Paper

New approach to computer aided design of coupled resonator filters

Piotr Kozakowski and Michał Mrozowski

Abstract — A gradient based optimization technique along with a new definition of cost function is applied to synthesis of coupled resonators filters. The cost function is defined using location of zeros and poles of the filter's transfer function. The topology of the structure is enforced on each step of optimization and its physical dimensions are used as independent variables.

Keywords* — *CAD, microwave filters, optimization.

1. Introduction

Coupled resonator filters have found many applications in communication systems. Synthesis techniques of this class of filters have been known for long time [1, 2]. Unfortunately, some of those synthesis techniques do not always converge with [6] others, such as those based on equivalent circuits, and provide only approximation of filter parameters. This results, in many cases, in the necessity to apply certain optimization methods to meet electrical specifications.

Recently, a gradient based optimization method along with relatively simple definition of the cost function was used for synthesizing coupled resonators filters and excellent results have been reported in [6]. Unfortunately, this technique requires at least rough synthesis serving as a starting point for the optimization method to ensure good quality of the final solution.

This paper presents new approach to the coupled resonator filters synthesis which allows to find physical parameters of filter with a given topology without any prior synthesis. The method is partly similar to one described in [6]. It uses gradient based optimization technique along with modified cost function. The cost function is based on direct analysis of zeros and poles location of filter's transfer function. Physical dimensions of the structure are then used as variables in the optimization procedure. The superiority of the proposed approach is evident as a starting guess for a gradient optimization method can be chosen almost at random.

To illustrate application of the method three kind of filters were synthesized:

– an E-plane metal insert filter of the fourth order (Fig. 1),

– a filter based on inductive irises of the third order (Fig. 2),

– same filter as above, but with rounded corners (Fig. 3).

The first two filters were analyzed using the mode-matching technique (MM) and the third one by means of the Finite Difference-Time Domain (FD-TD) method. We have chosen a gradient technique based on the Sequential Quadratic Programming (SQP) method as optimization tool. This method is implemented in the Matlab Optimization Toolbox.

2. Cost function

The centerpiece of the new CAD procedure is a new definition of cost function which involves quantities uniquely describing the filtering character of electrical prototype. To derive this cost function let us recall that for the generalized Chebyshev approximation of the filter composed of series of N coupled resonators the transfer function $S_{21}(\omega)$ of the electrical prototype is given by

$$\left|S_{21}\right|^2 = \frac{1}{1+\varepsilon^2 F_N^2(\omega)}, \tag{1}$$

where ε is a constant related to the passband return loss and $F_N(\omega)$ is the filtering function given by

$$F_N(\omega) = \cosh\left(\sum_{n=1}^{N} \cosh^{-1}(x_n)\right), \tag{2}$$

where

$$x_n = \frac{\omega - 1/\omega_n}{1-\omega/\omega_n}, \tag{3}$$

ω_n is the position of the nth transmission zero.

If all n transmission zeros are at infinity, the filtering function becomes a pure Chebyshev polynomial defined as

$$T_N(\omega) = \cosh\left(N \cosh^{-1}(x_n)\right). \tag{4}$$

$F_N(\omega)$ and $T_N(\omega)$ are rational functions. For instance $F_N(\omega)$ can be expressed as [7]

$$F_N(\omega) = 0.5\left[\frac{\prod_{n=1}^{N}(a_n+b_n) + \prod_{n=1}^{N}(a_n-b_n)}{\prod_{n=1}^{N}\left(1-\frac{\omega}{\omega_n}\right)}\right], \tag{5}$$

where

$$a_n = \omega - 1/\omega_n, \quad b_n = \left[(\omega^2-1)\left(1-1/\omega_n^2\right)\right]^{1/2}. \tag{6}$$

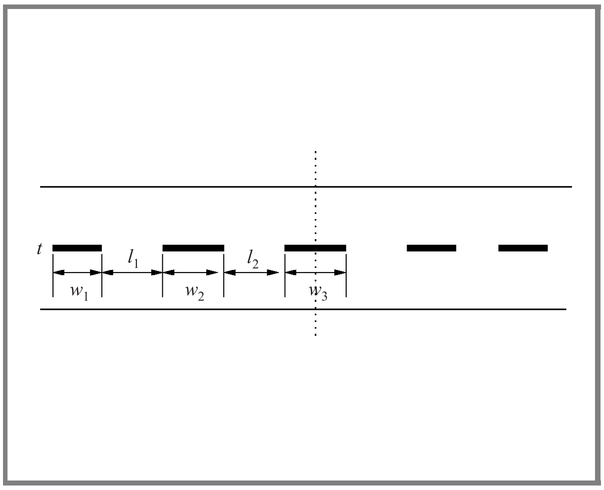

Fig. 1. E-plane metal insert filter (WR-75 waveguide, $t = 0.5$, $w_1 = 2.866$, $w_2 = 10.288$, $w_3 = 11.66$, $l_1 = 11.077$, $l_2 = 11.106$ – *all dimensions in mm*).

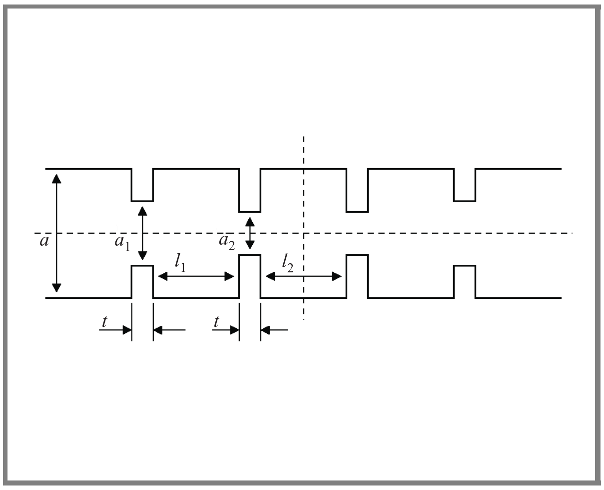

Fig. 2. Filter based on inductive irises in a rectangular waveguide (WR-75 waveguide, $a = 19.05$, $b = 9.525$, $t = 2$, $a_1 = 9.845$, $a_2 = 6.795$, $l_1 = 13.218$, $l_2 = 14.632$ – *all dimensions in mm*).

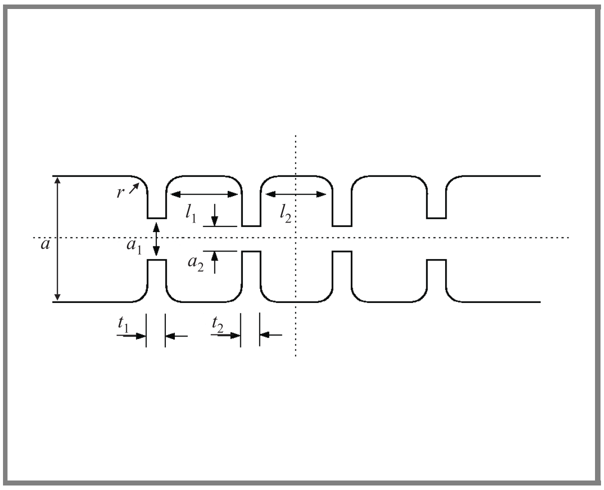

Fig. 3. Filter based on inductive irises in rectangular waveguide with rounded corners (WR-75 waveguide, $a = 19.05$, $b = 9.525$, $t_1 = 1.634$, $t_2 = 1.75$, $a_1 = 9.539$, $a_2 = 6.546$, $l_1 = 13.419$, $l_2 = 14.758$, $r = 1.5$ – *all dimensions in mm*).

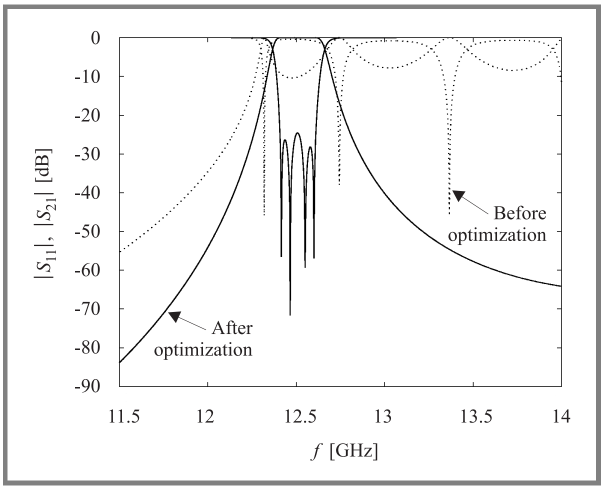

Fig. 4. Scattering parameters of E-plane metal insert filter.

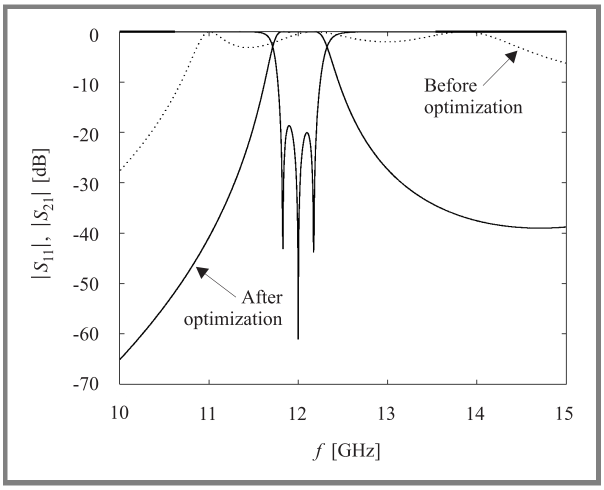

Fig. 5. Scattering parameters of filter based on inductive irises.

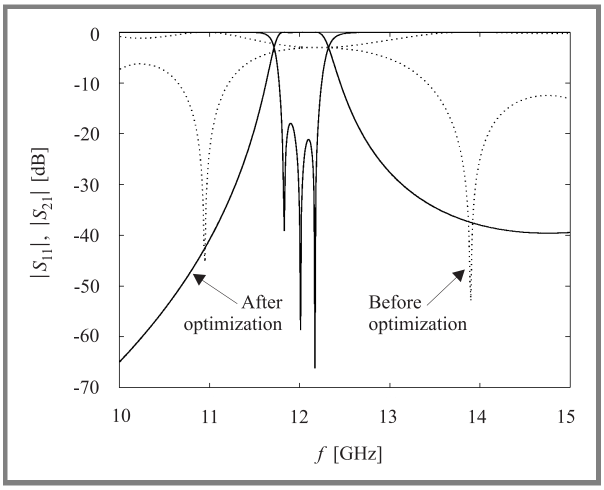

Fig. 6. Scattering parameters of inductive irises filter with rounded corners.

For the bandpass approximation the angular frequency ω is related to ω_0 and bandwidth

$$\Delta\omega \text{ by } \omega \to (\omega_0/\Delta\omega((\omega/\omega_0) - (\omega_0/\omega)).$$

Since F_N is a rational function so is S_{21} and up to the scaling factor they both are uniquely determined by the location of their poles and zeros. Calculating the poles (P_i) as roots of a denominator and zeros (Z_i) as roots of a numerator of the transfer function gives the cost function defined as follows:

$$C = \sum_{i=1}^{N} \left|Z'(i) - Z(i)\right|^2 + \sum_{i=1}^{N} \left|P'(i) - P(i)\right|^2, \qquad (7)$$

where (Z_i') and (P_i') are zeros and poles of a filter being optimized. The (Z_i') and (P_i') are calculated for every structure created by optimization procedure. For pure Chebyshev filters and other all-pole filters the cost function degenerates to the following form

$$C = \sum_{i=1}^{N} \left|P'(i) - P(i)\right|^2. \qquad (8)$$

To extract poles and zeros from the frequency response of the optimized filter the two procedures are possible depending on whether the time domain or the frequency domain software is used to calculate the electromagnetic response of the filter.

3. Pole extraction from time domain data for all-pole filters

Let us first deal with all-pole filters and the time domain techniques such as the Finite Difference-Time Domain method. Time domain techniques are in general ill-suited to repetitive analysis of filters required in optimization. This is because the simulation time for high-Q circuits can be too long. However, the analysis time can be substantially shortened and additionally for all-pole filters the poles required for forming the objective function can be extracted directly from time domain output transients without any need to compute frequency domain characteristics.

To this end we use the matrix pencil approach (MP) [4, 5]. In general, the MP technique builds a model of discrete time signal in the form of a superposition of $P/2$ damped sinusoids, where P is a model order. This can be written as

$$y(n\Delta t) \approx \sum_{i=1}^{P} A_i z_i^n, \qquad (9)$$

where $z_i = e^{p_i'\Delta t}$, Δt is the sampling time, n is the sample number, A_i is pole residue and p_i' is the location of pole in the complex plane. Hua and Sarkar [5] showed that poles of the model can be estimated by considering a matrix pencil

$$\underline{\underline{Y_1}} - \lambda \underline{\underline{Y_2}} \qquad (10)$$

with the following data matrices

$$\underline{\underline{Y_1}} = \begin{pmatrix} x(1) & x(2) & \cdots & x(L) \\ x(2) & x(3) & \cdots & x(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ x(K-L) & x(K-L+1) & \cdots & x(K-1) \end{pmatrix} \qquad (11)$$

$$\underline{\underline{Y_2}} = \begin{pmatrix} x(0) & x(1) & \cdots & x(L-1) \\ x(1) & x(2) & \cdots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(K-L-1) & x(K-L) & \cdots & x(K-2) \end{pmatrix}, \qquad (12)$$

where K is the number of samples and L is the pencil parameter. The rank of matrix $\underline{\underline{Y_1}} - \lambda \underline{\underline{Y_2}}$ is P except when $\lambda = z_i = e^{p_i'\Delta T}$. Hence, the poles p_i' can be determined by finding the rank reducing values λ of matrix pencil. Technically, this is amounts to solving an ordinary eigenvalue problem

$$(\underline{\underline{Y_1}}^{\dagger} \underline{\underline{Y_2}} - \lambda \underline{I}) \underline{a} = 0, \qquad (13)$$

where $\underline{a}$ is the generalized eigenvector of (10) and $\underline{\underline{Y_1}}^{\dagger}$ is the Moore-Penrose pseudoinverse of $\underline{\underline{Y}}_1$, which can be found using the singular value decomposition.

Keeping in mind that the method is applied to filter design, selection of model order P is straightforward, namely the number of damped sinusoids should be twice as large as the number of filter cavities (poles appear in complex conjugate pairs) or, in other words it should be set to the number of poles in the filter prototype.

So the location of poles on the complex plane is determined for each trial filter by computing the eigenvalues of matrix $\underline{\underline{Y_1}}^{\dagger} \underline{\underline{Y_2}}$. These eigenvalues are used to evaluate the cost function later. Once the poles are known, one might proceed to find the amplitudes of each damped sinusoid. This would be required for obtaining the frequency domain characteristics. However, this step may be skipped in case of all-pole filter optimization as the amplitudes are not used in the definition of cost function.

Matrices $\underline{\underline{Y_1}}$ and $\underline{\underline{Y_2}}$ are assembled based on desampled FD-TD sequences of field recorded at the filter output. Selection of time segment to be used for extracting poles is based on the technique accounting for signal dynamics described in detail in [3]. In general, the recorded samples are divided into early and late time response. The first part is discarded. The other part, determining the late time behavior of output transients provides the samples for the MP method. The criterion for signal separation on early and late time responses is based on investigation of normalized average energy passing though output port of the filter.

Application of the MP technique for filter optimization, as proposed in this paper, requires an additional step whose incorporation guarantees correct extraction of poles. Before the data matrices $\underline{\underline{Y_1}}$ and $\underline{\underline{Y_2}}$ are assembled, the time-domain signal is passed through a cascade of digital filters.

The bandwidth of a single component of the cascade should be somehow greater than the bandwidth of filter being designed. In general, the cascade should consists of low-order Chebyshev or Butterworth filters whose poles shall not coincide with the poles of filter being the optimization target. This technique is called the Band-Pass Matrix Pencil (BPMP) method [5].

4. Extraction of zeros and poles from frequency domain characteristics

When the filtering function has zeros that affect the stop band behavior of a filter, the approach outlined above can not be used. Also, while theoretically possible, it does not make any practical sense to use BPMP operating on time domain data to optimize all-pole filter when the frequency characteristics are known (e.g. computed by the mode-matching method). For these cases we propose to use the Cauchy method [8]. The Cauchy method is an interpolation technique which assumes that the approximated function is expressed as ratio of two polynomials. Coefficients of these polynomials are found by applying the total least squares technique to solve a set of complex linear equations involving the values of function to be interpolated at few sampling points. To set up the equations one uses the filter's frequency characteristics obtained using the full wave method. The Cauchy method yields then the polynomials in numerator and denominator whose roots are zeros Z_i's and poles P_i's which are subsequently used to form the objective function. Note, that the order of the polynomials in numerator and denominator are known from filter specifications. This implies that provided the structure topology permits it, the Cauchy method combined with optimization procedure should find the filter dimensions which guarantee that transfer function of the optimal geometry reproduces as closely a possible, the distribution of zeros and poles of ideal electrical prototype. Obviously, the Cauchy method should be used also in conjunction with time domain analysis for filters whose transfer function contains zeros.

5. Results

The methods outlined above were verified by synthesizing the three previously mentioned filters. In all cases a starting guess for the optimization was chosen at random within the range of possible dimensions for a given topology. The first two filters were analyzed using mode matching techniques and the poles were found by means of Cauchy method. For the filter with the rounded corners, FD-TD QuickWave software was used and the poles extracted directly from transients with the MP technique described above. Regardless of the numerical tool used and the poles extraction technique applied the optimization converged to solution meeting electrical specifications (Figs. 4–6). The number of cost functions evaluations during the gradient based optimization (SQP method) ranged from 420 to 580 depending on number of independent variables. Finally, it has to be pointed out that while this paper presents only all-pole designs, pseudo-elliptic filters with finite transmission zeros have also been designed. The results are presented in [9].

6. Conclusions

Using a gradient based optimization technique along with a new definition of cost function it is now possible to design resonator filters based on given topology without any prior synthesis. Due to the judicious choice of goal function and robust post-processing techniques used to extract zeros on poles of the filter response, the new algorithm appears to converge globally from an arbitrarily selected starting dimensions.

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