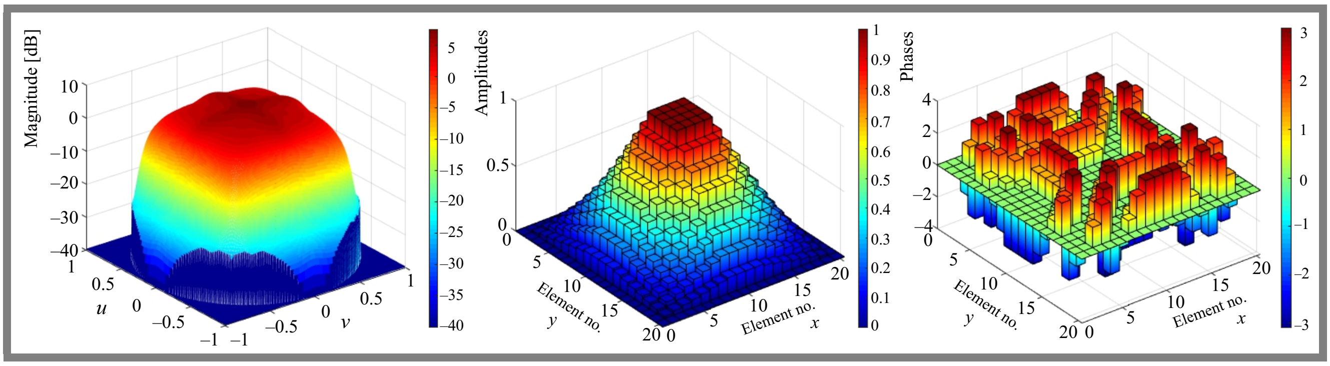

Fig. 8. Beam pattern and its corresponding trapezoid amplitudes and optimized phases for planar array with $FNBW_D = 80°$.

generating beam width with $FNBW = 40°$, the SLL improvement is more than –20 dB, directivity improvement is more than 2.6 dB, while for the case of $FNBW = 80°$, these improvements were –24 dB and 4 dB respectively.

The results of using the second proposed method of trapezoid amplitudes and optimized phases showed significant improvements in the radiation characteristics. Thus, the two proposed methods are the way of using the widened beams in LEO satellite systems to successfully provide global Internet access applications.

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