

# Precise measurement of complex permittivity of materials for telecommunications devices

Takayuki Nakamura and Yoshio Nikawa

**Abstract** — In order to obtain precise complex permittivity of the dielectric materials obtained from the perturbation method a correction curve is made using the electromagnetic field simulator which applies transmission line modeling (TLM) method. In this experiment, generated microwave power with the frequency of 2.45 GHz is applied to heat dielectric material while measuring temperature dependence of complex permittivity of dielectric material. To obtain these objectives cavity resonator with cooling system is designed. It is found from the result that the accurate temperature dependence of complex permittivity of the materials can be obtained by the method presented here.

**Keywords** — perturbation method, TLM method, cavity resonator, simulation model, temperature dependence of complex permittivity, microwave measurement.

## 1. Introduction

It is known that the complex permittivity of materials usually changes depending on the frequency, on the temperature and on the compositions. Therefore, in designing or developing microwave devices, it is very important to study the temperature dependence of complex permittivity of materials over the wide temperature range. One of the usual techniques to obtain complex permittivity of materials is the perturbation method using the cavity resonator [1]. When the perturbation method is applied, it is necessary to satisfy two conditions. One of them is to use a small dielectric material compared with the volume of the cavity resonator, and the other one is that EM field distribution is not changed after inserting the dielectric materials into the cavity resonator. If dielectric material is fragile, it is very difficult to prepare a thin sample. Thus, EM field distribution is usually disturbed by the insertion of dielectric materials into the cavity. Therefore, calculation error by the perturbation method is accrued according to the change of the EM field distribution changes, because the conditions of the perturbation method are not satisfied.

In this paper, correction curve is made by means of the EM field simulator, which applies the transmission line modeling method to reduce the error of complex permittivity of dielectric material obtained from the perturbation method. To measure the temperature dependence of complex permittivity of dielectric material and to heat the dielectric material simultaneously, microwave power with the frequency of 2.45 GHz is applied in this experiment, with

use of a network analyzer and amplifier. To improve accuracy of measurements, only the dielectric materials have to be heated. Therefore, a rectangular cavity resonator with cavity cooling system was designed.

The theory and applications of TLM for EM field simulation are reviewed by Johns and Hofer [3, 4]. The main advantage of the TLM method is to eliminate solving simultaneous numerous equations all over the structure. Therefore, the method consumes less computer memory and requires lower simulation time compared with the other simulators.

## 2. Theory

### 2.1. Perturbation theory

An example of cavity resonator is shown in Fig. 1. The dimensions of the cavity resonator in  $x$ ,  $y$  and  $z$  directions are defined as  $a$ ,  $b$ , and  $L$ , respectively.

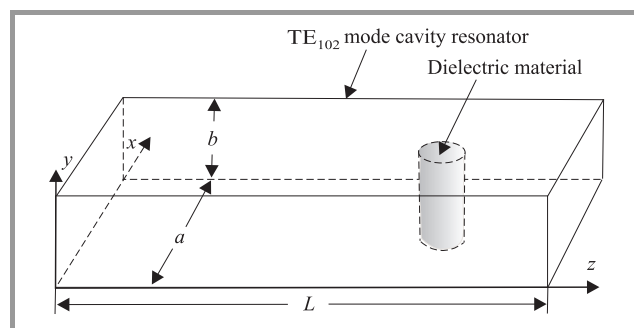


Fig. 1. Coordinates of cavity resonator.

If there is an electrical source with  $J_e = j\omega\epsilon_0(\epsilon_r' - 1) \cdot E$ , associated with a dielectric material inside the cavity, Maxwell's equations can be written as follows:

$$\nabla \times H = j\omega\epsilon_0 E + J_e, \quad (1)$$

$$\nabla \times E = j\omega\mu_0 H, \quad (2)$$

where  $\omega$  is the angular frequency,  $\epsilon_0$  and  $\mu_0$  are permittivity and permeability of the free space, respectively, while  $E$  and  $H$  are the electric and the magnetic field, respectively.

If there is no dielectric material inside the cavity resonator,  $J_e$  equals to zero. For such a case the resonant angular frequency is defined as  $\omega_0$ .

From Eqs. (1) and (2), Eq. (3) can be obtained as,

$$\begin{aligned} \int_V (E_0^* \cdot \nabla \times H + E \cdot \nabla \times H_0^* - H_0^* \cdot \nabla \times E + H \cdot \nabla \times E_0^*) dV = \\ = j(\omega - \omega_0) \int_V (\epsilon_0 E_0^* \cdot E_0 + \mu_0 H_0^* \cdot H_0) dV + \int_{\Delta V} J_e E_0^* dV, \end{aligned} \quad (3)$$

where  $V$  and  $\Delta V$  are volume of the cavity and volume of the material.  $E_0^*$  and  $H_0^*$  are complex conjugates of electric and magnetic field intensities,  $\omega$  and  $\omega_0$  are the angular frequencies before and after inserting dielectric materials into the cavity. From Gauss's theorem, we have  $n \times E^* = n \times E = 0$ . Hence Eq. (3) can be rewritten as follows:

$$\frac{\omega - \omega_0}{\omega_0} = j \frac{\int_{\Delta V} J_e E_0^* dV}{\omega_0 \int_V (\epsilon_0 E_0^* \cdot E + \mu_0 H_0^* \cdot H) dV}. \quad (4)$$

The assumption to use the perturbation theory is that the EM field distribution will not be changed after inserting dielectric material into the cavity resonator. Under this assumption,  $E$  equals to  $E_0$  and  $H$  equals to  $H_0$ . Thus Eq. (4) can be represented as follows:

$$\frac{\omega - \omega_0}{\omega_0} = - \frac{\int_{\Delta V} \{\epsilon_0 (\epsilon_r' - 1) |E_0|^2\} dV}{\int_V \{\epsilon_0 |E_0|^2 + \mu_0 |H_0|^2\} dV}, \quad (5)$$

where  $\epsilon_r'$  is the unknown relative complex permittivity to be obtained.

In this paper, we apply  $TE_{102}$  mode cavity. Therefore, only  $E_y$ ,  $H_x$  and  $H_y$  components of the EM field can have non-zero values in the empty cavity. They are described as follows:

$$\begin{aligned} E_y &= -C \frac{\omega \mu_0 k_x}{k_c^2} \sin(k_x x) \cos(k_y y) \sin(k_z z), \\ H_x &= C \frac{j \beta_g k_x}{k_c^2} \sin(k_x x) \cos(k_y y) \cos(k_z z), \\ H_y &= -jC \cos(k_x x) \cos(k_y y) \sin(k_z z), \end{aligned} \quad (6)$$

where  $k_x$ ,  $k_y$  and  $k_z$ , are  $m\pi/a$ ,  $n\pi/b$  and  $p\pi/L$ , respectively;  $\beta_g$  is  $p\pi/L$ ,  $a$ ,  $b$  and  $L$  are length of cavity resonator, and  $m$ ,  $n$  and  $p$  are the mode indices in the cavity.

The complex resonant angular frequency of the cavity resonator  $\omega$  is defined as follows:

$$\begin{aligned} \omega &= \omega_r + j\omega_i, \\ \frac{\omega_i}{\omega_r} &= \frac{1}{2 \cdot Q_L}, \end{aligned} \quad (7)$$

where subscripts  $r$  and  $i$  stand for the real and the imaginary parts.  $Q_L$  stands for the loaded  $Q$ .

If complex resonant angular frequency is changed by insertion of the dielectric material into the cavity resonator, the left side part of Eq. (5) changes, resulting in:

$$\frac{\omega - \omega_0}{\omega_0} \approx \frac{\omega_r - \omega_{r0}}{\omega_{r0}} + \frac{j}{2} \left( \frac{1}{Q_L} - \frac{1}{Q_{L0}} \right). \quad (8)$$

From Eqs. (5) and (8), the complex permittivity can be obtained.

## 2.2. Simulation of resonant frequency and $Q$ factor

With the perturbation method, it is assumed that the EM field distribution will not be changed after inserting the dielectric materials to the cavity. Therefore, the  $E_0$  equals to  $E_0^*$  and the  $H_0$  equals to  $H_0^*$  as adopted in Eq. (5). However, in the actual experiments, the EM field distribution is disturbed with insertion of the materials. As a result, the perturbation method always includes some error.

To simulate precise EM field and to obtain precise complex permittivity, a TLM simulator is applied to reduce measurement error as shown in Fig. 2.

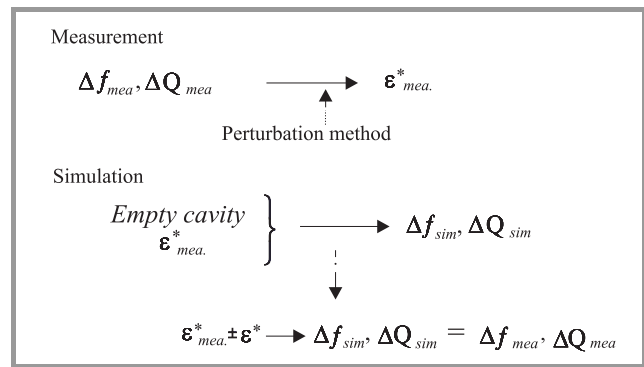


Fig. 2. Simulation method to reduce measurement error.

In Fig. 2,  $\Delta f_{mea}$ ,  $\Delta Q_{mea}$  and  $\Delta f_{sim}$ ,  $\Delta Q_{sim}$  represent the quantitative changes in resonant frequency and  $Q$  before and after inserting dielectric materials into the cavity both on the measurement and on the simulation,  $\epsilon_{mea}^*$  and  $\epsilon_{sim}^*$  are defined as complex permittivity obtained from the perturbation and simulation method, respectively.

At first, complex permittivity of the dielectric material is calculated by the perturbation method using measured resonant frequency and  $Q$  factor before and after inserting a dielectric material into the cavity resonator. Secondly, the empty rectangular cavity resonator was designed using TLM simulator (Micro-Strips, Fromerics). Then resonant frequency and  $Q$  are simulated using empty cavity resonator on the TLM method. Thirdly, the model of the rectangular cavity resonator with an inserted dielectric material is designed in the simulator. Then, resonant frequency and  $Q$  are

simulated by inputting characteristics of various complex permittivities of dielectric materials. After that, correction curve was made from calculated quantitative changes of resonant frequency and  $Q$  with and without placing the dielectric materials in the cavity resonator.

From these results, correction curves of the real and imaginary part of  $\epsilon^*$  are obtained which are shown in Figs. 3 and 4, respectively.

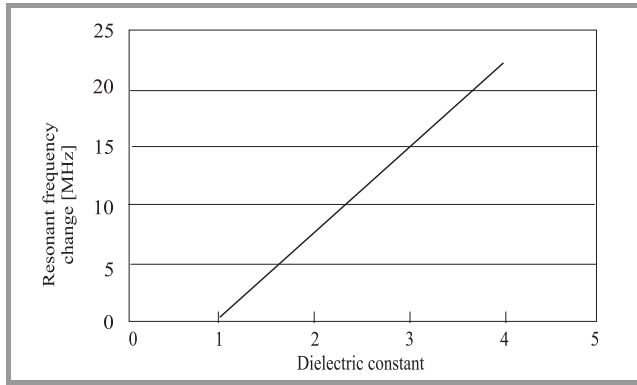


Fig. 3. Correction curve of real part.

Fourth, resonant frequency and  $Q$  are simulated with the adoption of the TLM simulator using the results of the complex permittivity on the perturbation method.

The calculated quantitative changes in resonant frequency and the  $Q$  for the TLM method are comparable with the result of quantitative changes in resonant frequency and  $Q$  for the perturbation method using correction curve. If these results are not fitting the correction curve, another value of complex permittivity will be input and the procedure will be repeated.

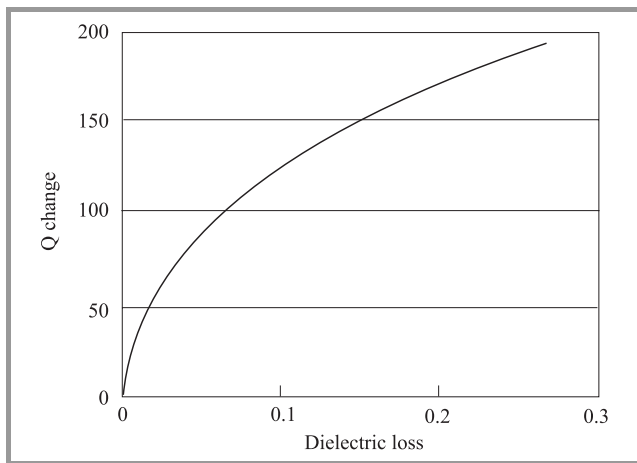


Fig. 4. Correction curve of imaginary part.

In the case, when the quantitative changes of resonant frequency and  $Q$  fits between the simulation and correction curves, the input parameter of complex permittivity is assumed to be a precise value.

### 3. Cavity resonator and measuring system

#### 3.1. Experimental system

The  $TE_{102}$  mode rectangular cavity resonator is shown in Fig. 5. In this experiment, the length of the cavity is 147.2 mm to generate  $TE_{102}$  mode. The cross-sectional size is  $110.0 \times 27.0 \text{ mm}^2$ . The coupling window and coupling disk are set at the optimum coupling position.

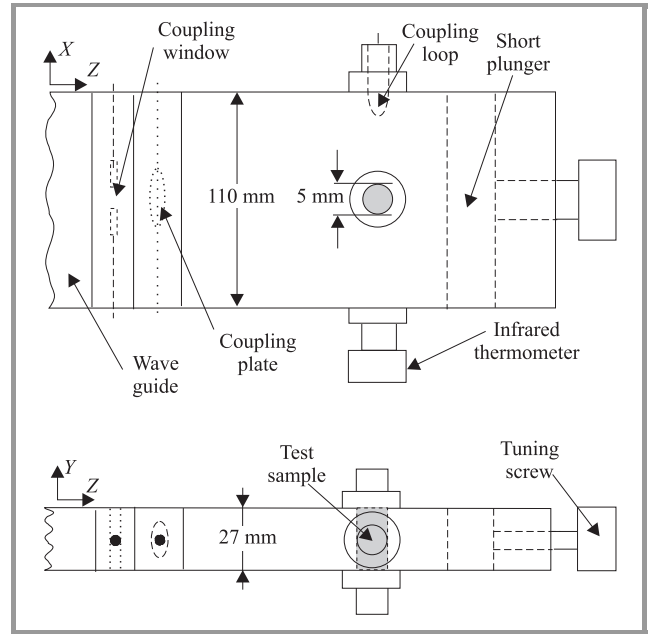


Fig. 5.  $TE_{102}$  mode rectangular cavity resonator.

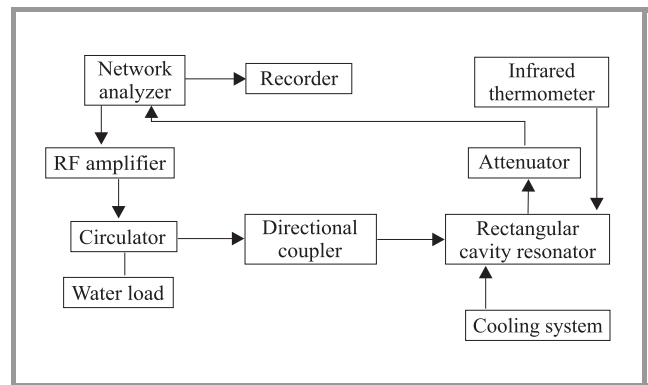


Fig. 6. Measuring system.

An infrared thermometer is used to measure surface temperature of the dielectric material from outside the cavity resonator through the small window on the cavity. The tuning screw can move the position of the short plunger to set initial resonant frequency [5, 6]. The dielectric material is inserted at the maximum EM field from the coupling window at  $3/4 \lambda$  wavelength. Coupling loop to measure transmission power is inserted at the position of maximum magnetic field in the cavity.

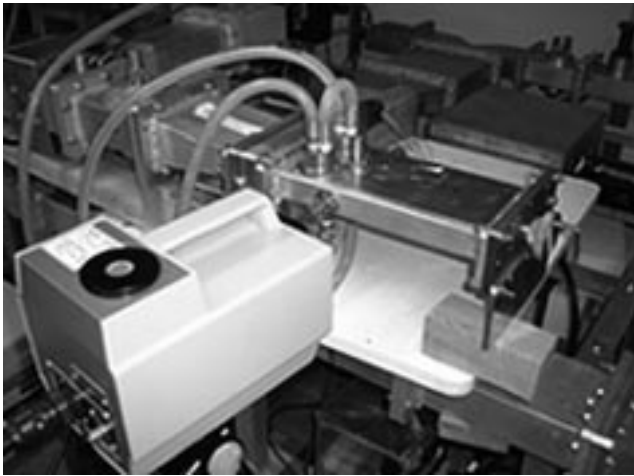


Fig. 7. Photograph of the measuring system.

The measuring system is shown in Figs. 6 and 7. Microwave power from the network analyzer (HP8753C) is amplified by means of a wideband high power amplifier (model A2325-5050-R R&K CO., LTD). An amplified microwave power is transmitted to the circulator, directional coupler and rectangular cavity resonator. The inserted coupling loop into the cavity resonator connects the network analyzer for measurement system transmitting power. A 30 dB attenuator, which can attenuate the equivalent power of amplifier gain, is set between rectangular cavity resonator and the network analyzer for protecting measuring circuit. The characteristics of resonant frequency and  $Q$  are measured by the network analyzer.

In the experiment, the center frequency of the signal fed to the cavity resonator was set at 2.45 GHz. Averaged duration time to tune of the resonant frequency was changed under the control of the sweeping span of the network analyzer. The temperature of dielectric material was measured by the infrared thermometer. In the experiment, little thick material was used intentionally for the evaluation of correction errors.

### 3.2. Simulation model of cavity resonator

A simulation model of cavity resonator is shown in Fig. 8. In the TLM method, the shape of cavity resonator is defined by metals, excitation source, the coupling loop, the dielectric material and the impedance wall. Excitation source and impedance wall are set at the waveguide side. Microwave power of  $TE_{10}$  mode is applied as excitation source. The impedance wall is used as the absorption wall and it has a defined optimum impedance. The waveguide and cavity resonator are separated by the coupling window. The dielectric material is set at maximum electric field at the  $3/4$  wavelengths from the coupling window. Measuring ports are set for calculation of the transmission property. Cavity was divided into an optimum number of cells. Es-

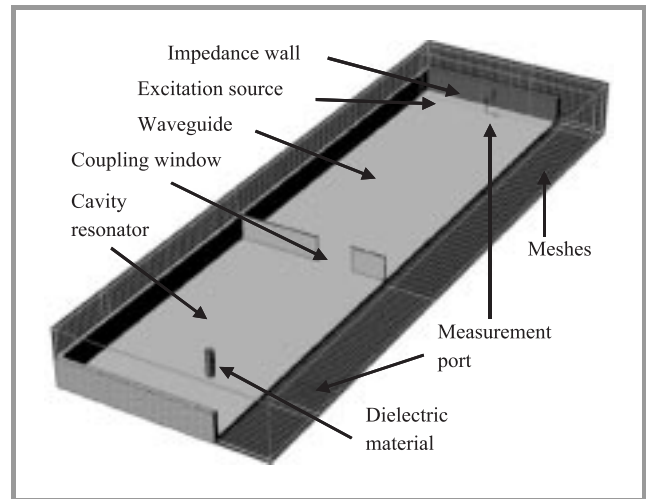


Fig. 8. Simulation model of cavity resonator.

pecially, it was noted that around coupling window the dielectric materials and coupling loop are divided into smaller cells for increasing accuracy of the simulation.

## 4. Results

Real part of the complex permittivity versus temperature of polyethylene-terephthalate (PET) is shown in Fig. 9. It is clear that the results obtained using the TLM simulator are a little smaller than the reference measurement data. Nevertheless, it is apparent that the error is smaller than that using the perturbation method.

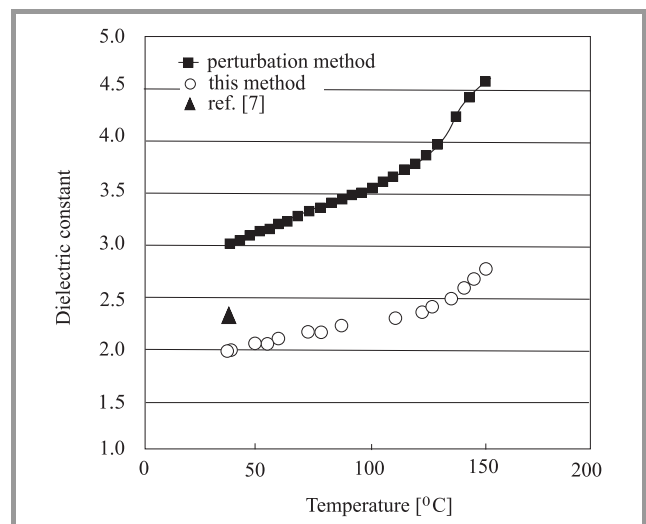


Fig. 9. Dielectric constant versus temperature of PET.

The results obtained from the perturbation method of dielectric loss versus temperature of PET are shown in Fig. 10 and the results obtained from TLM simulation and reference data are shown in Fig. 11. It is remarkable that the data obtained from TLM simulator almost fit to the reference data.

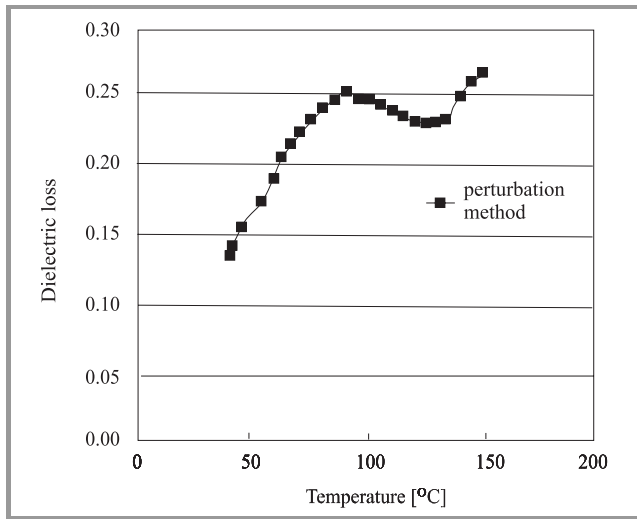


Fig. 10. Dielectric loss versus temperature of PET (perturbation method only).

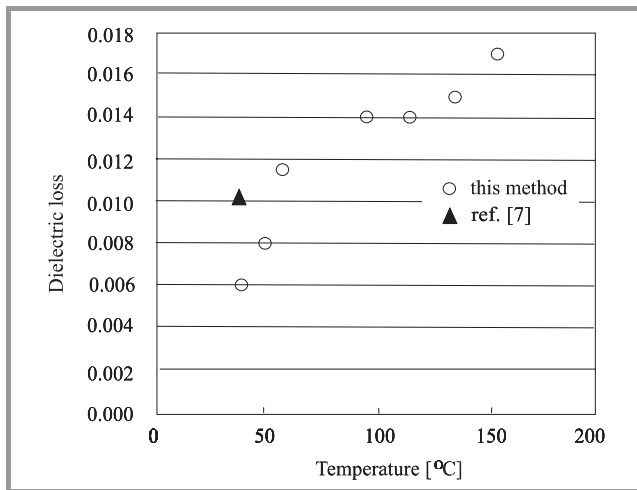


Fig. 11. Dielectric loss versus temperature of PET obtained by proposed method.

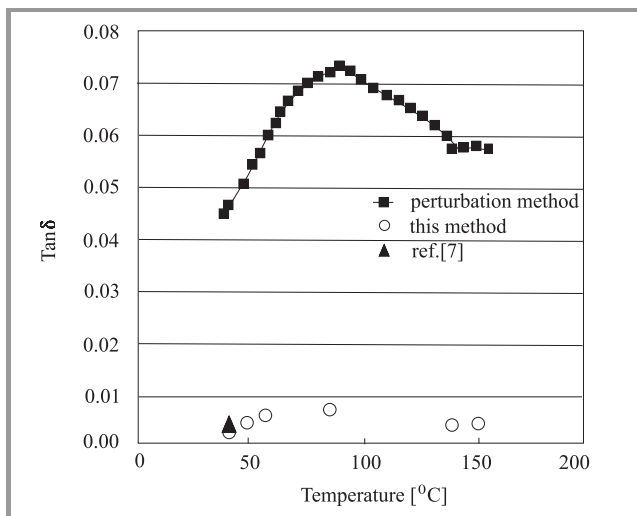


Fig. 12.  $\text{Tan}\delta$  versus temperature of PET.

The loss tangent versus temperature of PET is shown in Fig. 12. In the figure, the simulation data is similar to the reference one obtained at the room temperature. It is shown from these results that the TLM simulator can be used for the precise estimation of temperature dependence of complex permittivity.

### 5. Conclusions

The perturbation method is assumed to define that dielectric material is small enough to compare to the volume of the cavity resulting in EM field distribution not being changed after inserting the dielectric material into the cavity. But, for example, if we cannot use small dielectric material compared with the volume of the cavity because of the fragility of the material, the EM field distribution is changed by the insertion of dielectric material. Therefore, when perturbation method is applied, the correction error occurs because of the approximate calculation.

In this paper, the complex permittivity has been calculated using the TLM simulator. The designed correction curve obtained using the TLM simulator can reduce the calculation errors in complex permittivity of materials for the perturbation method. From these results, when TLM simulation and measurement with perturbation method are used together, the calculated data are expected to be similar to reference data.

In presented experiment, to measure temperature dependence of complex permittivity of the dielectric material, a microwave power with the frequency of 2.45 GHz was applied as measuring and heating power using a network analyzer and an amplifier. The averaged duration time to tune of the resonant frequency was changed by controlling of the sweeping span of the network analyzer. Thus stable temperature dependence of materials was measured. From these results, it is visible that the TLM method can be used to correct calculation error for the perturbation method.

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