Paper

Modeling of the inverse base width modulation effect in HBT transistor with graded SiGe base

Agnieszka Zaręba, Lidia Łukasiak, and Andrzej Jakubowski

Abstract—A model of the position of the edge of emitter-base junction in the base and collector current pre-exponential ideality factor in HBT transistor with a SiGe base is presented. The model is valid for transistors with nonuniform profiles of doping and Ge content. The importance of taking into account the dependence of the effective density of states in SiGe on local Ge content and that of electron diffusion coefficient in SiGe on drift field for modeling accuracy is studied.

Keywords— heterojunction bipolar transistor, SiGe, base width modulation.

1. Introduction

SiGe heterojunction bipolar transistors (HBT) are widely used in wireless and high-speed digital communications due to their many advantages over Si bipolar junction transistors (BJTs): higher current gain (β), cut-off frequency (f_T) and early voltage (V_A). SiGe HBTs are a low-cost alternative to GaAs technology. Moreover, they offer great flexibility in design of germanium content profile in the base.

In SiGe HBTs band gap grading gives rise to a drift field, which aids the minority carrier transport through the base. However, in transistors with steep Ge grading in the base, collector current (J_C) is more affected (in comparison with BJT) by the so-called "inverse base width modulation effect", e.g. [1]. This effect is connected with the collector current dependence on the position of the edge of emitter-base junction space-charge region (SCR) in the base (x_0), which varies with emitter-base voltage (V_{BE}) changes. In the HBT case, this position determines not only the width of electrically neutral base but also the germanium content at x_0. Shrinking of SCR in a forward-biased emitter-base junction increases the effective base width, which in turn lowers J_C. In addition, Ge content at x_0 decreases and so does the difference between emitter and base band gaps. This results in lower injection of minority carriers and lower values of J_C (i.e., β). A collector current pre-exponential ideality factor m is defined as [1]:

$$m = \frac{\mathrm{k}T}{q}\,\frac{1}{J_C}\,\frac{\mathrm{d}J_C}{\mathrm{d}V_{BE}} = 1 - \delta m < 1\,. \tag{1}$$

The inverse base width modulation effect in SiGe-base HBTs has been calculated numerically (e.g., [2]). For transistors with exponential doping profile in the base, an analytical model of m has been presented in [1]. In this model two important effects have been neglected: the dependence of the diffusion coefficient ($D_{n\mathrm{SiGe}}$) on the drift field (F) and the dependence of the effective density of states in SiGe ($N_{V\mathrm{SiGe}}$, $N_{C\mathrm{SiGe}}$) on local Ge content in the base (y_{Ge}). Moreover, this model needs a complicated procedure for determining the position of x_0 involving numerical simulations.

A model of x_0 for AlGaAs HBT with graded base has been derived in [3], but it is only valid for constant base doping. This model is not quite appropriate for SiGe-based HBT case, because of the differences in the energy band diagrams of those two transistor types. Moreover, it assumes constant effective density of states throughout the whole transistor, which is not true for Si/Si$_{1-x}$Ge$_x$/Si structure.

The aim of this paper is to present a new extensive model of x_0 and m. To determine x_0 the Poisson equation is examined with mobile charges in SCR taken into account. Moreover, nonuniform doping and Ge profiles in the base are considered for the first time.

Our model of m incorporates not only high-doping effects and the dependence of band gap and $D_{n\mathrm{SiGe}}$ on local y_{Ge} in the base (similarly to [1]), but also the dependence of the effective density of states in SiGe on $y_{\mathrm{Ge}}(x)$ and the dependence of $D_{n\mathrm{SiGe}}$ on the drift field in the base. Moreover, it is valid for any doping and germanium profiles in the base of HBT.

2. Model

In this paper we focus on n-p-n SiGe-based HBT properties, but the treatment in this section holds for all heterojunctions with nonuniform composition. The only assumption is constant electron affinity ($\chi(x) = \mathrm{const}$) along the whole structure, which is a good approximation for a Si/Si$_{1-x}$Ge$_x$ structure. Moreover, the presented models easily extended to include the dependence of χ on material composition (see, e.g., [3]).

In further considerations material parameters are defined as follows. Intrinsic carrier concentration in the SiGe base is given as [4]:

$$\begin{aligned} n^2_{i\mathrm{SiGe}}(x) &= \gamma(x) n^2_{i0\mathrm{Si}} \exp\left(\frac{\Delta E_{GEFF}(x)}{\mathrm{k}T}\right) \\ &= \gamma(x) n^2_{i0\mathrm{Si}} \exp\left(\frac{\Delta E_{G\mathrm{Ge}}(x) + \Delta E_{GAPP}(x)}{\mathrm{k}T}\right), \end{aligned} \tag{2}$$

where: $n_{i0\mathrm{Si}}$ – intrinsic carrier concentration in pure silicon, ΔE_{GEFF} – the effective band gap narrowing in the base due to the presence of Ge ($\Delta E_{G\mathrm{Ge}}$) and due to heavy doping effects (ΔE_{GAPP}). It is assumed that band gap depends

linearly on y_{Ge} (7.5 meV per 1% of Ge, e.g., [4]). The model of Klaassen-Slotboom-de Graaff [5] was chosen to describe ΔE_{GAPP}. The ratio of the effective density of states in a SiGe base to that in a silicon one is defined in the following way as a function of Ge content (for $y_{Ge} \geq 0.01$) [4]:

$$\gamma(x) = \frac{N_{C\mathrm{SiGe}}(x) N_{V\mathrm{SiGe}}(x)}{N_{C\mathrm{Si}} N_{V\mathrm{Si}}} = \exp\left(-\sqrt{5 y_{\mathrm{Ge}}(x)}\right). \quad (3)$$

We also assume that $N_{C\mathrm{SiGe}} = 2/3 N_{C\mathrm{Si}}$ [6].

The model of intrinsic carrier concentration in Si ($n_{i\mathrm{Si}}$) adopted here takes into account the apparent band gap narrowing due to high doping concentration (ΔE_{GAPP}).

2.1. Position of the edge of emitter-base junction space-charge region in the base (x_0)

The energy band diagram of emitter and base regions of a HBT with exponential doping profile ($N_A(x)$) and linearly graded Ge content ($y_{\mathrm{Ge}}(x)$) in the base and constant doping in the emitter region (N_D) is presented in Fig. 1.

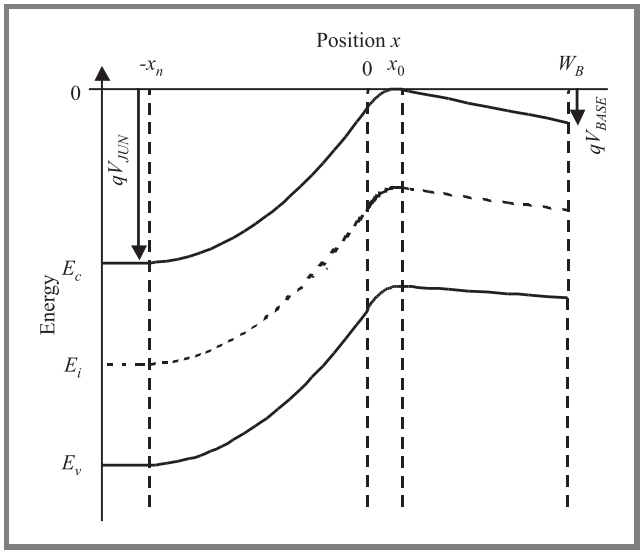

Fig. 1. Energy band diagram of emitter and base of a HBT with exponential doping profile and linearly graded Ge content in the base and constant doping in the emitter region.

As usual, $E_C(x) = -qV(x)$, $E_V(x) = -qV(x) - E_G(x)$. Moreover, we assume that $qV(x_0) = 0$. The built-in potential of the junction is

$$V_{JUN}(x_0) = V(-x_n) - V(x_0) = V_{BJ} - V_{BASE}(x_0), \quad (4)$$

were V_{JB} is the potential drop across the emitter-base junction and the base:

$$V_{JB} = V(-x_n) - V(W_B) = \frac{\mathrm{k}T}{q} \ln\left(\frac{N_A(W_B) N_E}{n_{i\mathrm{Si}}^2}\right) + \frac{E_G(W_B) - E_G(-x_n)}{q} + \frac{\mathrm{k}T}{q} \ln\left(\frac{N_V(-x_n)}{N_V(W_B)}\right) - V_{BE} \quad (5)$$

and V_{BASE} is the potential drop across the base:

$$V_{BASE}(x_0) = V(W_B) - V(x_0) = V(W_B) - 0 = \frac{\mathrm{k}T}{q} \ln\left(\frac{N_A(x_0)}{N_A(W_B)}\right) + \frac{E_G(x_0) - E_G(W_B)}{q} + \frac{\mathrm{k}T}{q} \ln\left(\frac{N_V(W_B)}{N_V(x_0)}\right). \quad (6)$$

For low injection level, it may be assumed that the concentration of majority carrier at SCR boundaries is $p(x_0) \approx N_A(x_0)$ and $n(x_n) \approx N_D(x_n)$. Therefore, the carrier concentration in the base part of SCR $(0 \leq x \leq x_0)$ may be approximated as:

$$p_p(x) = N_A(x_0) \frac{N_V(x)}{N_V(x_p)} \exp\left(\frac{E_G(x_0) - E_G(x) - qV(x)}{\mathrm{k}T}\right), \quad (7)$$

$$n_p(x) = \frac{n_{i\mathrm{SiGe}}^2(x_0)}{N_A(x_0)} \exp\left(\frac{qV(x)}{\mathrm{k}T}\right) \quad (8)$$

and in the emitter part of SCR $(-x_n \leq x \leq 0)$:

$$n_n(x) = N_D \exp\left(\frac{q\big(V(x) - V_{JUN}(x_0)\big)}{\mathrm{k}T}\right), \quad (9)$$

$$p_n(x) = \frac{n_{i\mathrm{Si}}^2}{N_D} \exp\left(\frac{q\big(V_{JUN}(x_0) - V(x)\big)}{\mathrm{k}T}\right). \quad (10)$$

Considering mobile charges in the junction SCR, the standard procedure (e.g., [3]) is applied to solve the Poisson equation for the base and the emitter SCR. As a result the following set of equations is obtained:

$$F(0^+)^2 - F(x_0)^2 = \frac{2\mathrm{k}T}{\varepsilon_{\mathrm{SiGe}}} \left\{ N_A(0) \frac{qV(0)}{\mathrm{k}T} + \big(p_p(0) - N_A(x_0)\big) + \big(n_p(0) - n_p(x_0)\big) \right\}, \quad (11)$$

$$F(0^-)^2 = \frac{2\mathrm{k}T}{\varepsilon_{\mathrm{Si}}} \left\{ -N_D \frac{q\big(V(0) - V_{JUN}(x_0)\big)}{\mathrm{k}T} + \big(p_n(0) - p_n(-x_n)\big) + \big(n_n(0) - N_D\big) \right\}. \quad (12)$$

To relate the electric field to SCR boundaries, we employ the depletion approximation and integrate the Poisson equation once to obtain equations $F(0^-) = f(N_D - x_n)$ and $F(0^+) - F(x_0) = f(N_A(x),\ x_0)$.

Combining the sets of equations described above with the condition that the electric flux density must be continuous, the value of x_0 may be calculated numerically using the Newton method (for more details see, e.g., [3]).

In our model the dependence of the dielectric constant in the base ($\varepsilon_{\mathrm{SiGe}}$) on $y_{\mathrm{Ge}}(x)$ is taken from [7]. Assuming, however, that x_0 is small and $\varepsilon_{\mathrm{SiGe}}$ changes only slightly between $x = 0$ and $x = x_0$, in our calculations we assume that in this region $\varepsilon_{\mathrm{SiGe}} = \mathrm{const} = 0.5\big[\varepsilon_{\mathrm{SiGe}}(0) + \varepsilon_{\mathrm{SiGe}}(x_0)\big]$.

2.2. Collector current pre-exponential ideality factor (m)

The collector current density of a SiGe HBT with arbitrary Ge and doping profiles may be expressed as [8]:

$$J_C = \frac{q\exp\left(\frac{qV_{BE}}{kT}\right)}{\int\limits_{x_0}^{W_B}\frac{N_A(x)}{n_{i\mathrm{SiGe}}^2(x)D_{n\mathrm{SiGe}}(x)}\mathrm{d}x+\frac{N_A(W_B)}{n_{i\mathrm{SiGe}}^2(W_B)v_{SAT}}} = J_{C0}\exp\left(\frac{qV_{BE}}{kT}\right), \tag{13}$$

where: v_{SAT} – saturation velocity of electrons.

The model of $D_{n\mathrm{SiGe}}$ in the SiGe base used in our analysis is described as $D_{n\mathrm{SiGe0}} = D_{nrel}D_{n\mathrm{Si0}}$, where $D_{n\mathrm{SiGe0}}$ is electron diffusion coefficient in SiGe for low drift fields and D_{nrel} is parameter dependent on Ge content [7]. The impurity-concentration-dependent diffusion coefficient $D_{n\mathrm{Si0}}$ in silicon is taken from [9]. The $D_{n\mathrm{SiGe}}$ dependence on the drift field F in SiGe is also defined similarly to the case of silicon [9].

Substituting Eq. (13) into Eq. (1) one obtains:

$$m = \frac{kT}{q}\frac{1}{J_C}\frac{\mathrm{d}J_C}{\mathrm{d}V_{BE}} = 1-\frac{kT}{q}\frac{N_A(x_0)}{n_{i\mathrm{SiGe}}^2(x_0)D_{n\mathrm{SiGe}}(x_0)}\frac{J_{C0}}{q}\frac{\partial x_0}{\partial V_{BE}}. \tag{14}$$

3. Results and discussion

As it was mentioned above, SiGe-based HBTs have many advantages over Si BJTs. In general, Ge gradients in the base are necessary to improve transistor speed, while HBTs with $y_{\mathrm{Ge}}(0) > 0$ have higher current gains. Optimization of $y_{\mathrm{Ge}}(x)$ is beyond the scope of this paper, but we examine some cases of interest from the point of view of emitter-base junction properties.

In the present study, we consider transistors with emitter doping concentration $N_D = 10^{18}$ cm^{-3} and with exponential doping profile in the base: $N_A(0) = 10^{19}$ cm^{-3}, $N_A(W_B) = 5\cdot 10^{17}$ cm^{-3}, $W_B = 30$ nm. Linearly graded Ge profiles with different $y_{\mathrm{Ge}}(0)$ and $y_{\mathrm{Ge}}(W_B)$ (meaning also different gradients) are considered.

First of all, we examine the model of x_0. This parameter was calculated for $V_{BE} = 0.3$ V in three ways:

- assuming depletion approximation in SCR;
- considering mobile charges in SCR and assuming that the effective densities of states in SiGe are the same as those in Si ($\gamma(x) = 1$);
- considering mobile charges in SCR and taking into account the dependence of the effective density of states in SiGe on local Ge content $\gamma(x) = f\left(y_{\mathrm{Ge}}(x)\right)$.

The results are shown in Fig. 2. The position of x_0 moves deeper into the base with the increase of the built-in electric field in the base (i.e., increase of Ge gradient), while W_B is moving towards the collector. Of course, the depletion approximation results in overestimation of x_0 and this overestimation increases with increasing V_{BE}.

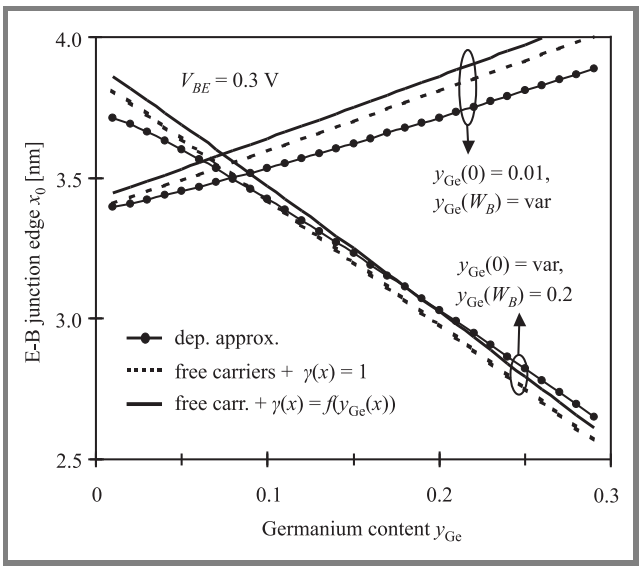

Fig. 2. The position of emitter-base junction space-charge region in the base calculated for different Ge content profiles.

In the case where $\gamma(x) = f\left(y_{\mathrm{Ge}}(x)\right)$, the calculated built-in electric field in the base is lower than that calculated assuming $\gamma(x) = 1$. Therefore x_0 moves towards the emitter (see "$y(W_B) = $ var" in Fig. 2).

When the value of x_0 is known it is possible to obtain the collector current pre-exponential ideality factor m. Using our full model we calculate m for 7 transistors with different $y_{\mathrm{Ge}}(x)$ linear profiles in the base (Fig. 3). As expected,

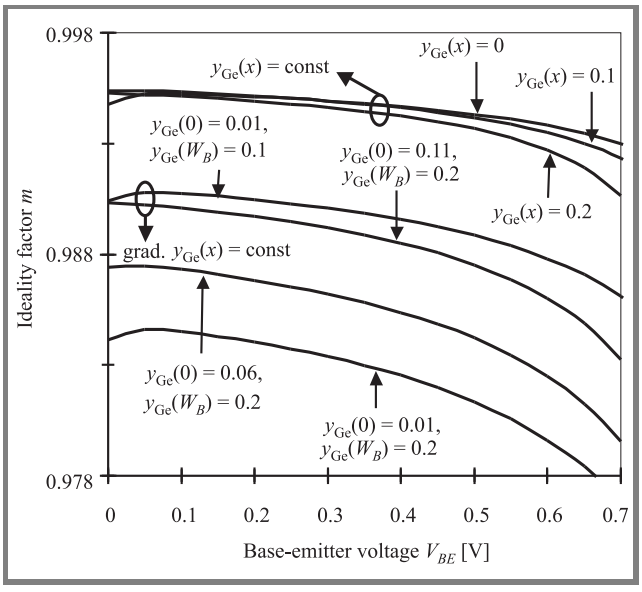

Fig. 3. The collector current pre-exponential ideality factor calculated as a function of emitter-base voltage for different Ge content profiles.

the increase of Ge gradient lowers m, i.e., collector current becomes much more dependent on V_{BE}.

The importance of including the dependence of $D_{n\mathrm{SiGe}}$ on F and that of $\gamma(x)$ on $y_{\mathrm{Ge}}(x)$ in J_C model of SiGe-based HBT has been demonstrated in [4]. The influence of these two effects on the collector current pre-exponential ideality factor m is discussed below.

To illustrate the influence of $D_{n\mathrm{SiGe}}$ reduction due to the drift field, the parameter m was calculated in two ways: with $D_{n\mathrm{SiGe}}$ either dependent or independent of the field. This yields two sets of δm values calculated from Eq. (1) – δm_{DF} and δm_D, respectively. To make further analysis more transparent we define the accuracy of δm calculations as:

$$\Delta\delta m_D = \frac{\delta m_{DF} - \delta m_D}{\delta m_{DF}}\,[\%]\,. \tag{15}$$

This value is plotted in Fig. 4 as a function of V_{BE} for 5 transistors. As seen, it is important to incorporate the investigated effects for transistors with high Ge gradients in the base. Surprisingly, for two transistors with the same Ge gradient but different $y_{\mathrm{Ge}}(x)$ content ("$y_{\mathrm{Ge}}(0) = 0.01$, $y_{\mathrm{Ge}}(W_B) = 0.1$" and "$y_{\mathrm{Ge}}(0) = 0.11$, $y_{\mathrm{Ge}}(W_B) = 0.2$") the calculated $\Delta\delta m_D$ values differ strongly. This is associated with the dependence of $D_{n\mathrm{SiGe}}$ on $y_{\mathrm{Ge}}(x)$ (for more information see [7]).

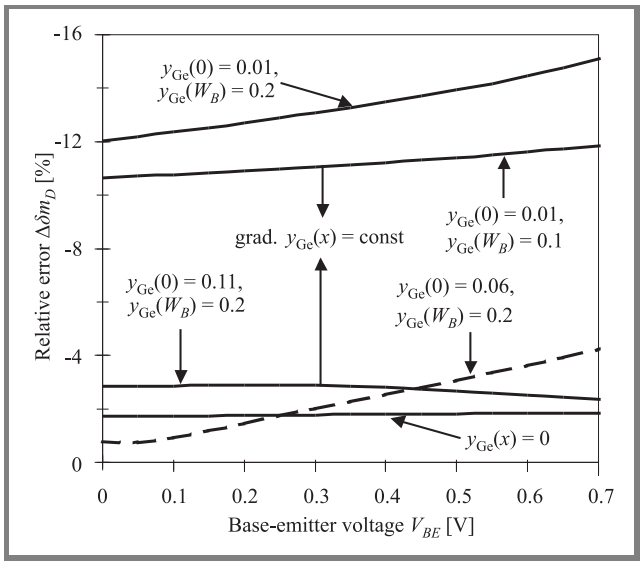

Fig. 4. Accuracy of modeling of the collector current pre-exponential ideality factor assuming that $D_{n\mathrm{SiGe}}$ is independent of the drift field for different Ge content.

The importance of including the dependence $\gamma(x) = f\big(y_{\mathrm{Ge}}(x)\big)$ in the model of m was studied in a similar way. Again, the calculation yields two sets of δm: first $\delta m_\gamma\big(\gamma(x) = f(y_{\mathrm{Ge}}(x)\big)$ and second $\delta m_1\big(\gamma(x) = 1\big)$. The results are plotted as a function of V_{BE} in Fig. 5. The highest error is obtained for transistors with high total germanium

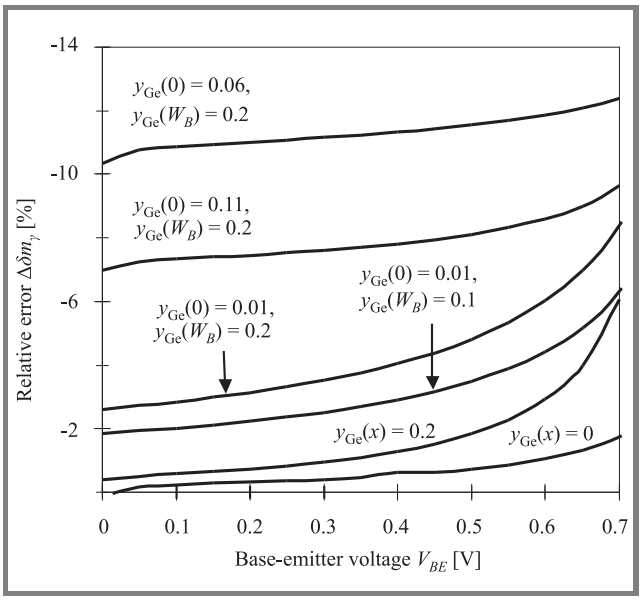

Fig. 5. Accuracy of modeling of the collector current pre-exponential ideality factor assuming $\gamma(x) = 1$ for different Ge content profiles.

content in the base ("$y_{\mathrm{Ge}}(0) = 0.06$, $y_{\mathrm{Ge}}(W_B) = 0.2$" and "$y_{\mathrm{Ge}}(0) = 0.11$, $y_{\mathrm{Ge}}(W_B) = 0.2$").

4. Conclusions

In this paper a new model of the position of the edge of emitter-base junction space-charge region in the base and the collector current pre-exponential ideality factor in SiGe-base HBTs was presented. This model is valid for any doping and germanium content profiles. It includes, for the first time, the dependence of the effective density of states in SiGe base on local Ge content and the dependence of the diffusion coefficient on the drift field in the base.

Both investigated parameters turned out to be sensitive to appropriate modeling of the dependence of the effective density of states in SiGe base on local Ge content.

It was found that in the case of modern HBTs with high built-in fields in the SiGe base collector current ideality factor should be modeled taking into account the dependence of diffusion coefficient on the drift field and on the local Ge content in the base.

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Lidia Łukasiak – for biography, see this issue, p. 65.

Andrzej Jakubowski – for biography, see this issue, p. 7.