Paper

Modelling and simulation of combat operations in SimCombCalculator application

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Abstract—The basis of the article is mathematical discrete problem connected with local military battle. The problem consists of two dependent problems. One of them shows how to find our troops allocation to enemy's troops. The battle damage assessment function for the problem is proposed. The second problem is connected with finding global strategy of attack. Local battle is described by modified Lanchester model. Two level logistic model is built for describing materials and munitions resupply management processes. This idea was used in computer simulator SimCombCalculator. The paper shows how to use this simulator for finding attack strategy from the platoon to brigade level.

Keywords— battlefield model, decision support, mathematical modelling, computer simulation.

1. Introduction

This article extends the previous research on mathematical models of military battlefield [1, 2, 4–6] and proposition of logistic support during the battle [3]. Many up-to-date mathematical and simulation models of military battlefield are dedicated to operation and strategy levels of decision processes. The idea of building battlefield simulator for lower levels of military structures is interesting from the practical point of view. Mathematical problems formulated for finding strategy of attack are usually complicated. Methods for solving these problems are complicated as well. They need high computer speed and memory. For lower levels of decision processes on the battlefield we have not so complicated problems. It comes from number of troops, not so large terrain, rather poor logistic processes, etc. Implication from these assumptions is chance to design computer simulator for commanders which will be used and perform on for example palmtop.

In the paper the problem of mathematical designing the attack against enemy in the local battle is shortly described. In order to prepare the best plan of attack in a short time there should be investigated special methodology. Mathematical or other formal models, optimization problems and methods for solving these problems are very important in that methodology.

In the model a battlefield area is divided into sectors small, quadratic and nearly homogeneous in the sense of size. Our site in the local battle has finite number of military units used in strategy of attack. Each military unit should be displaced to appointed target. The route of military unit to target is described by sequence of sectors.

Military units are described by many characteristics. We can divided them into subgroup: localization, military equipment, weapon, warfare, combat material, petrol and munitions, and so one. Each sector on the battlefield is characterized by enemy defense power in that sector. From this point of view routes of military units to targets are difficult for covering. This paper is prepared to show a few formal problems that should be solved during an attack designing:

– model of battlefield terrain reducing into sectors which depends on defense abilities;

– the problem of military units allocation to selected targets;

– the analysis of probabilistic characteristics of sectors crossing by military units;

– the problem of solving local battle;

– the problem of dynamic route modification;

– the problem of start and finish point and circumstances in simulation model.

It was built computer simulator SimCombCalculator (simulation of combat, calculator – SCC). Simulator is based on primitive database that contains information about both sides of conflict and terrain. Procedures connected with methods finding strategy of attack and user interface is made in Delphi environment.

2. The model of attack against enemy

We assume that strategy of attack made by single military unit against enemy is represented by i'ts target, located in particular sector, and sequence of many small and nearly homogeneous sectors as a way to this target.

Let

$$\overline{S} = \{1, 2, 3, \ldots, s, \ldots, S\} \quad (1)$$

is set of sector indexes (numbers). Information about sectors neighborhood is defined and described by matrix

$$\mathrm{D} = |d_{ij}|_{s\times s} \quad (2)$$

where $d_{ij} = 1$ when ith sector is in neighborhood (is so called "next") of jth sector and $d_{ij} = 0$ otherwise.

If we assume that number of our military units taking part in attack is N then: a_n is index of initial (starting) sector for nth military unit ($n = 1, \ldots, N$), b_n is index of final sector for nth military unit and there is sector in which target of nth military unit attack is ($n = 1, \ldots, N$), t_n is the moment of nth military unit start to mission ($n = 1, \ldots, N$), v_{ns} is mean time passed by nth military unit to cross the distance of sth sector area, ($n = 1, \ldots, N,\ s = 1, \ldots, S$).
So that

$$a = (a_1, a_2, \ldots, a_n, \ldots, a_N) \tag{3}$$

is vector of starting sectors for our military units,

$$b = (b_1, b_2, \ldots, b_n, \ldots, b_N) \tag{4}$$

is vector of final sectors for our military units (in which there are targets for military units),

$$t = (t_1, t_2, \ldots, t_n, \ldots, t_N) \tag{5}$$

is vector of starting moments of our military units.
We decide to make discrete intervals of time during an attack performance. So, let

$$T = \{1, 2, 3, \ldots, t, \ldots, T\} \tag{6}$$

is set of essential moments considered in our model.
The schedule of an attack strategy can be described by matrix

$$\mathrm{x} = \left[x_{nst}\right]_{N\times\overline{S}\times T}, \tag{7}$$

where $x_{nst} = 1$, when nth military unit is in sth sector while tth moment and $x_{nst} = 0$ otherwise.
In general point of view the schedule of an attack strategy described by matrix have to satisfy conditions given below:

$$x_{nst} \in \{0, 1\}, \quad n = \overline{1, N}, \quad t = \overline{1, T}, \quad s = \overline{1, S}, \tag{8}$$

in one moment each military unit should take place only in one sector

$$\sum_{s=1}^{S} x_{nst} = 1, \quad t = \overline{1, T}, \quad n = \overline{1, N}, \tag{9}$$

after obtaining final sector military unit don't move in our model

$$x_{nb_nt} \le x_{nb_n(t+1)}, \quad t = \overline{1, T{-}1}, \quad n = \overline{1, N}, \tag{10}$$

before beginning the mission military unit is continuously in start sector

$$x_{na_nt} = 1, \quad t = \overline{1, t_n}, \quad n = \overline{1, N}, \tag{11}$$

military unit should cross borders only of next sectors

$$x_{ns_1t} + x_{ns_2(t+1)} - \mathrm{d}_{s_1s_2} \le 1 \tag{12}$$

for $s_1, s_2 \in \overline{S},\ t = \overline{1, T{-}1},\ n = \overline{1, N}$, military unit cross sector not longer than v_{ns}

$$\sum_{t=1}^{T} x_{nst} \le v_{ns}, \quad s = \overline{1, S}, \quad n = \overline{1, N} \tag{13}$$

military unit should cross sector continuously

$$\sum_{k=1}^{v_{ns}} x_{ns(t+k)} \ge v_{ns}\left(x_{ns(t+1)} - x_{nst}\right) \tag{14}$$

for $t = \overline{t_n, T{-}v_{ns}},\ s = \overline{1, S},\ n = \overline{1, N}$.

3. Probabilistic parameters of sectors

The probability $\zeta_s(m)$ of a single military unit down and out in the sth sector when m enemy's military units are in this sector must be determined. It is done in two steps. At the first step, the probability of single military unit down and out is considered separately. At the second step, the allocation of targets (may be enemy's units) in every sectors to our military units is considered.
Let assume, that the one type of enemy's military units are in the sector. Than the probability of down and out the single military unit is known from for example Lanchester model or each other. We can estimate the number of required enemy's units that are needed for destroying our single military unit. The rule of military unit destroying is determined in dependence on type of our military unit and enemy's unit type.
We can estimate probability $\zeta_s(m)$ even from models presented in [1] and [2] where many schemes like Bernoulli scheme, Poisson scheme are used or described on the basis of semi-regenerative process $\varphi(t) = \eta_{v(t)}\left(t - S_{N(t)}\right)$ $t \ge 0$, where $v(t)$ is semi-Markov process with state space $Y = \{0, 1, 2, \ldots, m\}$, which describes a process of number military units changing in the sector, $S_{N(t)}$ is the last moment of $v(t)$ changing and $N(t)$ is the number of the process state changing in the $[0, t)$ interval. The $\eta_i(t)$, $(i \in Y)$ processes are independent with states, denoting of military units numbers, which are ready to fight. The $\{\eta_i(t), \tau_i,\ i \in Y\}$ is a class of death processes $\eta_i(t)$ on the interval $0 \le t < \tau_i$, particularly, there are homogenous Markov chains, which describe ways of regeneration and decreasing of units military means in dependence on semi-Markov process $v(t)$ state.

4. The Battle Damage Assessment Function (BDA-F)

Another problem is to determine the allocation targets to our military units. We suppose that two sides are on the battlefield. We consider situation in which weapon of our N military units are used to destroy M units of enemy's side. It is possible to accept such simplify conditions:

- weapons which belongs to decision maker are homogeneous in the sense of its destroying and additive potential;
- argument of BDA-F function is the destroying power potential which decision maker decide to attach in order to damage a specific target;
- BDA-F function of targets is increase function;
- zero value of destroying power potential causes zero-value of BDA-F function of target;
- BDA-F function accumulate target's destruction obtained during the local combat because it is a closed process;

– above a certain value of destroying power potential attached to a target it's BDA-F function is nearly equal to it's total value;

– we know the probability of destroying effectiveness; it depends on the number of elements of side A which try to destroy side B and depends on the number of element of side B being damaged;

– for every element we know that the value of BDA-F function indicate the level of element's losses adequate to a certain value of destroying power potential attached to this element;

– we can accept an assumption that each kind of destroying power potential can be represented by real number as multiplicity of a certain standard destroying power potential.

The last assumption comes form a real military method connected with a certain standard representation for every destroying power potential.
Assumptions listed above enable us to approximate BDA-F function of target, for the side A, by real function as follows [2, 5]:

$$h_n(y) = C_n\left(1 - \mathrm{e}^{-\alpha_n p_n y_n}\right), \quad n \in \overline{1,N}, \tag{15}$$

where: y_n – destroying power potential of our side allocated to the nth target of enemy's side, C_n – value of mth target, p_n – probability of successful attack, α_n – function which represents sensitiveness of nth target of side B to unitary destroying power potential allocated to it.

5. The military units allocation to targets

Let we assume that there are M enemy's objects as targets for our military units. For enemy's looses estimation we assume that we know global destroy potential of our units. We are looking for optimal allocation vector of destroy potential of our military units to targets

$$y = \left[y_m\right]_M, \tag{16}$$

where y_m is the global destroy potential of our military units attached to mth enemy's target in order to damage it. In the first step we assume that vector y has continuous variables as coordinates.
It is easy to show that the problem of finding the strategy of fighting is as follows: if K is the global destroying potential K of our military units we are looking for such $y^* \in Y$ which meets

$$\sum_{n=1}^{N} C_n \mathrm{e}^{-\alpha_n p_n y_n^*} = \min_{y\in Y} \sum_{n=1}^{N} C_n \mathrm{e}^{-\alpha_n p_n y_n}, \tag{17}$$

where

$$Y = \left\{ y \in R^N : \sum_{n=1}^{N} y_n \le K, y_n \ge 0, \;\; n = \overline{1,N} \right\}. \tag{18}$$

Lagrange function for this problem is as follows:

$$L(y,\, u) = \sum_{n=1}^{N} C_n \mathrm{e}^{-\alpha_n p_n y_n} + u_0 \left(\sum_{n=1}^{N} y_n - K \right) - \sum_{n=1}^{N} u_n y_n. \tag{19}$$

Kuhn-Tucker differential conditions for the problem above can be presented as below:

$$\nabla_{y_n} L(y,\, u) = -\alpha_n p_n C_n \mathrm{e}^{-\alpha_n p_n y_n} + u_0 - u_n = 0, \quad n = \overline{1,N}, \tag{20}$$

$$\nabla_{u_0} L(y,u) = \sum_{n=1}^{N} y_n - K \le 0, \tag{21}$$

$$\nabla_{u_n} L(y,u) = -y_n \le 0, \quad n = \overline{1,N}, \tag{22}$$

$$\left(\nabla_u L(y,u),\, u\right) = u_0 \left(\sum_{n=1}^{N} y_n - K \right) - \sum_{n=1}^{N} u_n y_n = 0, \;\; n = \overline{1,N}. \tag{23}$$

We can show [1] that the problem can be solved by very effective method that uses the final formula (it is effect of solving Kuhn-Tucker differential conditions) [2, 5]:

$$y_n^* = \frac{-1}{\alpha_n p_n} \ln \frac{\left(\mathrm{e}^{-K} \prod_{m=1}^{N} (\alpha_m p_m C_m) \right)^g}{\alpha_n p_n C_n}, \tag{24}$$

where

$$g = \frac{1}{\sum_{m=1}^{N} \frac{1}{\alpha_m p_m}}. \tag{25}$$

If all elements of vector y are nonnegative than this vector is optimal for our problem. If not, we put all negative elements of vector y into zero and for the other elements we use the formula given above.
Now we should allocate our military units to targets that give us similar destroying potential as we calculate from solution above. So, we modify vector y to discrete vector.

This provides us additionally to obtain values of vector b coordinates.
It is worth to remark that parameter p_n we can calculate as product of values $\zeta_s(m)$ from the shortest paths from starting sectors to target sectors. The problem of finding the shortest paths we can solve with one of the well-known methods in theory of graphs.

6. The criterion function for strategy of attack

We propose objective function for attack evaluation in the following form:

$$F(x) = \sum_{n=1}^{N} w_n \sum_{s=1}^{S} \sum_{t=2}^{T} x_{nst} \frac{1}{v_{ns}} \zeta_s \left(\sum_{k=1}^{N} x_{ks(t-1)} \right), \tag{26}$$

where

$$w_n \in [0,\, 1], \quad n = \overline{1,N}, \quad \sum_{n=1}^{N} w_n = 1, \tag{27}$$

are coefficients connected with values of targets which will be destroyed by particular military unit, $\zeta_s(m) : \{1, 2, 3, \ldots, N\} \rightarrow \Re$, is the function connected with probability of a single military unit (of m military units being in sth sector) success crossing.
We are looking for such x^*, which gives us

$$F(x^*) = \max_{x \in X} F(x)\,. \tag{28}$$

7. Recurrent method for solving the problem

The recurrent algorithm for searching the best attack schedule is proposed. At first we should have the shortest routes for each target. Then we can solve the problem of military units allocation to targets. Having solution of allocation problem we are able to looking for first estimation of optimal attack schedule. We solve problem (28) with conditions (8)–(14) for decision matrix (7). It is easy to show that this problem can be solve by well known dynamic programming method, because of recurrent formula

$$x_{ns(t+1)} \frac{1}{v_{ns}} \zeta_s \left(\sum_{k=1}^{N} x_{kst} \right). \tag{29}$$

The solution obtained is input data for improving last military units allocation. After that, we can improve last attack schedule, etc. Finally, if we achieve attack schedule good enough, for global criterion, we finish our searching.

8. Simulator

Computer simulator SimCombCalculator has possibility to choose number and kind of troops. It was built special editor that help us to define every military units on battlefield. For defining concrete or only type of military troops for both sides we can use windows given in Fig. 1.

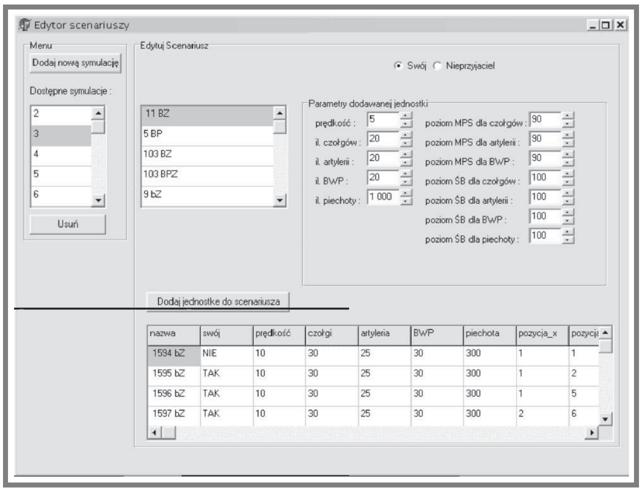

Fig. 1. Interface of SimCombCalculator scenario editor.

We can edit basic characteristics for every military units: number of unit, level of military structures, kind of unit, average speed, etc. (Fig. 2).

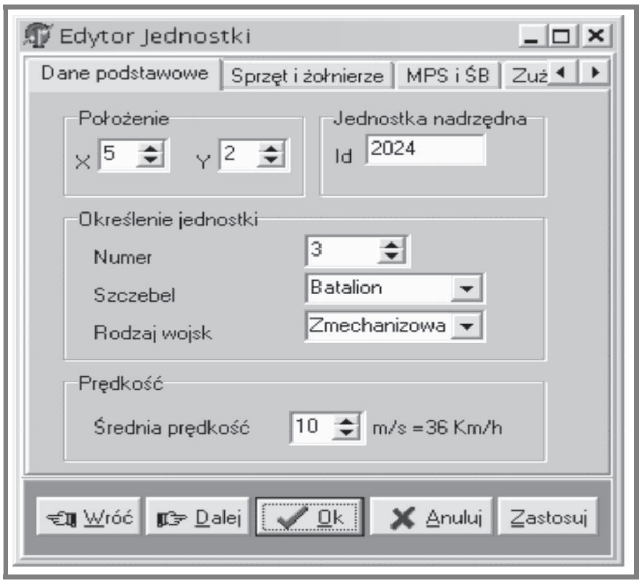

Fig. 2. Interface for setting basic characteristics of troops.

It possible to describe damage potential connected with weapons used by military units. Even level of materials can be defined for each military unit (Fig. 3).

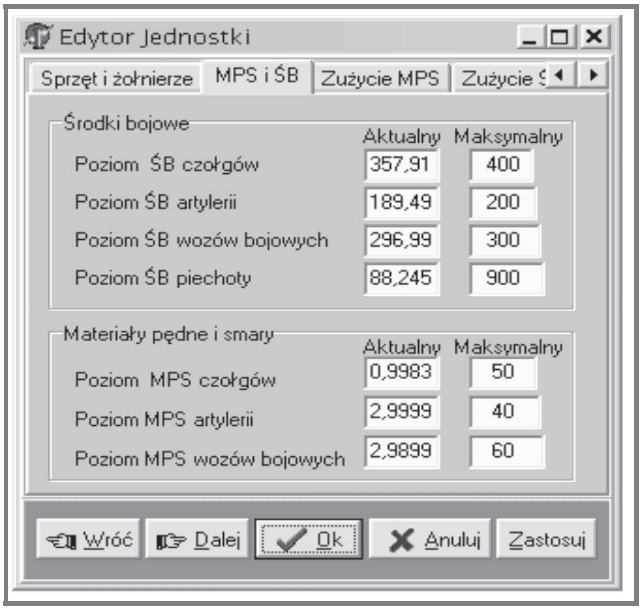

Fig. 3. Interface for any troop materials defining.

Number of tanks, artillery, infantry fighting vehicle, and people can be described (Fig. 4).
Attrition processes of petroleum oil and lubricants can be described (Fig. 5).
Attrition processes of munitions can be described as well (Fig. 6).
The palette in which troops can be located on the battlefield has following form (Fig. 7).

Palette is scalable so it is possible to make sectors more smaller (Fig. 8).
If digital map is necessary to use than simulator offer such possibility (Fig. 9).
If it is necessary basic information about troops can be displayed on windows (Fig. 10).
It is possible to put several parameters for simulation, for example: length of time quantum, frequency of steps, frequency of database recording, etc. (Fig. 11).
Results during the simulation processes are shown on the window (Fig. 12).
Many characteristics connected witch results of local battles can be displayed. We can see how many peoples, materials, and weapons has given military unit in selected moment in time (Fig. 13).

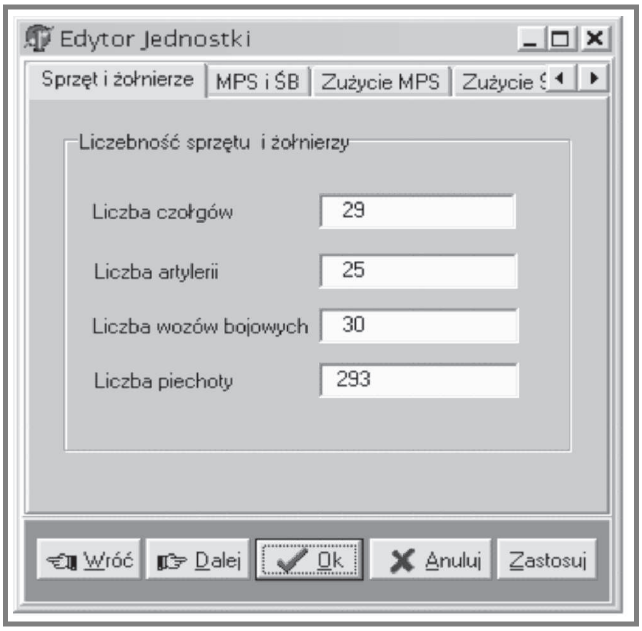

Fig. 4. Interface for any troop armament defining.

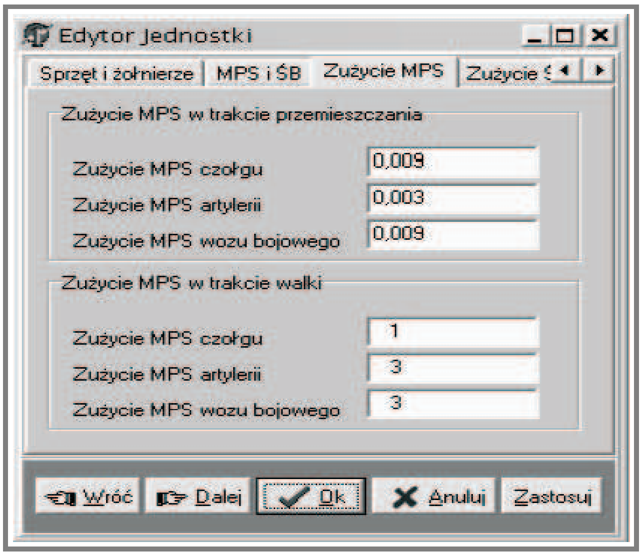

Fig. 5. Interface for main material attrition processes defining.

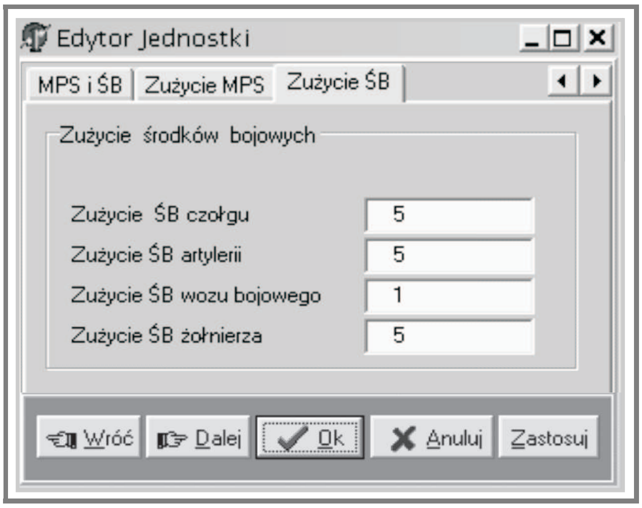

Fig. 6. Interface for people and munition attrition processes defining.

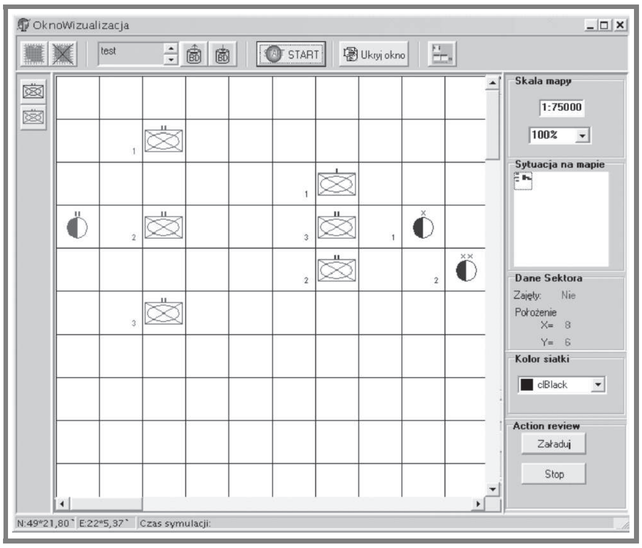

Fig. 7. Palette for locating military troops.

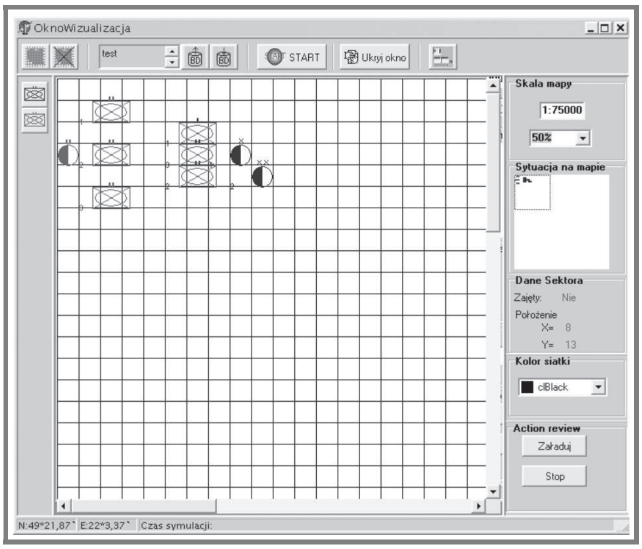

Fig. 8. Scalability of SimCombCalculator palette.

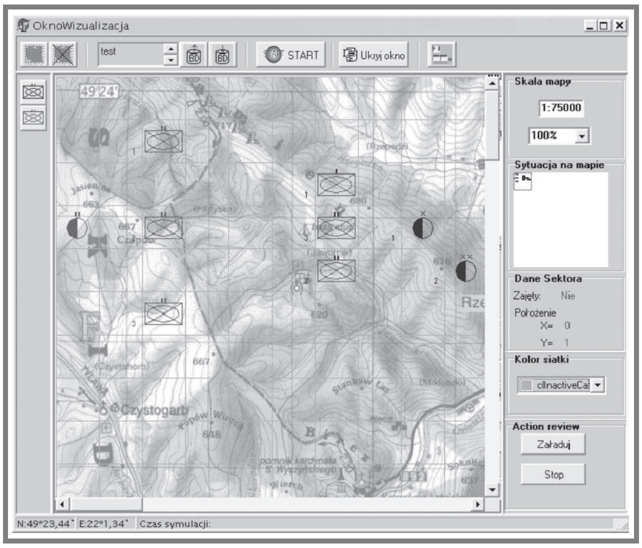

Fig. 9. Digital map as background of SimCombCalculator.

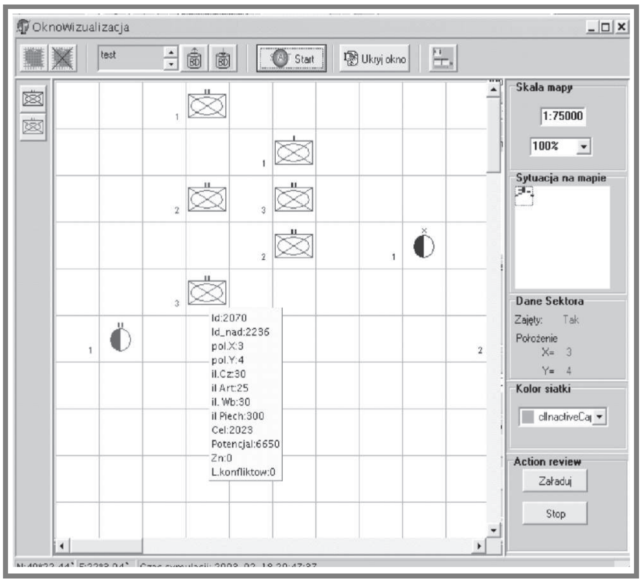

Fig. 10. Monitoring of information about individual troop.

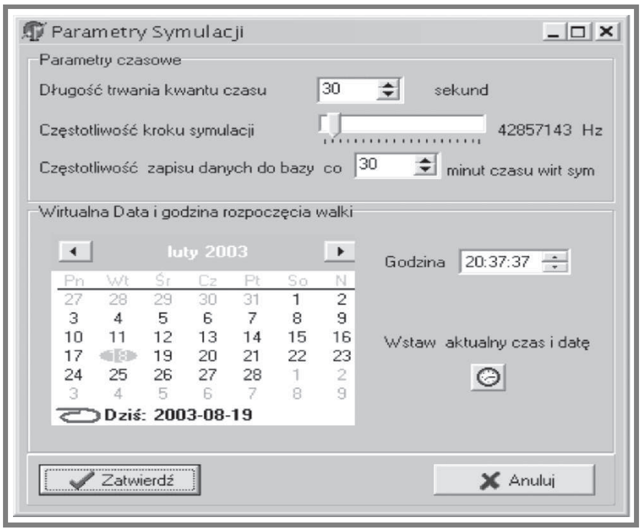

Fig. 11. Palette for simulation parameters setting.

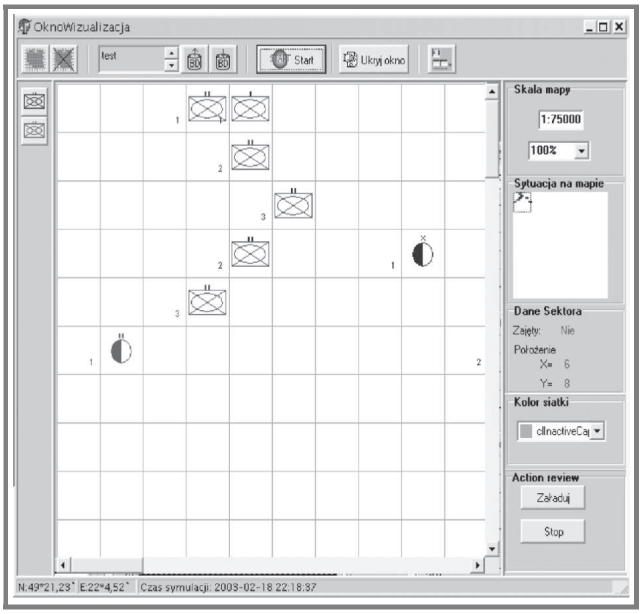

Fig. 12. The fight of troops.

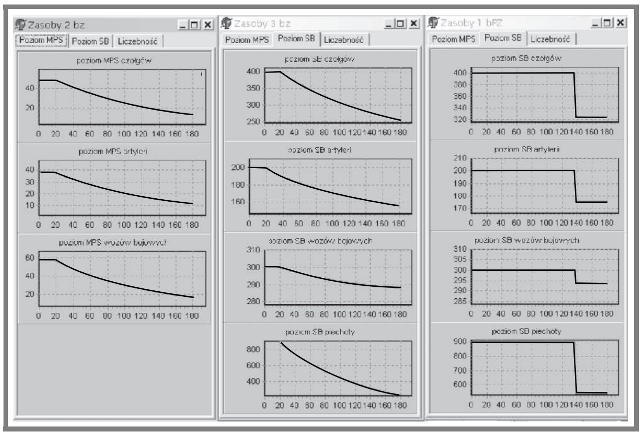

Fig. 13. Monitoring of troops material attrition.

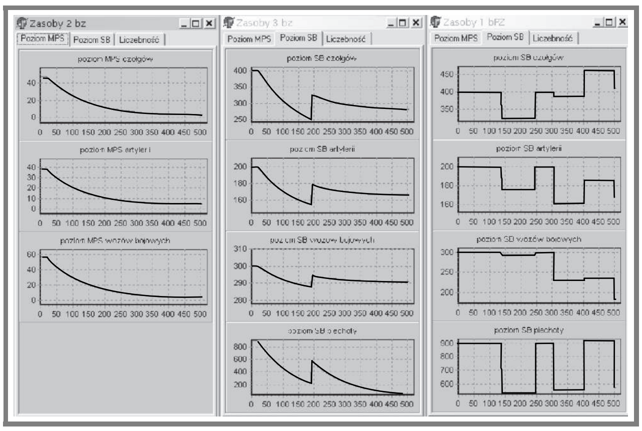

Fig. 14. Monitoring of material attrition in material bases.

Processes of resupply military units with people, materials and other things from company, battalion and brigade loading points can be shown during simulation (Fig. 14).

We can see elementary characteristics of military units during simulation (Fig. 15).

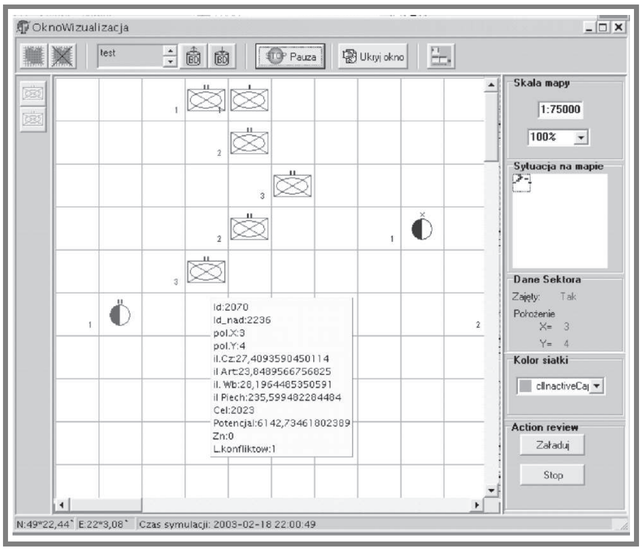

Fig. 15. Characteristics of military units during simulation.

It is possible to change every selected details during simulation.
The time to obtain the solution is good enough for it's practical use in simulation environment for battle description and analyse.

9. Conclusions

It is proven that simulation application for lower level of military processes on battlefield can be designed. Even complicated mathematical problems that are used in simulator procedures are perform in not too long time. Our application is rather prototype and professional equipment for lower level commanders should be built from the beginning. Thus the idea of building mathematical models of the battlefield and using computer environment for testing these models is important, necessary and possible at all.

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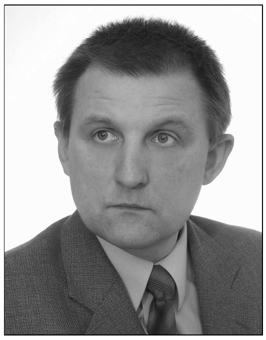

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