Paper

Designing WDM networks by a variable neighborhood search

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Abstract— With the ever-rising data volume that is demanded by the market, network planning in order to minimize the necessary investment while meeting the demands is constantly an important task for the network providers. Synchronous digital hierarchy (SDH) and wavelength division multiplex (WDM) form the core of many current backbone networks. In order to solve the provisioning and routing problem in such WDM networks, we develop a variable neighborhood search (VNS) metaheuristic. VNS is a metaheuristic that combines series of random and improving local searches based on systematically changed neighborhoods. An integer flow formulation is modeled in AMPL and solved by CPLEX in order to obtain optimal solutions as a reference for the heuristic.

Keywords— network design, WDM, SDH, variable neighborhood search.

1. Introduction

With the ever-rising data volume that is demanded by the market, network planning in order to minimize the necessary investment while meeting customer demands is constantly an important task for the network providers. Synchronous digital hierarchy (SDH) and wavelength division multiplex (WDM) form the core of many current backbone networks. When we speak of SDH, this also applies to its American counterpart synchronous optical network (SONET). Though there are some differences, this does not affect network planning and optimization in general. Many of these networks, especially in Europe, have a general mesh topology. Thus, we do not consider special ring topologies but an arbitrary mesh of fiber lines (links) that connects the locations of the network provider (nodes) where the actual traffic demands arise. WDM systems are only used point-to-point, as in most current commercial networks. Therefore, the routing and wavelength assignment problem (RWA) does not arise. Common line-speeds for SDH/WDM networks range from 622 Mbit/s up to 40 Gbit/s per channel. Thus, with state-of-the-art multiplexers that provide 80 or even 160 channels, WDM is currently the fastest commercially available transmission technology for long-range networks. A good overview of the SDH and WDM technology can be found in [6].

While SDH requires a dedicated pair of fibers for each transmission, WDM multiplexes several optical signals on a single pair of fibers. The costs for several discrete fibers for SDH compared to the multiplexer costs and only a single fiber pair for WDM make WDM suitable for longer distances and high bandwidth demands while discrete SDH lines are cost effective for short lines with limited bandwidth demand. However, the overall optimization problem includes additional equipment like cross-connects, port-cards, amplifiers and regenerators and is much more versatile. Apart from the equipment, also building costs for the installation of new fiber lines might be included. The resulting optimization problem is to find a cost-minimal combination of the equipment and the routing for a given static demand matrix (i.e., traffic forecast). Static demand is a valid approximation for such kind of networks because most of the demands are leased lines over a certain period. Even if they carry originally bursty Internet traffic it is so highly aggregated that an IP provider can lease a fixed capacity from the operator of the long-haul network. The general topology of the network is bounded by the available links, but of course not all of these links have to be used.

In this paper, we present a variable neighborhood search (VNS) metaheuristic and an integer programming formulation as a reference for the described network-planning problem. Section 2 explains the general problem and the integer model, Section 3 contains the detailed description of the VNS. Computational results for various problem instances are given in Section 4; Section 5 summarizes the results and provides an outlook on future research.

2. Problem description and integer model

The optimization problem at hand deals with a set of demands to be routed through an optical network. Associated with each demand is an origin node, a destination node and a size expressed in 2.5 Gbit/s units. Each demand can be routed either entirely on one or more discrete fiber pairs or over one or more channels of a WDM system. The demands can be switched from one system to another at the intermediate locations through digital cross-connects (DXC). Optical fibers joining pairs of nodes are used to carry the demands through the network. These connections from one node to another are called links or edges. The costs of an edge depend on the length, the required bandwidth and the transmission technology. SDH requires a pair of fibers and maybe additional amplifiers or regenerators for long ranges. WDM links are basically composed of a pair of multiplexer terminals, the fiber pair and possibly also amplifiers. Depending on the number of channels that are actually used, transponder pairs are needed. The number of transponders can be any number between one and the maximum capacity of the multiplexer. In real WDM systems, the transponders are sometimes added in blocks, e.g., ten at a time, but this detail has been omitted in the model. It is important that the costs of the fiber and of the amplification do not depend on the number of channels that are actually used, they are always per pair of terminals; this is a huge advantage of WDM for long-haul transmissions. An SDH line and a single channel of a WDM system each alike occupy one port in the DXC and thus each need one port-card at both ends. The goal of the network planner is to minimize the total cost of the additional fibers and the SDH/WDM equipment.

The basic model is derived from [7]. It considers demands of a specific granularity, e.g., 2.5 Gbit/s, and the entire demand between a pair of nodes has to be routed on the same path. All cross-connects have the same number of ports; if a higher capacity is required, several cross-connects can be stacked. The given infrastructure is composed of a set of nodes N that represents the switching locations of the provider. A set of undirected edges E connecting these nodes represents the fiber links. A set E' containing all arcs of a complete graph with node set N is derived from E. Furthermore, a set of demands D is given which contains the number of units for each single demand d_{st} from node s to node t.

The cost input consists of the following data:

C_e^{FS}	costs of an SDH line on edge e (fiber, amplifiers, regenerators)
C_e^{FW}	costs of a WDM line on edge e (fiber, amplifiers)
C^W	costs of the WDM multiplexer terminals on edge e
C^O	cost of a basic DXC system
C^C	cost of a WDM channel (a pair of transponders)
C^P	cost of a DXC port (port-card)

The capacity of the systems is defined as follows:

M^W	capacity of a WDM system (number of wavelengths)
M^O	capacity of a DXC (number of ports)

Decision variables:

f_e	number of SDH systems on edge e
w_e	number of WDM systems on edge e
v_e	number of channels used in the WDM systems on edge e
y_n	number of DXCs used in node n
z_{ij}^{st}	1 if demand (s,t) is routed along edge (i,j); 0 otherwise

Objective value:

Minimize $\sum_{e\in E}\Big((C_e^{FS} + 2C^P)f_e + (C_e^{FW} + C^W)w_e + (C^C + 2C^P)v_e\Big) + \sum_{n\in N} C^O y_n$

s.t.:

$$\sum_{j\in N} z_{ji}^{st} - \sum_{j\in N} z_{ij}^{st} = \left\{\begin{array}{rl} -1 & i=s \\ 0 & \forall i\neq s,t \\ +1 & i=t \end{array}\right\} \quad \forall (s,t)\in D, \quad (1)$$

$$\sum_{(s,t)\in D} d_{st}(z_{ij}^{st} + z_{ji}^{st}) \leq v_e + f_e \quad \forall e\in E \text{ with } i \text{ and } j \text{ adjacent to } e, \quad (2)$$

$$v_e \leq M^W w_e \quad \forall e\in E, \quad (3)$$

$$\sum_{e \text{ adjacent to } n} (v_e + f_e) \leq M^O y_n \quad \forall n\in N, \quad (4)$$

$$\sum_{(i,j)\in E'} (z_{ij}^{st} + z_{ji}^{st}) \leq H \quad \forall (s,t)\in D, \quad (5)$$

$$y_n \geq 0 \text{ and integer} \quad \forall n\in N, \quad (6)$$

$$f_e, w_e, v_e \geq 0 \text{ and integer} \quad \forall e\in E, \quad (7)$$

$$z_{ij}^{st}\in\{0,1\} \quad \forall (i,j)\in E', (s,t)\in D. \quad (8)$$

Constraints (1) guarantee the flow-conservation. The origin and the destination nodes of each demand both have one adjacent edge that is used; all other nodes have either none or two. Constraints (2) ensure that the demands that each edge carries are less or equal than the total capacity of the installed SDH and WDM systems. Constraints (3) match the number of available WDM channels with the maximum capacity of the installed WDM multiplexers. Constraints (4) adjust the capacity of the cross connects of each node to the capacity of its adjacent edges. Constraints (5) impose a hop limit for each demand. With sufficiently high values for H (between 8 and 15, depending on the size of the network), this set of constraints speeds up the computation times without sacrificing solution quality. Constraints (6) to (8) ensure the integrality and non-negativity of the decision variables. A more detailed description of a closely related model and many possible extensions are discussed in [4].

3. Variable neighborhood search

Variable neighborhood search [1–3] is a recent metaheuristic for solving combinatorial and global optimization problems based on a simple principle: systematic changes of neighborhoods within the search. Its development has been rapid, with a lot of papers already published and its applications have been numerous and successful. Many extensions have been made, mainly to be able to solve large problem instances. However, the main idea behind variable neighborhood search is to keep the simplicity of the basic scheme.

Let $\mathcal{N}_k$, $(k = 1, \ldots, k_{\max})$ be a finite set of neighborhood structures, and $\mathcal{N}_k(s)$ the set of solutions in the kth neighborhood of a solution s. Neighborhoods $\mathcal{N}_k$ may be induced from metric functions introduced into a solution space S. If $d(.,.)$ is this distance, then take increasing values d_k, $k = 1, ..., k_{\max}$ and set $N_k(s) = \{s' \in S : d(s, s') \leq d_k\}$.

Usually, a series of nested neighborhoods is obtained from a single neighborhood by taking $\mathcal{N}_1(s) = \mathcal{N}(s)$ and $\mathcal{N}_{k+1}(s) = \mathcal{N}(\mathcal{N}_k(s))$, for every solution s. This means that a move to the kth neighborhood is performed by repeating k times a move into the original neighborhood. A solution $s' \in S$ is a *local minimum* with respect to $\mathcal{N}_k$ if there is no solution $s \in \mathcal{N}_k(s') \subseteq S$ better than s' (i.e., such that $f(s) < f(s')$, where f is the objective function of the problem).

The variable neighborhood search metaheuristic is based on three simple facts:

1) a local minimum with respect to one neighborhood structure is not necessarily a local minimum with respect to another;

2) a global minimum is a local minimum with respect to all possible neighborhood structures;

3) local minima with respect to one or several neighborhood structures are usually similar to each other.

By applying the VNS principles to an improving local search we get the variable neighborhood descent (VND). The method consists of changing the neighborhoods systematically within a local search. The basic VND is presented in Fig. 1.

The final solution should be a local minimum with respect to all $k_{\max}$ neighborhoods, and thus the probability of reaching the global minimum is higher than by using a single structure. Beside this *sequential* order of neighborhood structures in VND, one can develop a *random* strategy by choosing the successive values for k at random.

Most local search heuristics use a single or sometimes two neighborhoods ($k_{\max} \leq 2$) in their descents. A usual strategy with two neighborhoods consists of performing local searches for the first neighborhood from points s' that belong to the second neighborhood of the current solution (i.e., $s' \in \mathcal{N}_2(s)$). The perturbation strategy consists of applying a local search using the first neighborhood and then perturbing the current solution by choosing a random solution in the second neighborhood to perform a new local search. The basic variable neighborhood search (BVNS) method uses deterministic changes in the neighborhood structure for perturbation or shaking. Its steps are given in Fig. 2.

The stopping condition may be the maximum CPU time allowed, the maximum number of iterations, or the maximum number of iterations between two improvements. The reduced variable neighborhood search (RVNS) method is obtained if random points are selected from $\mathcal{N}_k(s)$, without being followed by descent. It is useful for very large instances for which local search is costly.

The local search Step 2b in the basic VNS may be replaced by VND. This gives the general variable neighborhood search (GVNS) that is the version with the most recent success. Its steps are shown in Fig. 3.

Initialization.

Select the set of neighborhood structures $\mathcal{N}_k$, for $k = 1, \ldots, k_{\max}$, that will be used in the descent.

Find an initial solution s.

Iterations.

Repeat the following sequence until no more improvement is obtained:

(1) Set $k \leftarrow 1$.

(2) Repeat the following steps until $k = k_{\max}$:

(a) *Exploration of neighborhood.* Find the best neighbor s' of s ($s' \in \mathcal{N}_k(s)$).

(b) *Move or not*. If the solution thus obtained s' is better than s, set $s \leftarrow s'$ and $k \leftarrow 1$; otherwise, set $k \leftarrow k + 1$.

Fig. 1. Variable neighborhood descent.

Initialization.

Select the set of neighborhood structures $\mathcal{N}_k$, for $k = 1, \ldots, k_{\max}$.

Find an initial solution s.

Choose a stopping condition.

Iterations.

Repeat the following sequence until the stopping condition is met:

(1) Set $k \leftarrow 1$.

(2) Repeat the following steps until $k = k_{\max}$:

(a) *Shaking*. Generate a point s' at random from the kth neighborhood of s ($s' \in \mathcal{N}_k(s)$).

(b) *Local search*. Apply some local search method with s' as initial solution; denote the so obtained local optimum with s''.

(c) *Move or not*. If this local optimum is better than the incumbent, move there ($s \leftarrow s''$), and continue the search with $\mathcal{N}_1$ ($k \leftarrow 1$); otherwise, set $k \leftarrow k + 1$.

Fig. 2. Basic variable neighborhood search.

Initialization.

Select the set of neighborhood structures $\mathcal{N}_k$, for $k = 1, \ldots, k_{\max}$ for the shaking.

Select the set of neighborhood structures $\mathcal{N}'_j$, for $j = 1, \ldots, j_{\max}$ for the descent.

Find an initial solution s.

Choose a stopping condition.

Iterations.

Repeat the following sequence until the stopping condition is met:

(1) Set $k \leftarrow 1$.

(2) Repeat the following steps until $k = k_{\max}$:

(a) *Shaking.* Generate a point s' at random from the kth shaking neighborhood of s ($s' \in \mathcal{N}_k(s)$).

(b) *Descent.* Apply to s' the VND with $\mathcal{N}'_j, j = 1, \ldots, j_{\max}$ as neighborhoods to get a new solution s''.

(c) *Move or not.* If $f(s'') < f(s)$ set $s \leftarrow s''$ and $k \leftarrow 1$; otherwise, set $k \leftarrow k + 1$.

Fig. 3. General variable neighborhood search.

Several extensions of the VNS have also been proposed. The basic VNS is a first improvement descent method with randomization. It is transformed into a descent-ascent method if Step 2c sets also $s \leftarrow s''$ with some probability even if the solution is worse than the incumbent (or the best solution found so far). It is changed into a best improvement method by making a move to the best neighborhood k^* among all $k_{\max}$ of them.

3.1. Application to the design of WDM networks

In order to solve the problem of designing WDM networks, we propose a basic variable neighborhood search metaheuristic.

The mixed integer programming (MIP) formulation described above considers any path between an origin and a destination of a demand and does not limit the number of intermediate nodes. Based on that it may be used to find optimal solutions or at least to provide lower bounds of the problem. On the other hand, the variable neighborhood search developed in this work restricts the number of paths available to route each demand through the network to five reaching an upper bound of the problem.

The construction of a solution starts with the selection of a path for each demand requirement. Once each demand is assigned to a path, the cost of the resulting design is calculated. The cost is associated with the equipment that is required to satisfy the demands using the chosen paths. A solution is fully determined by a data structure that stores the path assignments and the equipment required in each element of the original network.

- *Initialization.* In order to perform the initialization step of the BVNS metaheuristic, the procedure that generates the initial solution and the neighborhood structure must be defined.
  - *Initial solution.* The initial solution is constructed by using the constructive procedure proposed in [8] that attempts to assign demands to paths in order to efficiently utilize the spare capacity in the original base network. The rationale behind this initialization is that spare capacity for channels in the final network design should be zero except for channels on WDM systems covering a segment without slack. The strategy acknowledges that spare capacity in the original network simply accounts for existing network infrastructure.
  - *Initial solution.* The kth neighborhood of the solution s, $\mathcal{N}_k(s)$, consists of all the solutions that can be reached from s by changing the paths assigned to k different demands.
- *Shaking.* This procedure generates a solution S' at random from the kth neighborhood of s ($s' \in \mathcal{N}_k(s)$).
- *Improvement method and move decision.* The improvement method is the local search procedure proposed in [8]. In order to run the improvement method from a given solution, the demands are ordered according to their unit cost. The demand ordering is important because the improvement method, which is based on changing one demand from its current path to another, starts with the demand that has the largest unit cost. The first candidate move is then to reassign the demand that is at the top of the unit cost list. If reassigning this demand leads to an improving move, the move is executed to change the current solution. If an improving move that involves reassigning the first demand in the list cannot be found, then the second demand is considered. The process continues until a demand is found for which a reassignment of paths leads to an improving move. If all the demands are examined and no improving move is found, the local search is abandoned.

4. Computational results

The (mixed) integer flow formulation is modeled in a modeling language for mathematical programming (AMPL) and solved by CPLEX in order to obtain optimal solutions as a reference for the heuristic. The flow formulation does not limit the number of paths for each demand; it searches the entire solution space thus providing lower bounds of the problem. For the larger problem instances, where CPLEX

Table 1
Overview of the computational results

Set	|N|	|E|	|D|	VNS cost	VNS time [s]	CPLEX cost	CPLEX time [s]	CPLEX bound	% deviation of VNS from CPLEX
Extant0D	12	17	15	3.69	0.01	3.69	0.4	opt	0
			21	6.21	0.46	6.21	2	opt	0
			44	14.68	0.05	14.36	11	opt	2.17
			66	13.20	0.05	11.83	22	opt	10.37
	12	33	15	3.69	0.09	3.69	7	opt	0
			21	7.51	1.32	6.03	67	opt	19.70
			44	14.45	0.05	13.66	4228	opt	5.46
			66	15.48	0.58	11.76	9024	opt	24.03
		46	15	3.69	0.02	3.69	12	opt	0
			21	7.99	0.01	6.03	165	opt	24.53
			44	14.79	0.04	13.16	6148	opt	11.02
			66	13.03	1.49	11.77	11194	opt	9.66
Example2D	17	26	27	23.85	1.33	22.47	3	opt	5.78
			36	81.84	8.32	81.84	31	opt	0
			81	97.47	36.62	96.65	470	opt	0.84
			135	173.53	97.19	170.15	29255	opt	1.94
		68	27	24.95	0.06	19.27	110	opt	22.76
			36	64.72	10.54	63.75	28841	opt	1.49
			81	80.75	30.01	76.66	216806	75.21	5.06
			135	145.07	53.96	138.40	81098	130.98	4.59
NationalD	50	63	45	38.14	2.61	34.07	854	opt	10.67
			65	50.91	18.72	48.65	77493	45.56	4.43
			91	60.11	58.08	55.64	95755	53.19	7.43
			112	42.88	0.27	42.88	39544	38.67	0

is not able to find optimal solutions, it can at least provide good bounds. The total provisioning costs provided by the VNS when solving instances whose number of nodes range from 12 up to 50 were compared with the results provided by CPLEX.

All the experiments were carried out on a Pentium4 with 2.4 GHz and the CPLEX calculations were performed with CPLEX version 8.1. The size of the branch and bound tree was limited to 400 MB, which was also the termination criterion for the computation. Table 1 shows a comparison of the objective values and runtimes for the heuristic and the CPLEX calculations. The bound is either the optimal solution of a linear relaxation or a CPLEX internal bound taken from the node log. The CPLEX runtimes include just the final integer run. In most cases, a linear relaxation has been computed beforehand and served as a starting solution for the integer calculation. The computation times of the relaxation were usually far below the times of the integer model.

For the smaller problem instances the computations were simply performed with the universal AMPL model. For the larger networks, some custom modifications to cut off suboptimal parts of the solution space had to be used in order to achieve better results. This includes upper bounds on certain variables, e.g., the number of DXCs in a node. Also an a priori prohibition for SDH on certain very long links where link costs compared to WDM are prohibitive even for a single channel could be applied on some problem instances.

The computational results reported in Table 1 corroborate that the variable neighborhood search metaheuristic is able to obtain solutions which are within a 10% of the lower bounds provided by CPLEX for many of the instances. The VNS results are promising since the solutions for the bigger instances are reached in a few seconds while CPLEX takes minutes and even an hour for one of the problems. Anyway, additional work to remove the number of paths constraint in the variable neighborhood search is required.

5. Summary and conclusions

In this paper, we have addressed the important and current problem in the telecommunications industry of designing WDM networks. We have described a node-arc formulation as the framework to develop a variable neighborhood search procedure and as a means for finding lower and upper bounds.

Our experiments with real and randomly generated data show the value of our proposed solution procedure when compared to the bounds generated by solving the MIP formulation with CPLEX. Our experiments show that small instances are better solved to optimality by way of a commercial MIP solver. However, for bigger instances the metaheuristic procedure is able to reach upper bounds which are within 10% of the lower bounds given by CPLEX in a few seconds. With regard to the usually much larger uncertainty in the demand forecast of the network operator, 10% is already a practical gap for real world applications.

Although the variable neighborhood search reaches acceptable solutions by restricting the number of paths available for the demands, an extension of our work will include solving the problem without limiting the number of paths. The rationale behind this is to carry out a real comparative analysis with the lower bounds obtained solving the MIP described in this paper. Other possible extensions are 1+1 protected demands and problem instances with existing spare capacities (capacity expansion problem). A comparison with other metaheuristics applied to similar problems, like the greedy randomized search procedure used in [5], on the basis of the same problem instances, is also planned for the future.

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