Regular paper

A newly developed random walk model for PCS network

Imdadul Islam and Siddique Hossain

Abstract— Different types of random walk models are prevalent in mobile cellular network for analysis of roaming and handover, being considered as important parameters of traffic measurement and location updating of such network. This paper proposes a new random walk model of hexagonal cell cluster, exclusively developed by the authors and a comparison is made with two existing models. The proposed model shows better performance in context of number of probability states compared to existing models.

Keywords— cell cluster, random walk, subarea n, state transition, probability matrix, expected number of steps.

1. Introduction

In a mobile cellular network, the service area is represented as an array of hexagonal cells in a continuous fashion. In any cell two types of offered traffic take place: one is new call arrival and the other is handover arrival. The latter one solely depends on mobility of users hence mobility is an important parameter for measurement of quality of service (QoS) of a network. In teletraffic engineering mobility is measured as probability $P_{i,j}$, i.e., probability of an mobile station (MS) to make transition from cell i to cell j. User's mobility is random and usually does not follow any particular probability distribution function, but could be analyzed based on random walk model summarized in [1, 2] where model in [2] shows better performance than that of [1] in context of number of states. This paper proposes a new random walk model considering that any user makes transition from its current cell to any neighboring cells with equal probability, i.e., 1/6 like existing model but our aim is to reduce number of states of state transition diagram hence gives less process time in detection of state of an MS. In Section 2 previous two models are depicted in a nutshell and referred to as "model 1" and "model 2", but the proposed model is summarized in detail and designated as "proposed model".

2. Methodology

Any MS can make transition from its present cell to any one of surrounding cells with equal probability of 1/6 for hexagonal cell structure shown in Fig. 1. Each cell in a mobile cellular network has its own identification number hence in random walk model each cell has to be designated by a number based on certain criteria. Two dimensional cell identification technique is used in both previous and proposed model based on [1–3]. Probability of transition of mobile stations from one cell to another is depicted by both state transition and probability matrix. Finally, the expected number of steps to make transition from each cell to most peripheral cell is detected to get idea of mobility at a glimpse.

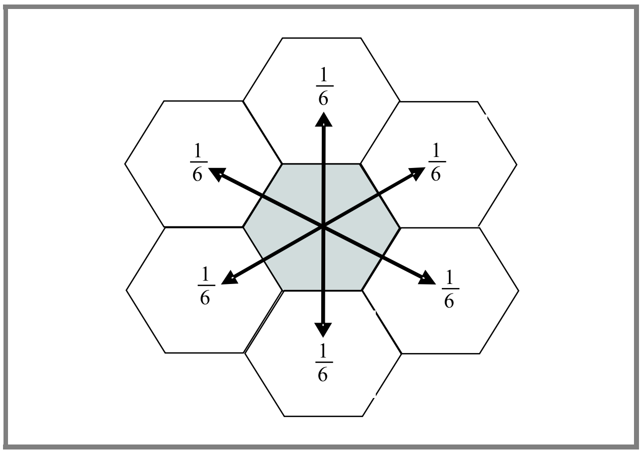

Fig. 1. State transition of hexagonal cell cluster/pattern.

2.1. Model 1

In this model [1] the entire cell pattern is divided into different subareas with respect to the cell at the center of the cluster labeled (0, 0) and this cell at the centre called subarea 0. All the cells surrounding (0, 0) are marked as (1, 0) and designated as subarea 1. All the cells surrounded by subarea 1 is called subarea 2 and cells are marked as (2, 0) and (2, 1) in an alternate fashion. Similarly cells of subarea 3 are marked as (3, 0), (3, 1) and (3, 2) and so on, is given in Fig. 2. In recursive form the cells surrounded by subarea x are called subarea $x+1$. This type of two dimensional model was first proposed in [3] and modified by the same author in [1]. In this model, the cell cluster/pattern is symmetrical in six wedges marked in alternate shade of white and dark. Cells in a single wedge suffice for analysis since they are symmetrical.
Here the number of distinguished cells increases by one with each increment of level, hence number of states for n subarea cell pattern would be

$$1+2+3+4+\ldots+(n-1)+n = n(n+1)/2. \quad (1)$$

This model is summarized in [1] hence state transition diagram and probability matrix is avoided since authors are

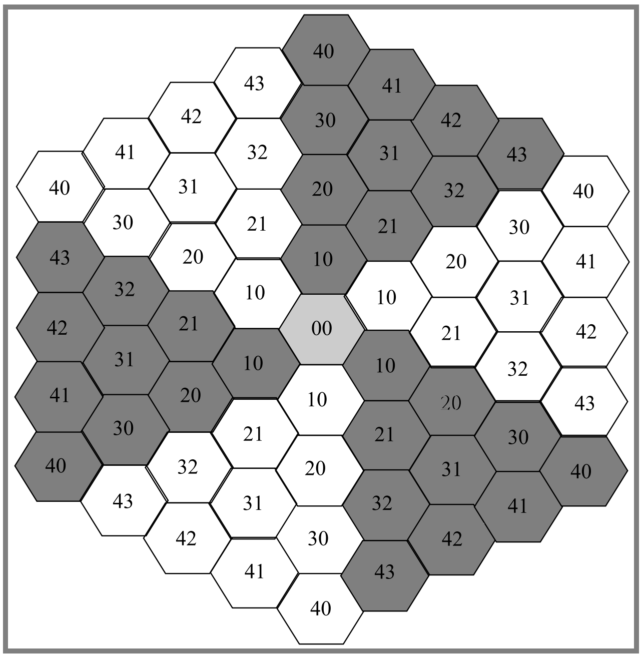

Fig. 2. Cell cluster of 4 subareas defined in model 1.

only interested in the proposed model developed by themselves in details.

2.2. Model 2

Advanced form of previous model is proposed in [2], where marking of cells is a little bit different than that of [1] given in Fig. 3. Like in model 1, the cell at the center of the cluster is called subarea 0 and the cells surrounded by that cell are called subarea 1, and are marked as (1, 0). Cells

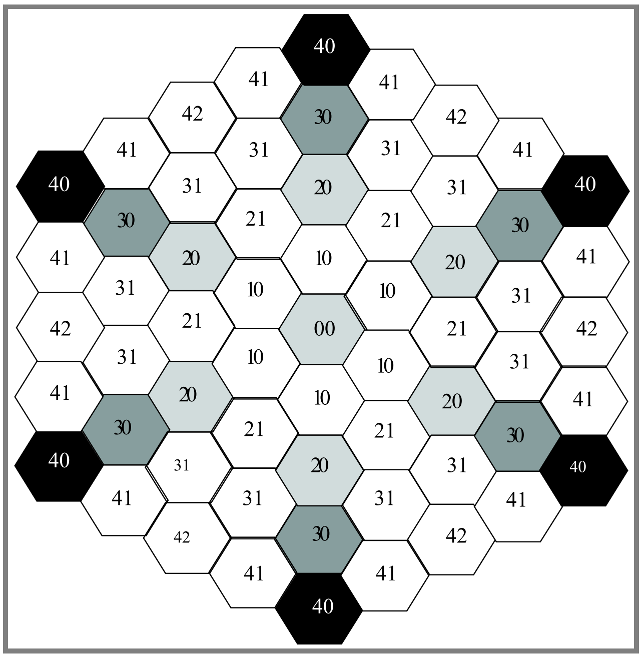

Fig. 3. Cell cluster of 4 subareas defined in model 2.

of subarea 2 are marked like [1] but that of subarea 3 are marked as (3, 0), (3, 1) and (3, 1). Subarea 4 is marked as (4, 0), (4, 1), (4, 2) and (4, 1) and so on. This model also shows the same symmetrical characteristics like the previous one. Here number of states for a cell cluster of n subarea is evaluated as

$$1+(1+1)+(2+2)+(3+3)+\ldots+(k+k)$$
$$=\frac{(n^2+2n+4)}{4}; \quad \text{where } n=2k, \text{ i.e., } n \text{ is even,} \quad (2)$$

$$1+(1+1)+(2+2)+(3+3)+\ldots+(k+k)+k+1$$
$$=\frac{(n^2+2n+5)}{4}; \quad \text{where } n=2k+1, \text{ i.e., } n \text{ is odd.} \quad (3)$$

State transition diagram and probability matrix of this model are also excluded for the same reason as mentioned in previous section.

2.3. Proposed model

This is the model proposed by authors where the number of states is reduced compared to [1, 2] at the expense of complexity of determination of probability of transition from one state to another. Here the cell at the center is marked as (0, 0) and called subarea 0, cells at subarea 1 are marked alternately (1, 0) and (1, 1), cells at subarea 2

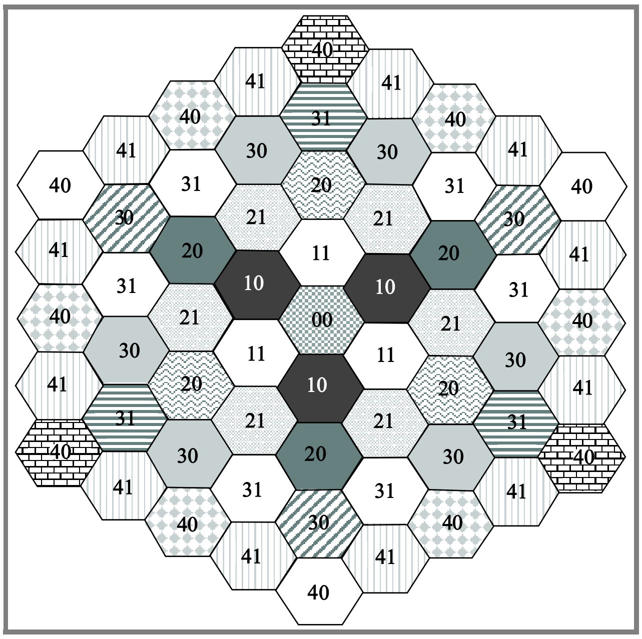

Fig. 4. Cell cluster of 4 subareas defined in the proposed model.

are alternately marked as (2, 0), (2, 1), that of subarea 3 as (3, 0) and (3, 1) and so on. The symmetric cells are marked with same brightness as shown in Fig. 4 but its symmetrical characteristics are different from both of previous two. Here number of states for a cell cluster of n subarea is

$$1+\big(2+2+2+\ldots+(n-1)\text{th } term\big)+1$$
$$=2\big(1+1+1+\ldots+n\text{th } term\big)=2n. \quad (4)$$

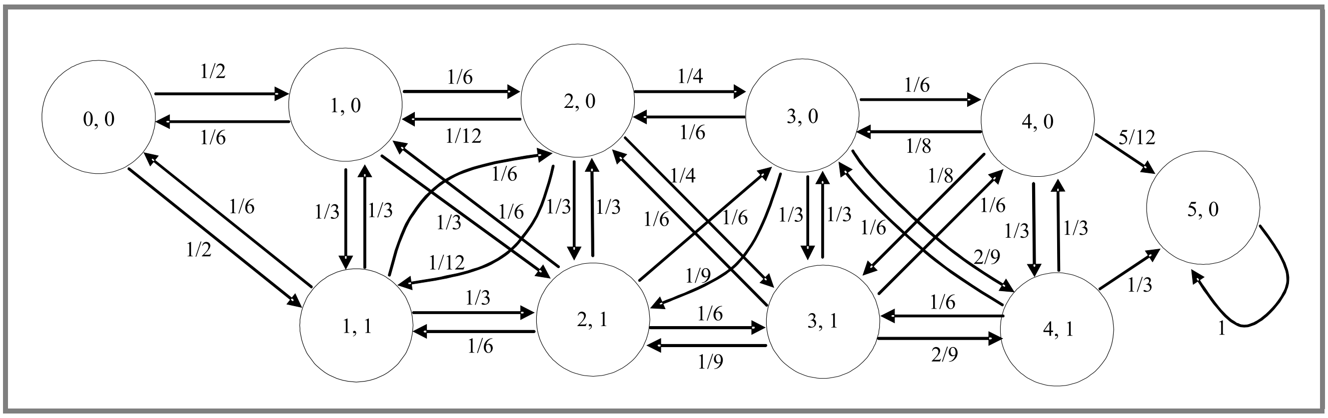

Fig. 5. State transition for 4 subareas of proposed model.

Figure 5 shows the state transition diagram for a cell pattern/cluster of 4 subareas, which resembles a finite stochastic process. Probability of transition from any arbitrary state (p, q) to $(p \pm 1, q \pm 1)$ doesn't remain constant in generalized form like [1, 2]. Probability of transition between very few states is constant, but in most of the cases probability is evaluated based on symmetrical relation among subarea i, subarea $(i+1)$ and number of cells s_i or $s_{(i+1)}$ of subarea i or subarea $(i+1)$. Probability of transition between any two states of cell cluster of n subarea is derived as

1. $P_{00,1i} = \frac{1}{2}$ for $i = 0$ and 1.
2. $P_{i0,(i+1)0} = \begin{cases} \frac{1}{6} & \text{for } 1 \le i \le n-1, \quad i \text{ is odd} \\ \frac{1}{4} & \text{for } 2 \le i \le n-2, \quad i \text{ is even.} \end{cases}$
3. $P_{i0,(i+1)0} = \begin{cases} \frac{1}{6} & \text{for } 1 \le i \le n-1, \text{ and } i \text{ is odd} \\ \left[0\times3+\left(\frac{s_i}{2}-3\right)\frac{1}{6}\right]\frac{2}{S_i} & \text{for } 2 \le i \le n\text{–}2, \text{ and } i \text{ is even.} \end{cases}$
4. $P_{i0,(i+1)1} = \left[3 \times \frac{1}{3} + \frac{1}{6}\left(\frac{s_i}{2} - 3\right)\right]\frac{2}{s_i}$ for $1 \le i \le n-2$.
5. $P_{i0,i1} = \frac{1}{3} = P_{i1,i0}$ for $1 \le i \le n-1$.
6. $P_{i1,(i-1)0} = \frac{1}{6}$ for $1 \le i \le n-1$.
7. $P_{i1,(i+1)1} = \frac{1}{6}$ and $P_{(i+1)1,i1} = \left[0 \times 3 + \frac{1}{6}\left(\frac{s_{i+1}}{2} - 3\right)\right]\frac{2}{s_{i+1}}$ for $1 \le i \le n-1$.
8. $P_{i1,(i+1)0} = \frac{1}{6}$ and $P_{(i+1)0,i1} = \left[0 \times 3 + \frac{1}{6}\left(\frac{s_{i+1}}{2} - 3\right)\right]\frac{2}{s_{i+1}}$ for $1 \le i \le n-1$.
9. $P_{(n-1)0,n0} = \frac{5}{12}$ and $P_{(n-1)1,n0} = \frac{1}{3}$.
10. $P_{n0} = 1$.

Probability matrix P of state transition in generalized form is like

$$P = \begin{bmatrix} P_{00,00} & P_{00,10} & P_{00,20} & \dots & P_{00,n0} \\ P_{10,00} & P_{10,10} & P_{10,11} & \dots & P_{10,n0} \\ P_{11,00} & P_{11,10} & P_{11,11} & \dots & P_{11,n0} \\ \dots\dots & \dots\dots & \dots\dots & \dots & \dots\dots \\ \dots\dots & \dots\dots & \dots\dots & \dots & \dots\dots \\ P_{n-11,00} & P_{n-11,10} & P_{n-11,11} & \dots & P_{n-11,n0} \\ P_{n0,00} & P_{n0,10} & P_{n0,11} & \dots & P_{n0,n0} \end{bmatrix}. \quad (5)$$

Element of the matrix, $P_{ij,pq}$ is the probability of transition between cell (i, j) to cell (p, q). Probability transition matrix of the proposed model of 4-subarea cell cluster is derived as

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{12} & \frac{1}{12} & 0 & \frac{1}{3} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{9} & 0 & \frac{1}{3} & \frac{1}{6} & \frac{2}{9} & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{9} & \frac{1}{3} & 0 & \frac{1}{6} & \frac{2}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{3} & \frac{5}{12} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6)$$

Now a random walk moves from state (a, b) to (a', b') with s steps summarized in [1, 2] as

$$P_{sab,a'b'} = \begin{cases} P_{ab,a'b'} & \text{for} \quad s = 1 \\ P^{(s)}_{ab,a'b'} - P^{(s-1)}_{ab,a'b'} & \text{for} \quad s > 1\,. \end{cases} \quad (7)$$

Where $P^{(s)}$ is evaluated by recurrence formulae

$$P^{(s)} = \begin{cases} P & \text{for} \quad s = 1 \\ P \times P^{(s-1)} & \text{for} \quad s > 1 \,. \end{cases} \tag{8}$$

Expected number of steps for a MS to leave subarea n is evaluated as

$$N(a,b) = \sum_{s=1}^{\alpha} s \times P_{sab,n0} \,, \tag{9}$$

where $N(a,b)$ is equivalent to $L(x,y)$ of [2]. Taking maximum value of $s = 300$, the values of $N(a,b)$ are derived from different starting states based on [4, 5] shown in Table 1.

Table 1
The values of $N(a,b)$ derived from different starting states

$N(a,b)$	$P_{00,50}$	$P_{10,50}$	$P_{11,50}$	$P_{20,50}$	$P_{21,50}$	$P_{30,50}$	$P_{31,50}$	$P_{40,50}$	$P_{41,50}$
$N(a,b)$	19.5	18.5	18.5	15.64	16.44	12.16	12.16	6.44	7.2

It is obvious that only $N(1,0) = N(1,1)$ and $N(3,0) = N(3,1)$, i.e., very few states show symmetry compare to [1, 2].

3. Conclusion

In model 1 [1] the number of states for n-subarea cell cluster is $n(n+1)/2$, greater than that of model proposed in [2]. The model 2 [2] of n-subarea cell pattern has $(n^2+2n+4)/4$ states when n is even and $(n^2+2n+5)/4$ states when n is odd. It is obvious that $n(n+1)/2 > (n^2+2n+4)/4$ or $(n^2+2n+5)/4$, i.e., model 2 is better than model 1 in context of number of states. In proposed model, the number of states is $2n < (n^2+2n+4)/4$ or $(n^2+2n+5)/4$ for $n > 5$. Therefore the proposed model yields smaller number of states for a network of 6 subareas or more, i.e., performance of our model is better than that of [1, 2] for a large mobile cellular network. Any network planner can use our model quite comfortably since the process time to estimate any probability of state of any MS would be smaller in comparison to any one of the existing models.

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